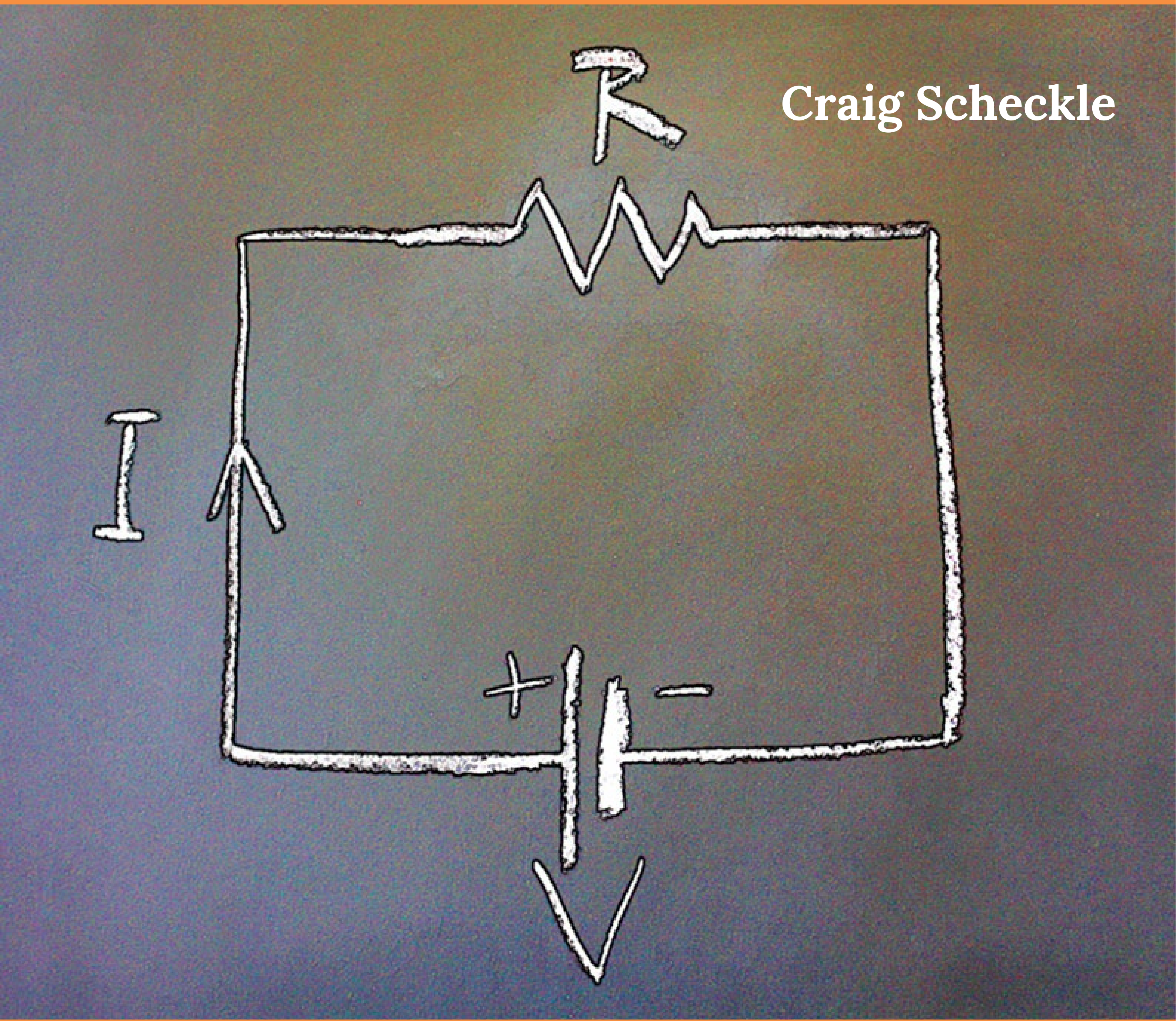
Craig Scheckle



Three Phase Electrical Circuit Analysis

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CRAIG SCHECKLE

THREE PHASE ELECTRICAL CIRCUIT ANALYSIS

Three Phase Electrical Circuit Analysis
1st edition

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CONTENTS

PREFACE

The purpose of this book is to introduce the concepts of three phase electrical circuit analysis to undergraduate engineering students, probably in their first or second year of tertiary education. Its purpose is to supplement the prescribed text book. Some pre-requisite knowledge of electrical circuits is required, and is reviewed briefly in Chapters 1 and 2, which cover the generation of AC sources and the analysis of single phase AC circuits. Those students who are familiar with these concepts can proceed to Chapter 3, but it is advised that students should read these first two chapters to become familiar with the notation used in the rest of this book.

The subsequent chapters of the book cover the analysis of three phase circuits connected in both Delta and Star configuration. (These connections are sometimes referred to as ∏ and Wye connections in other texts). Chapter 3 covers the analysis of balanced Star connected circuits, with and without a neutral connection. Chapter 4 examines balanced Delta connected circuits. Chapter 5 shows how unbalanced loads can be analysed in Delta and Star configuration, with the final section explaining how an unbalanced Star connected circuit without a neutral connection can be analysed without the use of advanced mathematics.

The author, Craig Scheckle, graduated from Wits University in 1981 with a degree in Electrical Engineering (BSc (Eng) Electrical). He did his training at the South African Transport Services and in 1987 he joined Sappi Fine Papers as the Electrical Engineer at the Stanger Mill. In 1987 he registered as a Professional Engineer, in 1988 he attained the Government Certificate of Competency in Electrical Engineering and in 1989 he completed the Management Development Program through UNISA. In 1990 he was promoted to Engineering Manager at Sappi Fine Papers, Adamas Mill in Port Elizabeth. In 1991 he moved to Delta Motor Corporation as a Plant Engineer and in 1997 was appointed as a Maintenance Manager at Volkswagen. In 2005 he left industry to join the Nelson Mandela Metropolitan University as a lecturer in the Mechatronics Department. He retired at the end of 2016.

1 AC GENERATION

1.1 OBJECTIVE:

The aim of this chapter is to explain how AC emfs (electromotive-forces) are generated, to explain why the AC waveform is sinusoidal, and how to determine the correct average value of the sine wave to be able to analyse the circuit. The generation of three phase sources is also explained, and shows how they can be connected.

1.2 SINGLE PHASE AC GENERATION:

When electricity was first discovered to be a useful source of energy, DC cells were produced based on an electro-chemical principle. These energy sources were very effective, but the energy source only lasted as long as the chemical reaction did, and then had to be replenished or replaced. (This is still applicable to modern electro-chemical DC sources). A more sustainable source of electrical energy was needed.

In the early 19th century Michael Faraday discovered that if an electrical conductor is moved through a magnetic field, an electro-motive force (emf) is produced. (Hughes 2002, p. 193) This is illustrated in Figure 1.1

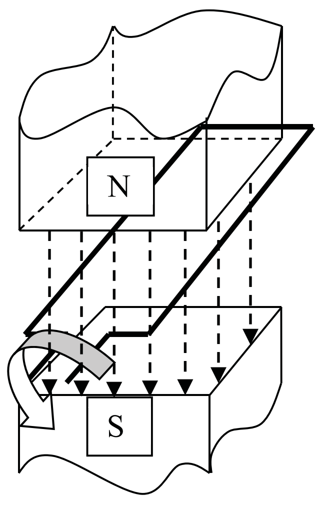

Figure 1.1

The diagram shows that a conductor formed into the shape of a loop (known as a coil or winding) is rotating inside a static magnetic field. It is illustrated this way because most AC sources are generated like this because it is the most convenient way of moving the coil through a magnetic field. A source of energy is needed to rotate the coil, and this source could be in the form of a diesel internal combustion engine or wind in the case of small generators, but usually a steam turbine is used in the case of large generators. The energy sources for generating the steam could be coal, diesel or other hydro-carbon fuels, or also the more modern nuclear energy sources. The basic principle is that a fuel is burnt to boil water (to generate steam), which is then used to drive a turbine, which in turn rotates the electrical coils inside a magnetic field. The means by which the coils are rotated is beyond the scope of this book. For now assume that the coils are rotated by some mechanical device.

Faraday also found that it was only the vector component of the velocity that was perpendicular to the magnetic field that produced an emf. Figure 1.2 shows a coil rotating inside a magnetic field at various points during a single revolution.

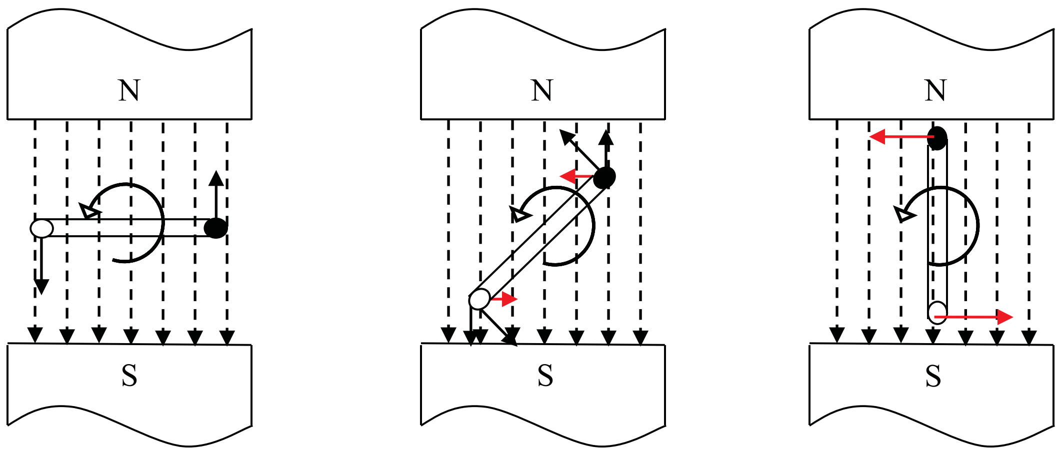

Figure 1.2a **Figure 1.2b** **Figure 1.2c**

In Figure 1.2a the direction of the velocity of the conductor is parallel to the magnetic flux, and so no emf is produced. In Figure 1.2b the coil has rotated by 45°, and the vector component of the velocity perpendicular to the flux is shown by the red arrow. Since this velocity is not zero an emf is produced (as discovered by Faraday). In Figure 1.2c, the coil has rotated by 90°, and so the total vector velocity is perpendicular to the flux and the emf produced is at its maximum (or peak) value. When the coil has rotated by 180°, the emf produced is again 0 (Figure 1.2a), and when rotated by 270°, the black dot on the winding is now at the bottom. The magnitude of the emf is again at its maximum, but of the opposite polarity because the direction of the velocity is in the opposite direction (Figure 1.2c). If a graph is then plotted showing the magnitude of the emf on the vertical scale, and the angle of rotation on the horizontal scale, the resultant waveform of the emf as a function of the angle of rotation is shown in Figure 1.3.

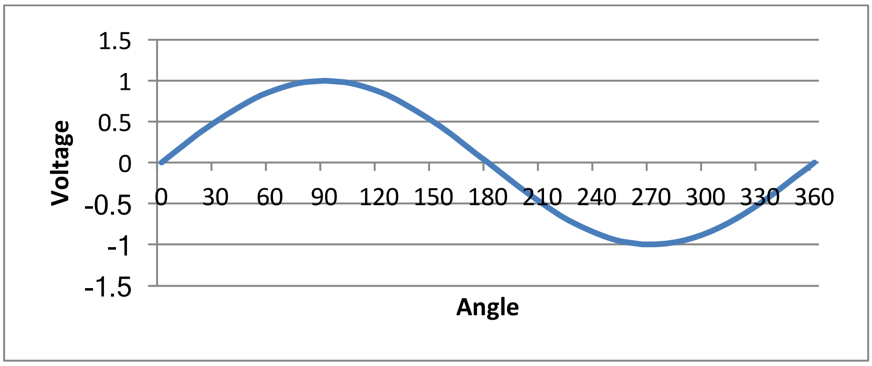

Figure 1.3

If the coil is then rotated a second time, the waveform of the emf is repeated. See Figure 1.4.

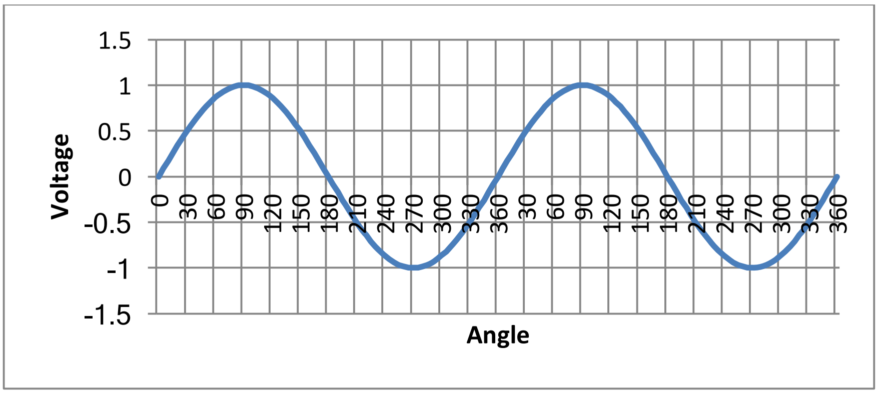

Figure 1.4

The cycle of the emf generated is repeated for each revolution of the coil.

In these graphs the horizontal axis is the angle measured in degrees. This axis could also be time, but usually it is shown as an angle measured in radians. This is done for convenience sake because any calculus required in an analysis would require the angles to be in radians. The most common scale for this axis is ωt, where ω is the angular frequency ($\omega=2\pi f$), where f is the frequency in cycles per second, and t is time.

The magnitude of the emf produced is proportional to the number of turns in each coil. Figure 1 shows a loop of 1 turn of conductor. If the number of loops in the coil is increased, then the total emf produced would increase proportionately. The magnitude of the emf is also proportional to the magnitude of the magnetic flux density, which is produced by the magnetic north and south poles of the stationary magnet.

The frequency of the induced emf is directly proportional to the speed at which the coil is rotated. If the coil is rotated at a speed of 3000 RPM, (= 50 revolutions per second), then the frequency of the generated emf would also be 50 cycles per second (= 50 Hz).

What is more important at this stage is to consider the effect that a sinusoidal emf waveform would have on how to analyse an electrical circuit.

It would be useful to first review how a simple DC circuit would be analysed. Figure 1.5 shows a 12 VDC source connected to a series connected resistor circuit.

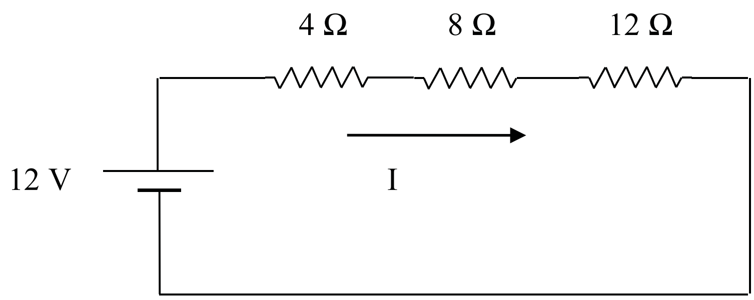

Figure 1.5

In this case the total resistance connected to the source is 24 Ω, and so from Ohm's law the current would be;

$$I = \frac{V}{R} = \frac{12}{24} = 0{,}5\ A$$

The volt drop across each resistor is;

$$V_{4\Omega} = (I)(R) = (0{,}5)(4) = 2V$$
$$V_{8\Omega} = 4V$$
$$V_{12\Omega} = 6V$$

Kirchoff's voltage law states that the sum of the voltages in the circuit is zero.

i.e. $12V - 2V - 4V - 6V = 0$

The power dissipated in each resistor is then;

$$P_{4\Omega} = I^2R = (0{,}5)^2{\times}4 = 1W$$
$$P_{8\Omega} = (0{,}5)^2{\times}8 = 2W$$
$$P_{12}\Omega = (0{,}5)^2{\times}12 = 3W$$

And the total power delivered by the source;

$P_{Source} = V \times I = (12)(0{,}5) = 6W$ (= the total power dissipated in the resistors)

In this case if the source was an AC source the same principle would apply, but what value of V should be used? Since V is varying continuously, using the average value would not work because the calculation of the average value would be;

$$V = \frac{1}{2\pi}\int_0^{2\pi} V_{Peak} Sin(\omega t) d(\omega t) = 0$$

This is obviously wrong because it implies that no power is provided by the source, which we know is not correct. The effective value of the sine wave is the value of voltage that would produce the same power dissipated in a resistor as a DC source would. This is known as the root mean square (RMS) value and is calculated as follows;

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_{Peak} Sin\omega t)^2 d\omega t}$$

Solving this integral will yield the result;

$$V_{RMS} = \frac{V_{Peak}}{\sqrt{2}}$$

For an AC source to deliver the same power as the circuit shown in Figure 1.5, an AC source with an RMS value of 12 V would be required. The required AC source needed is therefore;

$$v = 17 \sin(\omega t) \text{ V}$$

The notation used in this book is that if the voltage is given as

$$v = 17 \sin \omega t \text{ V}$$

implies that 17 is the peak value of the sine wave. If the voltage is given as

V = 12 V

implies that 12 is the RMS value. $(12 = \frac{17}{\sqrt{2}})$

1.3 THREE PHASE AC GENERATION:

The major cost of installing power generators is the cost of the plant to generate the steam. It therefore makes good economic sense to generate more than one phase from the same turbine. Also, by referring to Figure 1.2, it can be seen that there are times when the motion of the conductor is parallel to the magnetic field. No power is generated during these periods, which means that the machine is not being used effectively. This can be solved by installing an additional two coils, offset by 120^{o}, so that power is generated all the time. For this reason it is common to produce a three phase AC supply. Other advantages of three phase systems over single phase systems are;

- Three times the power can be transmitted on one transmission line, compared to a single phase system.
- The constant power flow associated with 3 phase systems (compared to the pulsating power flow in a single phase system) leads to the wires being used more effectively and so lees copper is needed, leading to reduced cost.
- The thinner conductors also have lower mass so the transmission line structures can be smaller.
- Three phase motors are self starting and also provide higher torque.
- Three phase rectifiers produce a lower harmonic content and thus a better quality DC supply and less harmonic filtering is required.

The generation of a three phase supply is the same principle as a single phase supply. Consider the drawing in Figure 1.2, but instead of having one coil rotating in the magnetic field, three independent coils are fitted as shown in Figure 1.6.

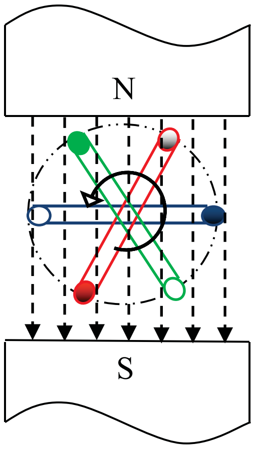

Figure 1.6

The three coils are mounted with a 120° angle between them. The AC waveform generated by each coil is then 120° removed (or displaced) from the other two. As the coils rotate the same principle applies as described for Figure 1.2. The three coils are electrically isolated. The generated waveform is illustrated in Figure 1.7

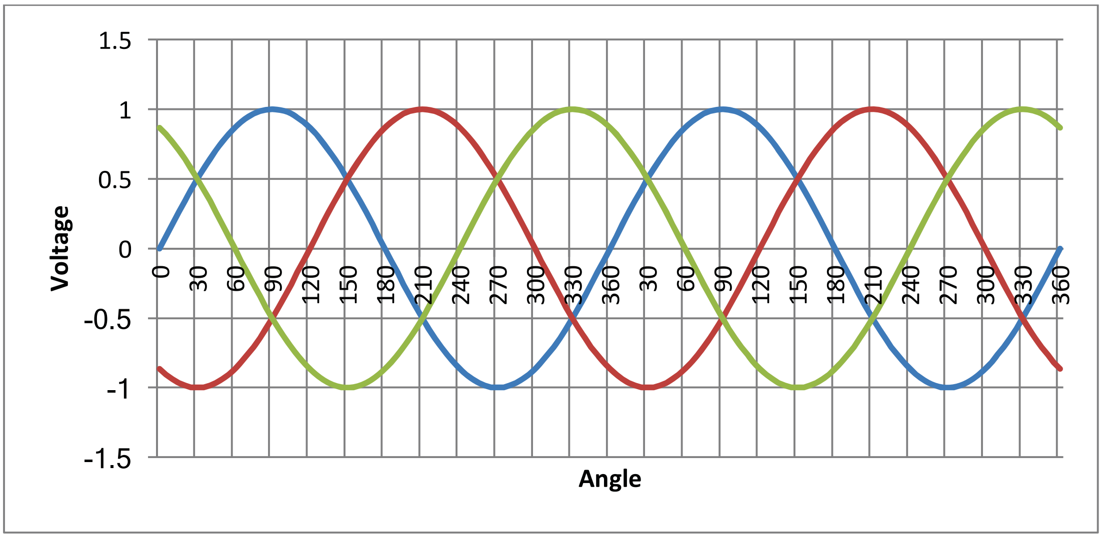

Figure 1.7

The three generated phases are displaced by 120°, as can be seen in both Figures 1.6 and 1.7 It can be seen by inspection that the potential difference between any two phases is greater than the voltage generated in each phase. This is caused by the 120° phase shift. By comparing Figures 1.6 and 1.7 at 0°, it can be seen that the blue coil is at zero in both diagrams, the green coil has passed its peak and dropping to zero, and the red coil is approaching its negative peak.

There are two different ways of connecting the three phases as shown in Figure 1.8, known as Star connected (Figure 1.8a) and Delta connected (Figure 1.8b). The letters A, B and C are referred to as phase A, phase B and phase C and N is the neutral connection. There is no neutral in a delta connection.

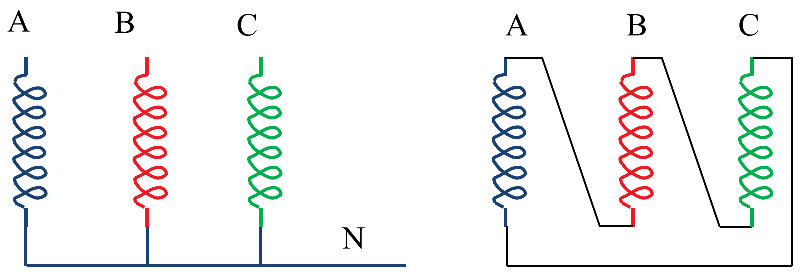

Figure 1.8a Star connection

Figure 1.8b Delta connection

In order to examine the effect of a three phase system, it becomes very cumbersome to use the three sine waves as shown in Figure 1.7. A rotating vector, called a phasor is used to simplify the analysis.

This is best illustrated by example. Consider a generator providing 220 $V_{(RMS)}$ per phase, and connected in Star as shown in Figure 1.8a. We will use phase A as the reference. The reference is drawn on the positive x-axis representing angle 0.

This is shown as V_{AN} in Figure 1.9

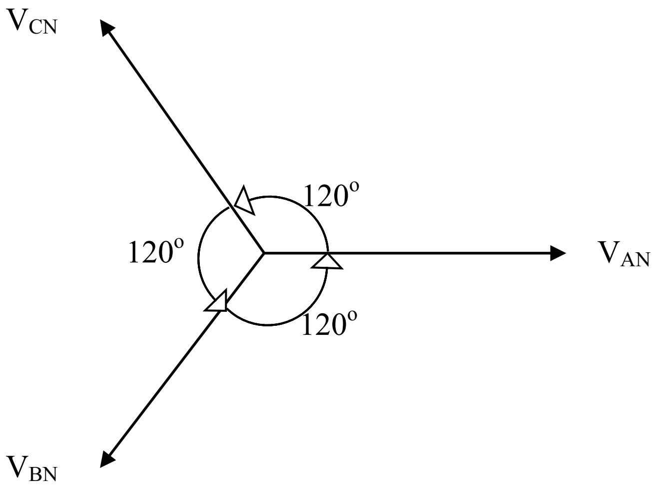

Figure 1.9:

These vectors rotate anti-clockwise, so the next vector to reach the reference is phase B, 120° later. Phase C reaches the reference 240° later (or 120° before). This is referred to as "rotation ABC". The three phases are then represented as;

$$V_{AN} = 220\angle 0^o$$

$$V_{BN} = 220\angle -120^o$$

$$V_{CN} = 220\angle -240^o = 220\angle +120^o$$

In mathematical form this would be written as;

$$V_{AN} = 311\sin(\omega t)$$

$$V_{BN} = 311\sin(\omega t - 120^o)$$

$$V_{CN} = 311\sin(\omega t + 120^o)$$

In this example it was given that the potential difference between phase and neutral was 220 V. We now need to determine what the potential difference between two phases would be. The potential difference between phase A and phase B would be;

$$V_{AB} = V_{AN} - V_{BN}$$

V_{AN} and V_{BN} are both vectors and so we refer to the phasor diagram in Figure 1.10.

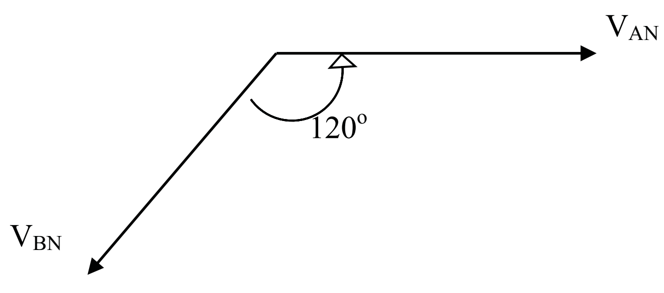

Figure 1.10

Since $V_{AB} = V_{AN} - V_{BN}$, then $V_{AB} = V_{AN} + (-V_{BN})$. Refer to Figure 1.11

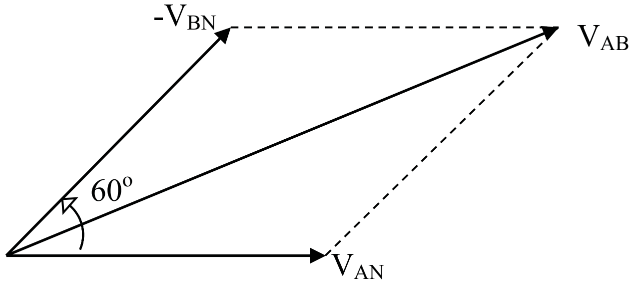

Figure 1.11

V_{AB} is then the vector sum of V_{AN} and $-V_{BN}$. To determine the magnitude of V_{AB}, the phasor diagram is re-drawn in Figure 1.12 to simplify the explanation of the geometry.

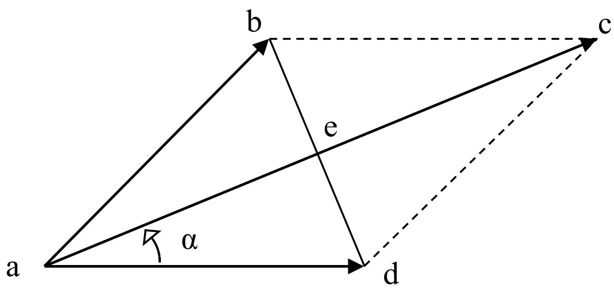

Figure 1.12

Line ab is parallel to line dc,
Line ad is parallel to line bc, and
The length of ab = the length of ad.

abcd is therefore a rhombus, and so line ac bisects the 60º angle shown in Figure 1.11. α is therefore 30º. The line bd is the perpendicular bisector of ac, and so the angle ∠aed is 90º. The length of ae is then;

$$ae = ad\cos 30^o = \frac{\sqrt{3}}{2}ad$$

Since ae = ec, the length of ac is then $\sqrt{3}ad$

Referring this back to Figure 1.11, $V_{AB} = \sqrt{3}V_{AN}\angle 30^o$

In this example the generated voltage was 220 V per phase. It is therefore possible to increase this to 380 V simply by changing the connection of the three phases. A star connected system has 220 V per phase, and a delta connected system has 380 V between phases.

The normal way to represent star and delta connected systems is shown in Figure 1.13.

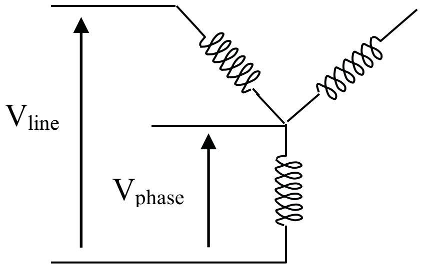

Figure 1.13a: Star

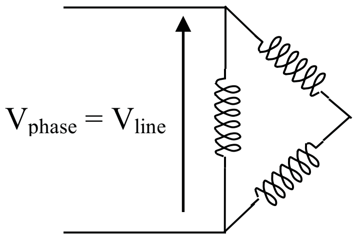

Figure 1.13b: Delta

A more detailed study of how star and delta connected circuits are analysed will be covered in later chapters of this book.

1.4 EXERCISES

1.4.1 A single turn coil is rotated at a speed of 3600 RPM in a static magnetic field. What is the frequency of the generated emf?

1.4.2 A coil is rotating in a magnetic field and generates an emf of $155{,}56\ \sin(377)t\ Volts.$ What is the RMS value of the emf, and what is the frequency of the generated emf?

1.4.3 A 12 V DC source is connected to a 12Ω resistor and a 6Ω resistor, which are connected in parallel. Determine the total current delivered by the source, and the total power consumed by the load.

1.4.4 Three independent coils are physically displaced by 120° and rotated in a static magnetic field at a speed of 3600 RPM. The three coils are referred to as A,B and C, and the neutral is N. Each coil has the same number of turns. The three coils are Star connected and phase A generates an emf of 110 V relative to the neutral. The phase rotation is ABC. If V_{AN} is used as the reference, determine the magnitude and phase angle of V_{CA} . What is the frequency and period of the generated emf?

2 SINGLE PHASE CIRCUITS

2.1 OBJECTIVE:

The purpose of this chapter is to review the concept of impedance in a circuit, the phase displacement between the voltage and the current, and the power factor of a circuit. The concepts of apparent power, real power and reactive power are also reviewed. It is best to review these concepts using a single phase system before examining three phase circuits.

2.2 IMPEDANCE:

The basic principle of any electrical circuit is that the applied voltage causes a current to flow, and the magnitude of the current is determined by the resistance of the circuit. This was discovered by George Ohm in 1827. (Boylestad 2007, p. 102). In the case of AC circuits the effect of inductance and capacitance (called reactance) also needs to be taken into account. The resistance of the circuit, combined with the reactance is known as impedance.

A current flowing through an inductor causes a magnetic field, and the voltage applied to a capacitor causes an electric field. These magnetic and electric fields make no contribution to the work being done by the circuit, and so for the purposes of analysis are represented by $\sqrt{-1}$. an imaginary number. This is done by multiplying these quantities by $\sqrt{-1}$ In mathematical texts is represented by the symbol i. However in electrical systems the symbol i represents current, and so to avoid confusion, the symbol j is used to represent $\sqrt{-1}$.

A combination of a real number and an imaginary number is known as a complex number and is illustrated in Figure 2.1.

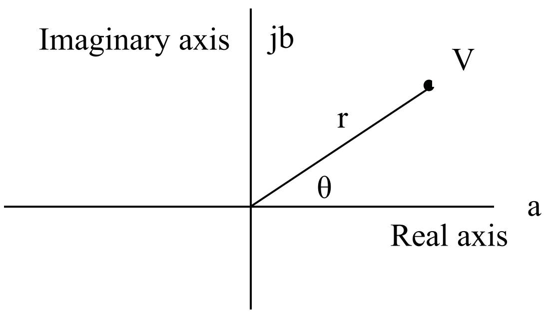

Figure 2.1

The number can be represented in rectangular co-ordinates as;

$$V = a + jb$$

Or in polar co-ordinates as;

$$V = r\angle\theta$$

Conversion from one system of co-ordinates to the other is then done using Pythagoras' theorem and trigonometry.

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$a = r\cos\theta$$

$$b = r\sin\theta$$

Inductive reactance is defined as

$$X_l = j\omega L$$

where $j = \sqrt{-1}$, ω is the angular frequency ($\omega = 2\pi f$), and L is the inductance, measured in Henry.

Capacitive reactance is defined as;

$$X_C = \frac{1}{j\omega C}$$

where $j = \sqrt{-1}$, $\omega = 2\pi f$, and C is the capacitance, measured in Farad.

The impedance in a series circuit is then;

$Z = R + X_L + X_C$ (Note that X_L and X_C are both imaginary numbers)

2.3 RLC CIRCUITS:

2.3.1 RESISTIVE LOAD:

Figure 2.2 shows an AC source connected to a purely resistive circuit.

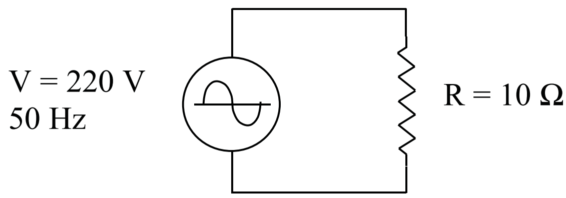

Figure 2.2

The current in the circuit can be determined from Ohm's law;

$$I = \frac{V}{R} = \frac{220}{10} = 22A$$

And the power dissipated is $P = VI = (220)(22) = 4840W = 4{,}84kW$
Note that RMS values of the voltage and current (which are both sinusoidal) must be used for this calculation.

The waveform of the voltage and current is shown in Figure 2.3.

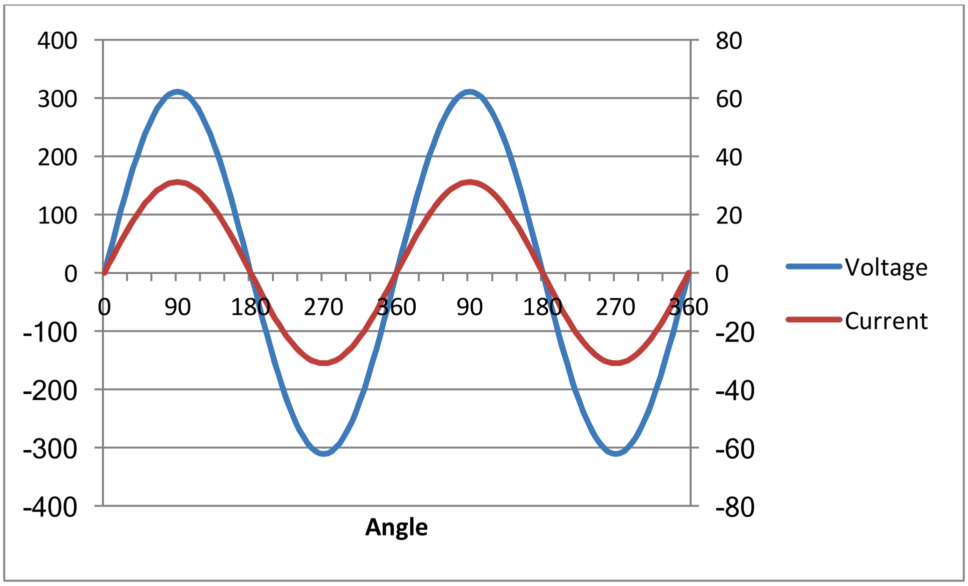

Figure 2.3

Figure 2.3 shows that for the period 0° to 180° both V and I are positive and so the resultant power (VxI) is positive. For the period 180° to 360°, both V and I are negative, and the resultant power (VxI) is also positive. The positive value of power indicates that the transfer of energy is from the source to the load. In this case where the load is purely resistive, all of the power is transferred from the source to the load.

2.3.2 INDUCTIVE LOAD:

Figure 2.4 shows a circuit where the load is purely inductive.

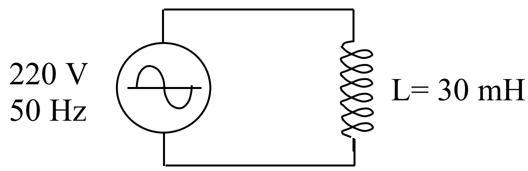

Figure 2.4

The impedance in this case is $Z = 0 + X_L = j\omega L = j(2\pi)(50)(30x10^{-3}) = j9{,}42\Omega$

Using the voltage as the reference, the current is then;

$$I = \frac{V}{Z} = \frac{220\angle 0}{9{,}42\angle 90} = 23{,}35\angle -90$$

The wave form of the voltage and current is then;

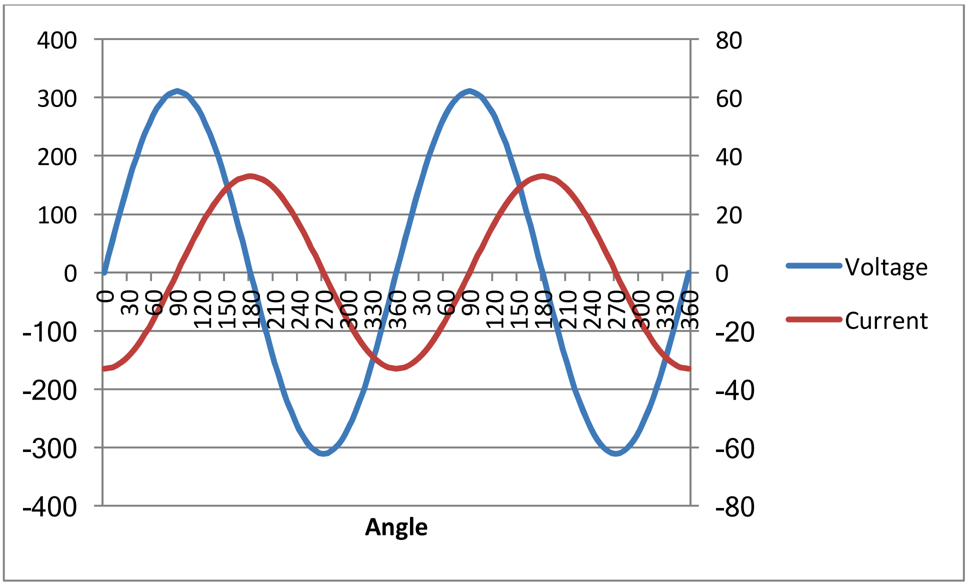

Figure 2.5

Figure 2.5 shows that for the period 90° to 180°, V and I are positive, so the power transfer is from the source to the load. From 180° to 270°, V is negative and I is positive, so the power transfer is from the load to the source. From 270° to 360°, both V and I are negative and the power transfer is again from the source to the load. From 360° to 90°, V and I are again of opposite polarity and the power transfer is from the load to the source. The nett result is that for a quarter of the cycle energy is transferred from the source to the load, stored as magnetic energy, and then for the next quarter cycle, the magnetic energy is recovered.

(Boylestad 2007, p. 840) Over a complete cycle no real work is done because the current lags the voltage by 90°. In mathematical terms this can be expressed as;

$$P = VI\cos\theta = VI\cos(-90^o) = 0$$

2.3.3 CAPACITIVE LOAD:

Figure 2.6 shows a circuit where the load is purely capacitive.

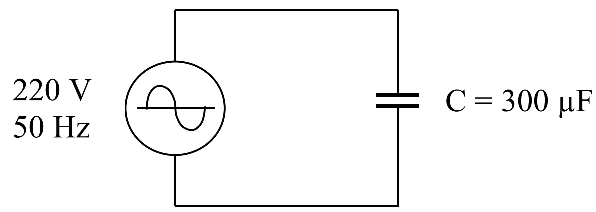

Figure 2.6

The impedance in this case is $Z = 0 + \frac{1}{j\omega C} = \frac{-j}{(2\pi)(50)(300x10^{-6})} = -j10{,}6\ \Omega$

The current is then $I = \frac{V}{Z} = \frac{220\angle 0}{10{,}6\angle -90} = 20{,}75\angle 90$

The V-I waveform is then;

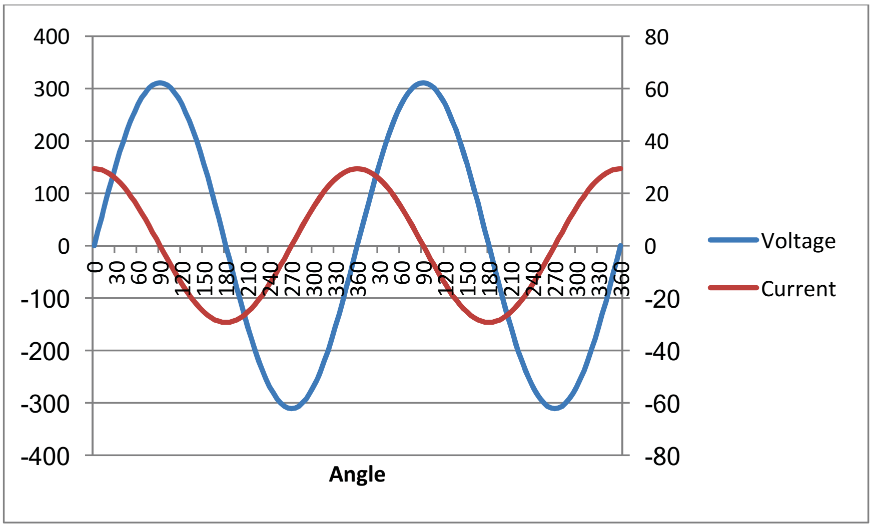

Figure 2.7

In this case the current is leading the voltage by 90°, and the power transfer is the same principle as for an inductive load. For a quarter of a cycle energy is transferred from the source to the load, stored as an electric field in this case, and for the next quarter cycle this energy is recovered. The nett result is that no real work is done. In mathematical terms this is;

$$P = VI\cos\theta = VI\cos 90 = 0$$

2.4 AC POWER:

AC power consists of apparent power, reactive power and real power. Determining these quantities in an AC circuit is best described by example.

Example 1:

Figure 2.8 shows a series RLC circuit connected to a single phase AC source. The source has an RMS value of 220 V at 50 Hz, the resistor is 68 Ω, an inductance of 100 mH, and a capacitor of 470 μF.

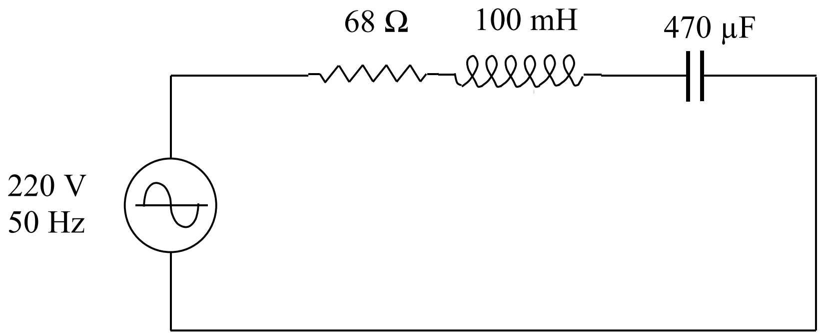

Figure 2.8

$$X_L = j\omega L = j(2\pi)(50)(100x10^{-3}) = \text{j}31{,}4\ \Omega$$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j(2\pi)(50)(470x10^{-6})} = \frac{-j}{(147x10^{-3})} = -\text{j}6{,}8\ \Omega$$

The impedance of the circuit is then;

$$Z = R + X_L + X_C = 68 + j31{,}4 - j6{,}8 = 68 + j\ 24{,}6\ \Omega$$

It is convenient to convert these rectangular co-ordinates to polar co-ordinates.

so $\quad Z = 68 + j24{,}6 = 72{,}3\angle 19{,}89^o$

If we choose the voltage to be the reference, then $V = 220\angle 0^o$

and $\quad I = \frac{V}{Z} = \frac{220\angle 0}{72{,}3\angle 19{,}89} = 3{,}04\angle -19{,}9\ A$

The phasor diagram for this circuit is shown in Figure 2.9.

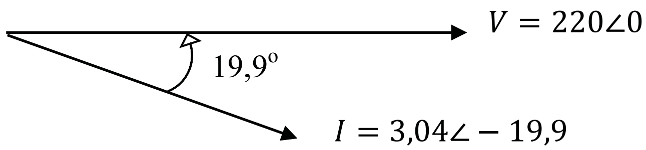

Figure 2.9

Figure 2.10 shows the same information, but plotted as a sinusoidal waveform.

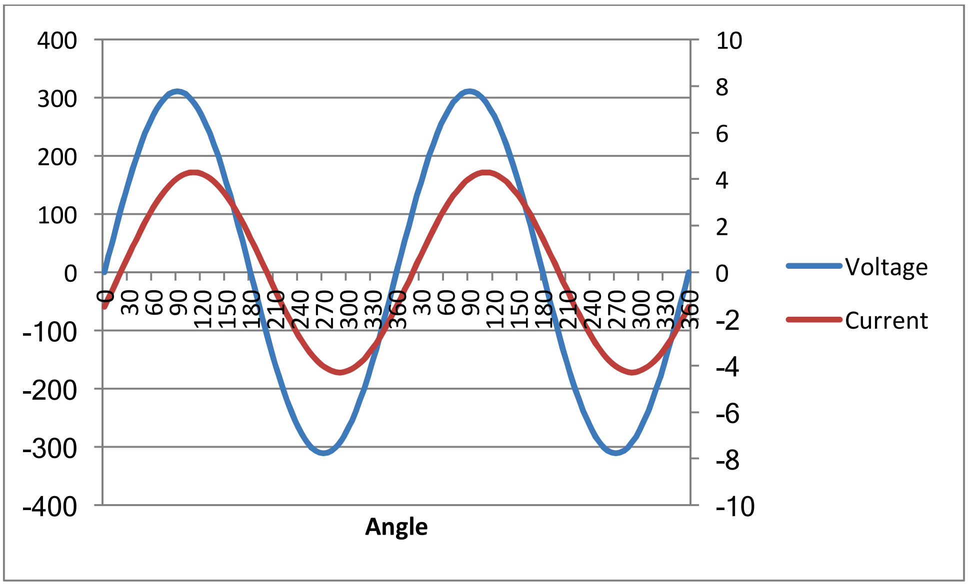

Figure 2.10

Both diagrams show the phase displacement of approximately 20°, with the current lagging the voltage. By examining Figure 2.10 it can be seen that between 0° and 20°, the voltage is positive and the current is negative. From 180° to 200°, I is positive and V is negative. This could imply a negative impedance, but this is not the correct interpretation. The correct interpretation is that since power is the product of V and I, the power during these periods is negative. This means that the power transfer is from the load to the source. This is called re-active power, and is the magnetic/electric field energy being recovered. It is not useful power and cannot do any work. This concept is best described by a power triangle as shown in Figure 2.11.

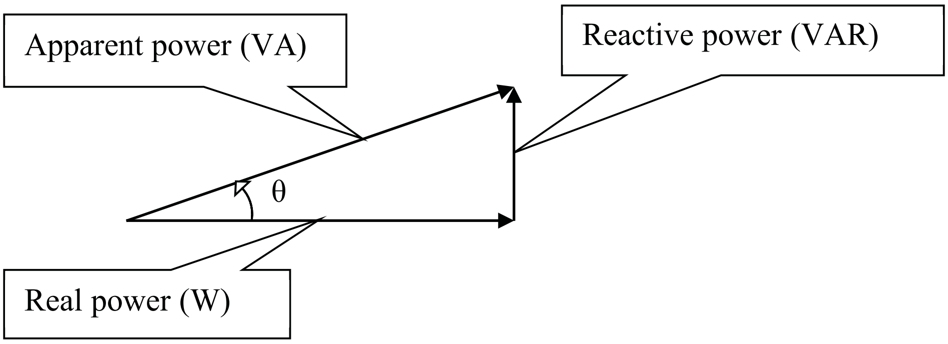

Figure 2.11

The apparent power $S = VxI = 220\ x\ 3{,}04 = 668{,}8\ VA$

The reactive power $Q = VI\sin(\theta) = (220)(3{,}04)\sin(19{,}9) = 227{,}64\ VAR$

The real power $P = VI\cos\theta = (220)(3{,}04)\cos(19{,}9) = 628{,}86\ W$

The power factor is defined as $\mathrm{PF} = \cos\theta$.

It is important to note at this stage that the angle θ can vary from -90° to +90°, and that $\cos\theta$ will then vary from 0 to 1 regardless of whether θ is positive or negative. When stating the value of power factor it is then important to state whether it is a leading power factor (for a predominantly capacitive circuit) or a lagging power factor (for a predominantly inductive circuit).

In this case the angle θ is easy to identify because it is simply the angle between the voltage and the current. In three phase systems this angle is not as easy to identify (as will be seen in later chapters) An alternative method to determine the power is to recognise that the real power is dissipated by the resistance, and the reactive power processed by the reactance, so;

The real power $P = I^2R = (3{,}04^2)(68) = 628{,}43\,W$

The apparent power $Q = I^2X = (3{,}04^2)(24{,}6) = 227{,}34\,VAR$

The minor discrepancies in the final answer are due to rounding errors in the calculation. It can be seen that there is no possibility of getting the angle θ confused using this method.

Example 1 shows a circuit where the reactance is predominately inductive. If the impedance is changed, the circuit could become predominately capacitive, and the results would be significantly different. This is illustrated in example 2.

Example 2:

Consider the same circuit as shown in Figure 2.8, but the value of the capacitor has changed from 470 μF to 470 μF. The circuit is redrawn in Figure 2.12. The analysis is then repeated with this new value of capacitance.

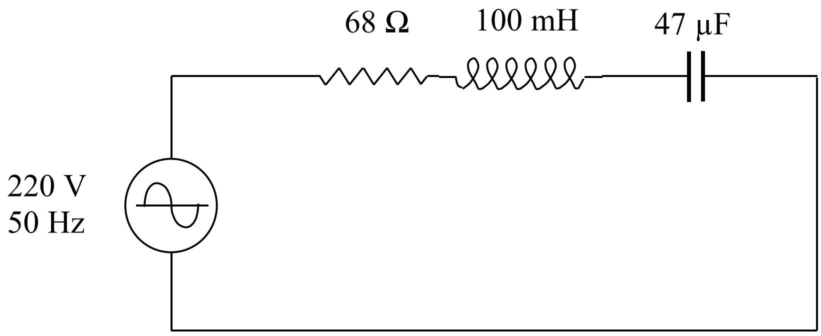

Figure 2.12

$$X_L = j\omega L = j(2\pi)(50)(100x10^{-3}) = \mathrm{j}31{,}4\ \Omega$$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j(2\pi)(50)(47x10^{-6})} = \frac{-j}{(14{,}7x10^{-3})} = -\mathrm{j}67{,}7\ \Omega$$

The impedance of the circuit is then;

$$Z = R + X_L + X_C = 68 + j31{,}4 - j67{,}7 = 68 - j36{,}3\ \ \Omega$$

And in polar co-ordinates,

$$Z = 68 - j36{,}3 = 77{,}08\angle{-28{,}09^o}$$

If we choose the voltage to be the reference, then $V = 220\angle 0^o$

and $\quad I = \frac{V}{Z} = \frac{220\angle 0}{77{,}08\angle{-28{,}09}} = 2{,}85\angle 28{,}09\ A$

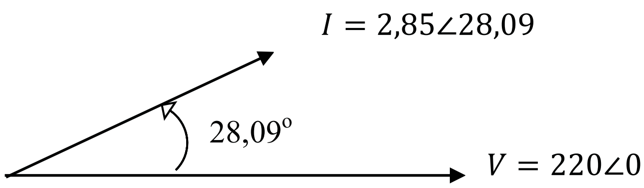

Figure 2.13

In this example the reactance is predominately capacitive so the current leads the voltage.

The power factor is $\cos\theta = \cos(28{,}09) = 0{,}88$ (Leading)

and the real power $\quad P = VI\cos\theta = (220)(2{,}85)(0{,}88) = 551{,}76\ W$

(or $P = I^2R = (2{,}85^2)(68) = 552{,}33W$) Again the minor discrepancy is due to rounding errors in the calculation.

The reactive power can be calculated by;

$$P_{Reactive} = VI\sin\theta$$

$$P_{Reacive} = (220)(2{,}85)\sin(28{,}09^o)$$

$$P_{Reactive} = (220)(2{,}85)(0{,}47) = 294{,}69\,VAR$$

The apparent power is;

$$P_{Apparent} = VI = (220)(2{,}85) = 627\,VA$$

The power triangle is shown in Figure 2.14. Note that in this case the reactive power has a negative value due to the leading power factor.

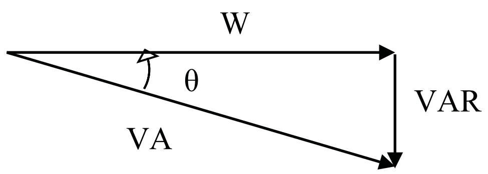

Figure 2.14

Figure 2.15 shows the voltage and current waveforms. Again during the periods where V and I are of opposite polarity the power transfer is from the load to the source and represents the reactive power. This is shown more clearly in Figure 2.16, where the instantaneous power is plotted as a function of angle. The negative value of power is the reactive power. The average power which is also shown on the graph is the product of the RMS values of V and I, and multiplied by the power factor. In mathematical terms this would also be the integral of the power over one cycle, divided by the period.

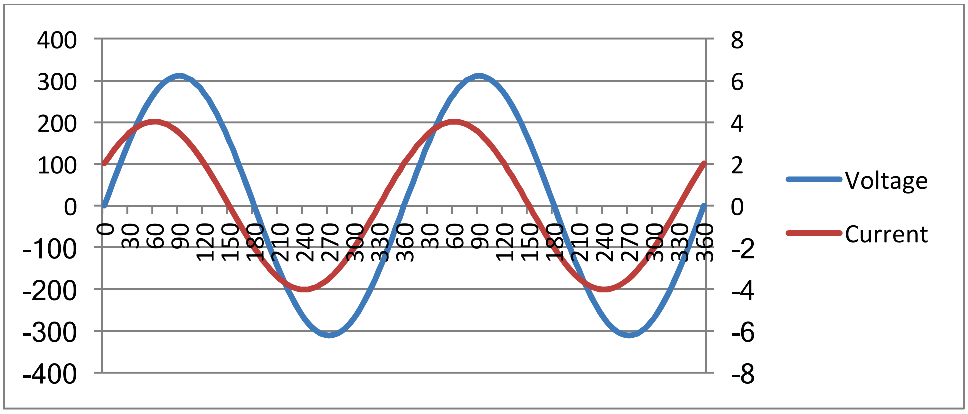
400
300
200
100
0
-100
-200
-300
-400
8
6
4
2
0
-2
-4
-6
-8
0
30
60
90
120
150
180
210
240
270
300
330
360
Voltage
Current

Figure 2.15

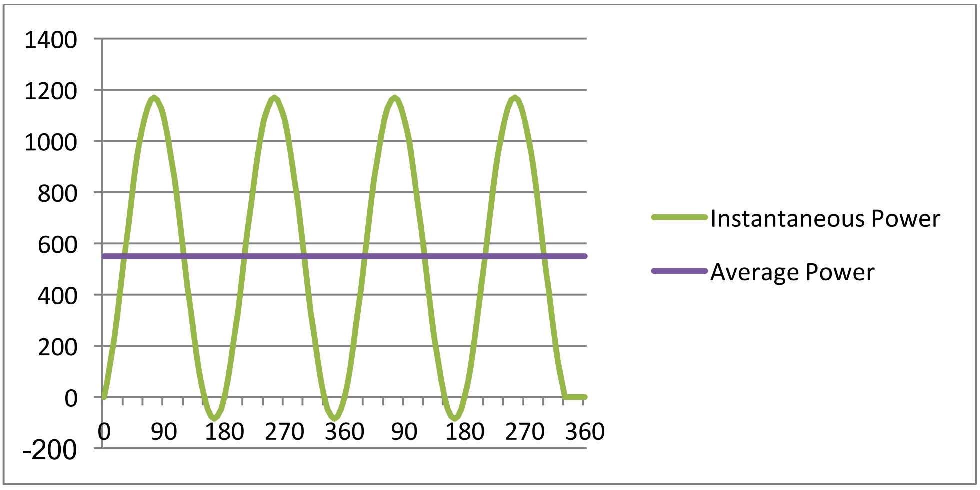

Figure 2.16

Example 3:

Illustrating the application of Kirchoff's laws is best shown by analysing a parallel connected circuit. This principle will be used in later chapters when Delta connected three phase circuits are examined.

Figure 2.17 shows a 220 V 50 Hz single phase supply connected to a parallel connected network. To determine the power delivered by the source we need to know the applied voltage, the current I_1 and the phase angle between the voltage and the current. The power delivered is then;

$$P = VI\cos\theta$$

The applied voltage V is given so it would be convenient to use this as the reference. To calculate I_1 the equivalent impedance of the circuit is needed.

This can be done by grouping the 72 Ω resistor and the 470 μF capacitor and calling this Z_1. Z_2 is then the combination of the 100 Ω resistor and 50 mH inductor, and Z_3 is the combination of the 39 Ω resistor, the 100 mH inductor and the 680 μF capacitor.

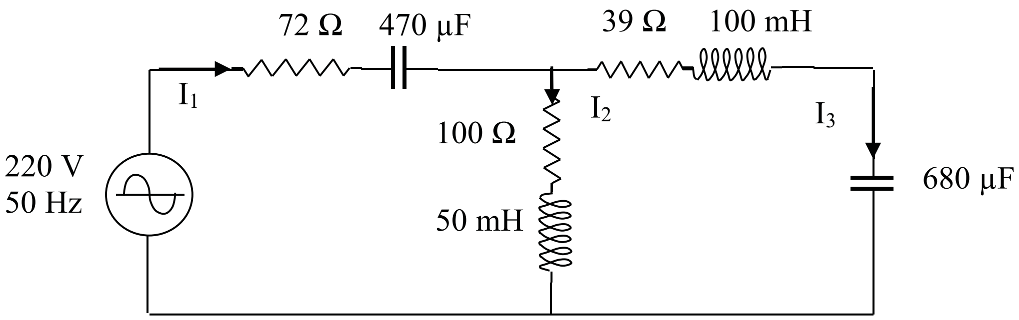

Figure 2.17

$$Z_1 = R + \frac{1}{j\omega C} = 72 + \frac{-j}{(2\pi)(50)(470x10^{-6})}$$

$$\mathbf{Z_1} = \mathbf{72 - j6,77}\ \Omega = \mathbf{72,32\angle - 5,37}\ \Omega$$

$$Z_2 = R + j\omega L = 100 + j(2\pi)(50)(50x10^{-3})$$

$$\mathbf{Z_2} = \mathbf{100 + j15,71}\ \Omega = \mathbf{101,23\angle 8,93}\ \Omega$$

$$Z_3 = R + j\omega L + \frac{1}{j\omega C} = 39 + j(2\pi)(50)(100x10^{-3}) + \frac{-j}{(2\pi)(50)(680x10^{-6})}$$

$$= 39 + j31{,}42 - j4{,}68 = 39 - j26{,}74 = 47{,}29\angle - 34.44\ \Omega$$

$$\mathbf{Z_3} = \mathbf{39 - j26,74} = \mathbf{47,29\angle - 34,33}\ \Omega$$

At this point in the calculation it is always useful to have these values in both polar and rectangular notation because both forms will be needed at a later stage.

Referring again to Figure 2.18 it can be seen that Z_2 and Z_3 are in parallel. If we define Z_4 as these two impedances in parallel, then;

$$Z_4 = \frac{(Z_2)(Z_3)}{Z_2 + Z_3} = \frac{(101{,}23\angle 8{,}93)(47{,}29\angle - 34{,}33)}{(100 + j15{,}71) + (39 - j26{,}74)}$$

$$= \frac{4787{,}17\angle - 25{,}4}{139 - j11{,}03} = \frac{4787{,}17\angle - 25{,}4}{139{,}44\angle - 4{,}54}$$

$$\mathbf{Z_4} = \mathbf{34,33\angle - 20,86} = \mathbf{32,08 - j12,22}\ \Omega$$

The equivalent impedance of the whole circuit is then;

$$Z_{eq} = Z_1 + Z_4 = (72 - j6{,}77) + (32{,}08 - j12{,}22)$$

$$\mathbf{Z_{eq} = 104{,}08 - j18{,}99 = 105{,}80\angle - 10{,}34\ \Omega}$$

The current I_1 is then $\frac{V}{Z_{eq}}$

$$I_1 = \frac{V}{Z_{eq}} = \frac{220\angle 0}{105{,}80\angle - 10{,}34} = 2{,}08\angle 10{,}34 = 2{,}05 + j0{,}373A$$

Figure 2.18 shows a simplified version of the original circuit.

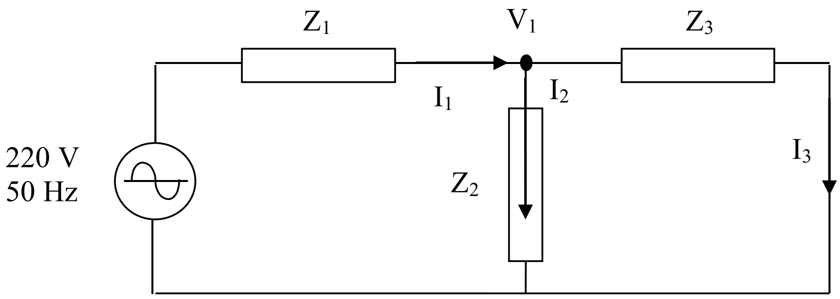

Figure 2.18

Kirchoff's voltage law states that the sum of the voltages in any closed circuit is zero. Therefore in Figure 2.18 the volt drop across $Z_1 + V_1$ equals the applied voltage.

The volt drop across Z_1 is $(I_1)(Z_1)$;

So $V_{Z1} = I_1 Z_1 = (2{,}08\angle 10{,}34)(72{,}32\angle -5{,}37) = 150{,}43\angle 4{,}97 = 149{,}86 + j13{,}03\ V$

V_1 is then;

$$V_1 = 220\angle 0 - 150{,}43\angle 4{,}97$$

$$= (220)\text{-}(149{,}86 + j13{,}03) = 70{,}14 - j13{,}03\ V = 71{,}34\angle -10{,}52\ V$$

The current I_2 is then;

$$I_2 = \frac{V_1}{Z_2} = \frac{71{,}34\angle -10{,}52}{101{,}23\angle 8{,}93} = 0{,}705\angle -19{,}45 = 0{,}67 - j0{,}235\ A$$

And I_3 is;

$$I_3 = \frac{V_1}{Z_3} = \frac{71{,}34\angle -10{,}52}{47{,}29\angle -34{,}33} = 1{,}51\angle 23{,}81 = 1{,}38 + j0{,}610\ A$$

Kirchoff's current law states that the sum of the currents at a node is zero. Therefore at node V_1,

$$I_1 = I_2 + I_3$$

Since all of these quantities are known, it would be a good idea to check the calculations at this point.

$$I_2 = 0{,}67 - j0{,}235$$

$$I_3 = 1{,}38 + j0{,}610$$

And so;

$$I_1 = (0{,}67 - j0{,}235) + (1{,}38 + j0{,}610) = 2{,}05 + j0{,}375\ A = 2{,}08\angle 10{,}37^o$$

The original calculation of I_1 showed a result of 2,05 +j0,373. Again it can be seen that there is a minor discrepancy in the calculated results, but this can be attributed to rounding errors in the calculations.

The real power delivered to the circuit is then;

$$P = VI_1 \cos\theta$$

So $P = (220)(2{,}08)\cos(10{,}37^o) = 450{,}13$ W

2.5 EXERCISES

2.5.1 The real power dissipated in an AC circuit is $P_{Real} = VI\cos\theta$. Show that the real power can also be calculated as $P_{Real} = I^2R$

2.5.2 A series connected AC circuit consists of a resistance of 4 Ω and an inductive reactance of 8 Ω. Determine the equivalent impedance in polar co-ordinates.

2.5.3 A series connected 50 Hz AC circuit has a resistance of 120 Ω and a capacitance of 100 μF. Determine the equivalent impedance of the circuit.

2.5.4 A series connected RLC circuit comprises a resistance of 470 Ω, an inductance of 10 mH and a capacitance of 100 μF. The supply has an RMS value of 220 V at a frequency of 1 kHz. Determine the real power consumed, the reactive power consumed and the apparent power consumed.

2.5.5 A series connected RLC circuit comprises a resistance of 470 Ω, an inductance of 10 mH and a capacitance of 100 μF. The supply has an RMS value of 220 V at a frequency of 10 Hz. Determine the real power consumed, the reactive power consumed and the apparent power consumed.

3 BALANCED STAR CONNECTED THREE PHASE CIRCUITS

3.1 OBJECTIVE:

This chapter shows how to analyse a balanced three phase Star connected load. It also explains the difference between phase quantities and line quantities, and how to calculate the total power delivered to the load. It also explains the advantages and disadvantages of Star connected systems, and when a Star configuration should be used.

3.2 ASSUMPTIONS AND NOTATION:

A balanced system is one in which the supply voltage has an equal magnitude in each phase with a phase displacement of 120° between them, and the impedance in each phase is equal. It is reasonable to assume that the source is always balanced. For this to happen all that is required is for the three coils in the generator to be physically displaced by 120° (which is always the case), and each coil to have the same number of turns. The only time this will not happen is under fault conditions, which is beyond the scope of this book.

At this point it would be useful to explain the notation used in this book. Upper case letters are used for the source, while lower case letters are used for the load. The current from the source to the load in phase A is then represented by $I_{Aa,}$ The current in phase B is then I_{Bb}, and so on. The notation Aa implies from A to a. Where the currents are shown in the circuit, the direction of the arrow indicates the assumed direction of the current for the purposes of analysis. It is normal to show all these currents as flowing from the source to the load. If the calculated value of the current is then found to be a negative value, this means that the assumed direction was wrong, but the magnitude of the current is correct. Referring to Figure 1.7 (in chapter 1), it can be seen by inspection that at least one of the currents will be flowing in the opposite direction to that assumed.

Similarly the symbol V_{AN} means the potential at A relative to N, with the upper case letters referring to the source. V_{cn} would then refer to the potential at c relative to n, with the lower case letters referring to the load.

It is also necessary to differentiate between phase quantities and line quantities. The phase quantities V_{Phase} and I_{Phase} refer to the voltage and the current within the source or the load. The line quantities V_{Line} and I_{Line} refer to the voltage and the current in the circuit connecting the source to the load. In this case it is clear to see that $I_{Line} = I_{Phase}$ and $V_{Line} \neq V_{Phase.}$ These quantities are shown in Figure 3.1

The source is represented as an inductor because the emf is generated by rotating a coil in a magnetic field, as explained in Chapter 1.

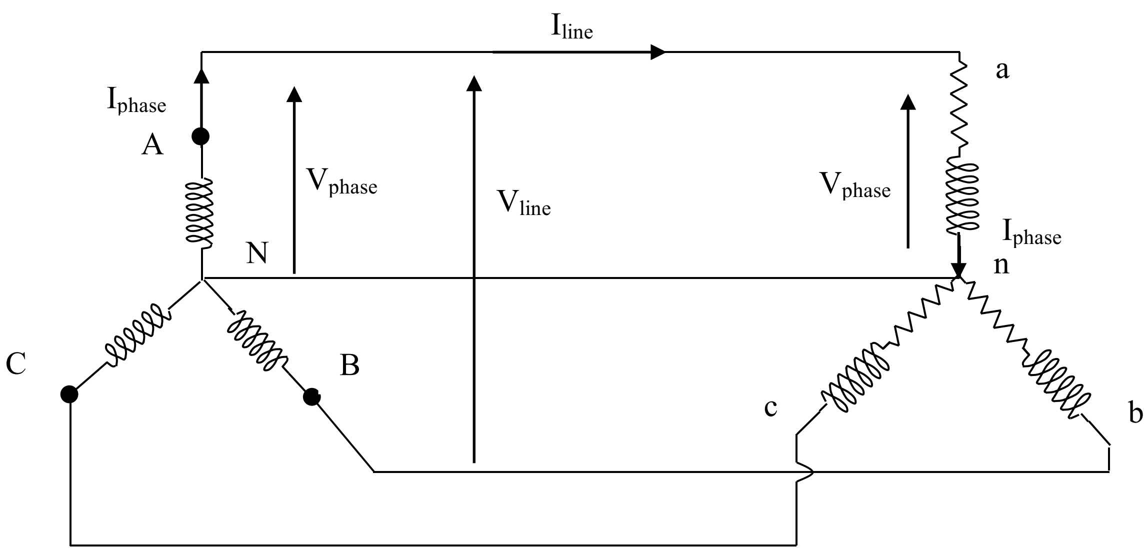

Figure 3.1:

3.3 BALANCED STAR CONNECTED THREE PHASE LOAD:

A Star connected system has three "live" phases and a neutral connection and so is a 4-wire system. It is used when a neutral connection is needed (usually for safety reasons, where the Star point is earthed) but has the disadvantage that that since 4 wires are needed, more copper is needed and so becomes more expensive.

Since a Star connected load requires the three live phases and a neutral, the source in this case must also be Star connected.

Example 1:

Figure 3.2 shows a Star connected source connected to a Star connected load. The load in this case is a balanced series connected RL circuit with an impedance value of 2 + j6 Ω per phase. The phase rotation is ABC. In this first example it is assumed that the electrical connection between the source and the load has negligible impedance. The phase voltage in the source is 220 V per phase, and we will use V_{AN} as the reference.

If V_{AN} is the reference, and the phase rotation is ABC, then;

$$V_{AN} = 220\angle 0^o\ Volts$$

$$V_{BN} = 220\angle{-120^o}\ Volts$$

$$V_{CN} = 220\angle{-240^o} = 220\angle{+120^o}\ Volts$$

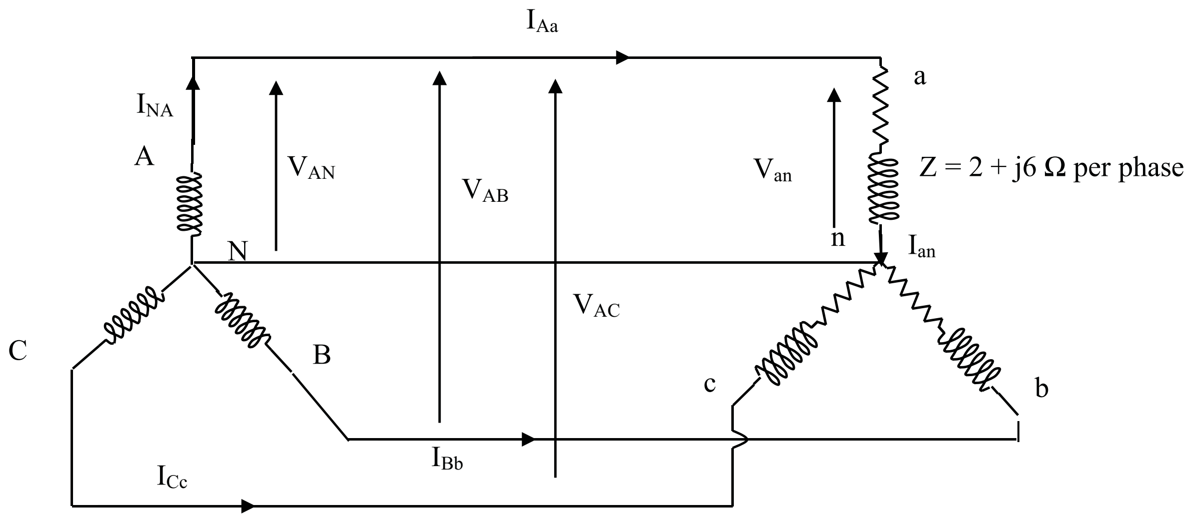

Figure 3.2:

In Chapter 1 (Figure 1.11) it was shown that $V_{AB} = \sqrt{3}V_{AN}\angle{+30^o}$

If V_{AN} is the reference, then;

$$V_{AB} = \sqrt{3}(220)\angle 30^o = 381{,}05\angle 30^o \cong 380\angle 30^o \; Volts$$

Similarly $V_{BC} = \sqrt{3}V_{BN}\angle{+30^o} = 380\angle{-90^o} \; Volts$

And $V_{CA} = \sqrt{3}V_{CN}\angle{+30^o} = 380\angle 150^o \; Volts$

Note that the factor of $\sqrt{3}$ and the phase shift of 30° is only applicable to balanced three phase systems. (Refer to the geometry in Figure 1.12 in Chapter 1)

It is known that $V_{AN} = 220\angle 0^o$. Then it can be seen by inspection that $V_{an} = 220\angle 0^o$. The impedance of the load is given as $2 + j6\ \Omega$ per phase. Then the current I_{an} can then be determined from;

$$I_{an} = \frac{V_{an}}{Z} = \frac{220\angle 0^o}{2 + j6} = \frac{220\angle 0^o}{6{,}325\angle 71{,}57^o} = 34{,}783\angle - 71{,}57^o \; Amps$$

Similarly;

$$I_{bn} = \frac{V_{bn}}{Z} = \frac{220\angle{-120^o}}{6{,}325\angle 71{,}57^o} = 34{,}783\angle - 191{,}57^o = 34{,}783\angle 168{,}43\ Amps$$

And;

$$I_{cn} = \frac{V_{cn}}{Z} = \frac{220\angle 120^o}{6{,}325\angle 71{,}57^o} = 34{,}783\angle 48{,}43^o\ Amps$$

The magnitude and phase angle of these quantities is best illustrated with a phasor diagram as shown in Figure 3.3

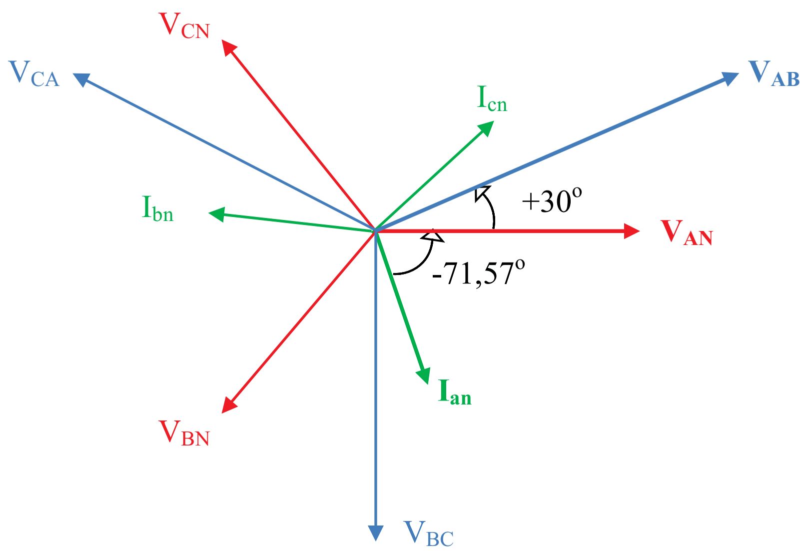

Figure 3.3:

It should be noted that the line voltages (blue) are equal in magnitude and displaced from each other by 120°. The phase voltages (red) are also equal in magnitude and displaced from each other by 120°. Similarly the phase currents (which for a Star connected system are equal to the line currents), the magnitudes are equal, and phase displaced by 120°.

If we now apply Kirchoff's current law at the node "n" in Figure 3.2;

$$I_{an} = 34{,}783\angle{-71{,}57^o} = 10{,}997 - j32{,}999$$

$$I_{bn} = 34{,}783\angle 168{,}43^o = -34{,}076 + j6{,}976$$

$$I_{cn} = 34{,}783\angle 48{,}43^o = 23{,}080 + j26{,}023$$

The sum of the currents is then

$$I_{an} + I_{bn} + I_{cn} = (10{,}997 - j32{,}999) + (-34{,}076 + j6{,}976) + (23{,}080 + j26{,}023)$$

Therefore $I_{Nn} = I_{an} + I_{bn} + I_{cn}$

$$= (10{,}997 - 34{,}076 + 23{,}080) + j(-32{,}999 + 6{,}976 + 26{,}023)$$

$$= (0{,}001) + j(0) \cong 0$$

The significance of this is that because the supply voltage is balanced, and the load is balanced, the phase currents are also balanced. The sum of the phase currents at the neutral connection "n" is therefore zero. The same applies to the neutral connection at the source node "N" (Refer to Figure 3.2). Since there is no current in the neutral connection between the source and the load, in theory it is then not necessary to have any physical connection between the two neutral points. For a balanced load it is then not necessary to have any physical copper connection between the neutral points and both can then be connected directly to earth at a considerable reduction in the cost of the installation. (The earth connection of the neutral point in both the source and the load is necessary so that a path for the current can be provided under abnormal or fault conditions, where the load currents are no longer balanced)

The power consumed by the load in this example is then, for phase A;

$$P_A = V_{an} I_{an} \cos\theta_A = (220)(34{,}783)\cos(71{,}57^o) = 2419\,W$$

Where θ_A is the angle between V_{an} and I_{an}

Similarly for phase B

$$P_B = V_{bn} I_{bn} \cos\theta_B = (220)(34{,}783)\cos(71{,}57^o) = 2419\,W$$

θ_B is the angle between $V_{bn}\ and\ I_{bn} = (-120^o) — (-191{,}57^o) = 71{,}57^o$

And for phase C

$$P_C = V_{cn} I_{cn} \cos\theta_C = (220)(34{,}783)\cos(71{,}57^o) = 2419\,W$$

$$\theta_C = 120^o - 48{,}43^o = 71{,}57^o$$

The total power delivered by the source is then the sum of the power consumed by the three phases;

$$P_{Total} = 7257\,W$$

The total power consumed can also be written as;

$$P_{Total} = 3V_{Phase}I_{Phase}\cos\theta$$

Or in line quantities;

$$P_{Total} = \sqrt{3}V_{Line}I_{Line}\cos\theta$$

Note that for this last equation the voltage and the current are line quantities, but θ is the angle between the phase values. [Stevenson Page 33]

An alternative method to calculate the real power is to remember that real power is consumed by the resistance in the load and so the power consumed is then;

$$P = I^2R = 34{,}783^2(2) = 2419\, W\, per\, phase$$

By using this method it is not possible to choose the wrong angle θ.

Example 2:

In this example we will consider a predominately capacitive load as shown in Figure 3.4. The supply voltage is 220 V per phase, and again we will use V_{AN} as the reference. The supply is balanced and the load is also balanced with an impedance of 2 – j6 Ω per phase. We will determine the currents in each phase and the total power consumed by the load. The phase rotation is ABC. Again it is assumed that the connection between the source and the load has negligible impedance.

If V_{AN} is chosen as the reference and because the source and load are both Star connected with the neutral connected, then clearly;

$$V_{an} = 220\angle 0^o$$

$$V_{bn} = 220\angle {-120^o}$$

$$V_{cn} = 220\angle {+120^o}$$

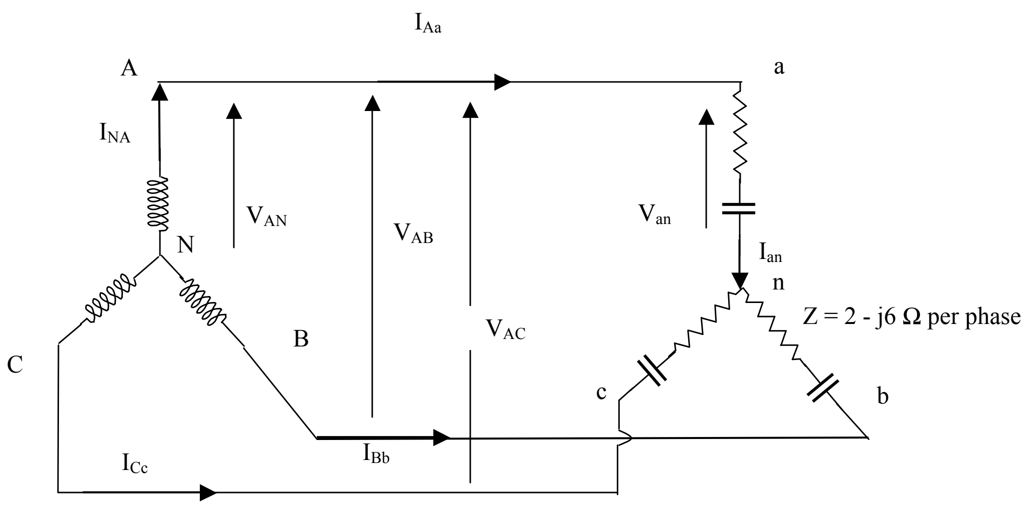

Figure 3.4:

$$I_{an} = \frac{V_{an}}{Z} = \frac{220\angle 0^o}{2 - j6} = \frac{220\angle 0^o}{6{,}325\angle {-71{,}57^o}} = 34{,}783\angle 71{,}57^o\ Amps$$

From example 1 it can be seen that because the load is balanced the phase currents are equal in magnitude but phase displaced by 120°. I_{bn} and I_{cn} can then simply be stated as;

$$I_{bn} = 34{,}783\angle{-48{,}43^o}\ Amps$$
$$I_{cn} = 34{,}783\angle{-168{,}43^o}\ Amps$$

The phasor diagram is shown in Figure 3.5

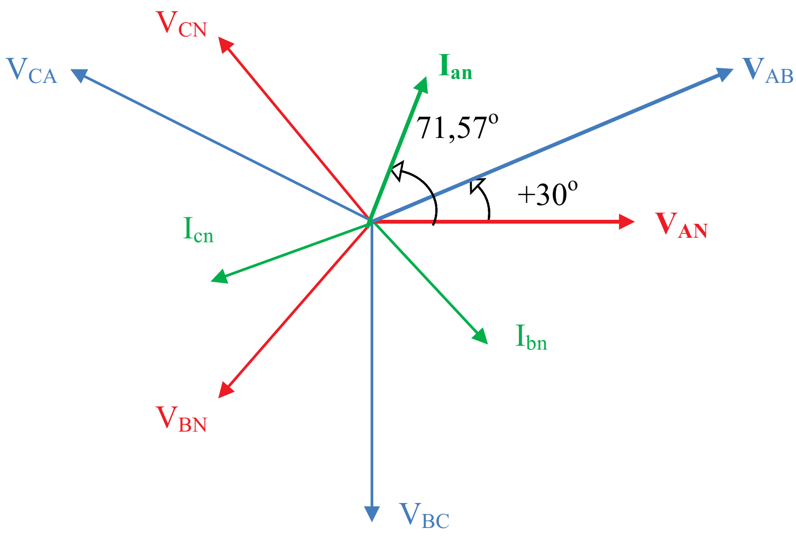

Figure 3.5:

The total real power transferred from the source to the load is then, using phase values;

$$P_{Real} = 3V_{Ph}I_{Ph}\cos\theta = (3)(220)(34{,}783)\cos(71{,}57^o) = 7258\ W$$

Or using line values;

$$P_{Real} = \sqrt{3}V_L I_L\cos\theta = \sqrt{3}\left(220\sqrt{3}\right)(34.788)\cos(71{,}57^o) = 7258\ W$$

Or considering the real value of the impedance in the load;

$$P_{Real} = 3I^2R = (3)(34{,}783^2)(2) = 7259\ W$$

Again the minor discrepancy in the answer is due to rounding errors in the calculation.

If we apply Kirchoff's current law at the node "n" (Figure 3.4) the sum of the three currents is $I_{an} + I_{bn} + I_{cn}$

$$I_{an} = 34{,}783\angle 71{,}57 = 10{,}997 + j32{,}999$$

$$I_{bn} = 34{,}783\angle -48{,}43 = 23{,}079 - j26{,}023$$

$$I_{cn} = 34{,}783\angle -168{,}43 = -34{,}076 - j6{,}976$$

And so $I_{nN} = (10{,}997 + j32{,}999) + (23{,}079 - j26{,}023) + (-34{,}076 - j6{,}976)$

$$I_{nN} = (10{,}997 + 23{,}079 - 34{,}076) + j(32{,}999 - 26{,}023 - 6{,}976) = 0 + j0$$

Again it can be seen that for a balanced load the neutral current is zero.

Example 3:

In the previous two examples it was assumed that the impedance between the source and the load was negligible. In normal electrical engineering practice this is not always a good assumption because the electrical connection between the source and the load can often exceed a distance of 100 m, if not longer, and so the impedance of this connection should not be ignored. In this example we will consider the effect of the impedance of the connection between the source and the load. We will consider the same predominately capacitive load as in example 2, but where the connection from the source to the load (such as a transmission line) has an impedance of 2 + j6 Ω. It can be seen that the inductive reactance of the line has the same capacitive reactance of the load and so the circuit becomes resonant. Again in this example the load is balanced and so the neutral current is zero, so the impedance of this connection can be ignored because there is no current flowing in this part of the circuit. The line impedance has a significant impact on the real power transferred from the source to the load and the calculations will illustrate the importance of power factor correction.

The circuit is shown in Figure 3.6

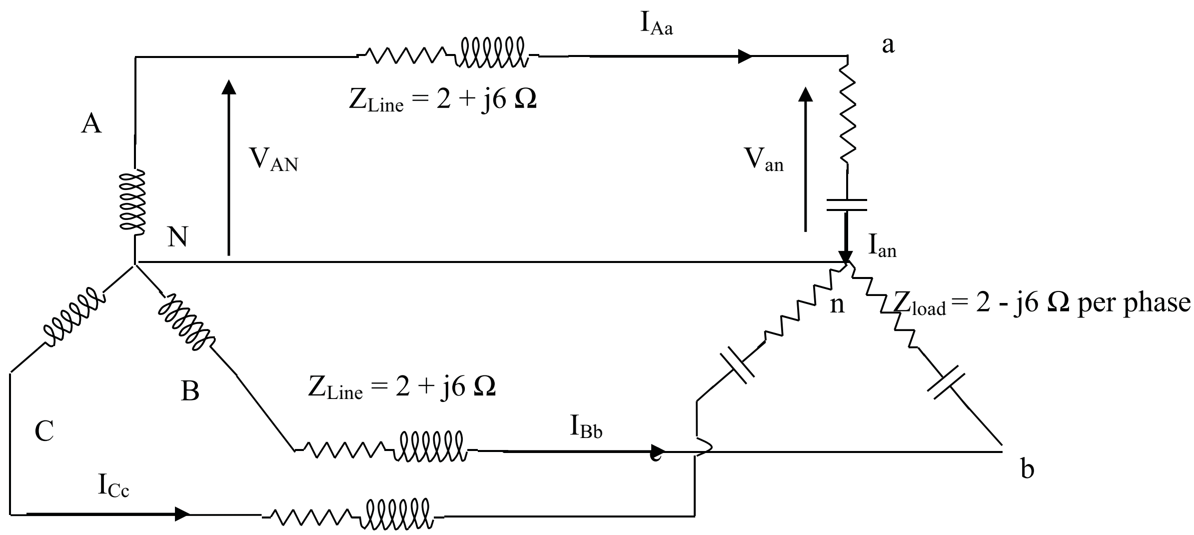

Figure 3.6

If the source is 220 V per phase, and using V_{AN} as the reference;

$$V_{AN} = 220\angle 0^o$$

$$V_{BN} = 220\angle{-120^o}$$

$$V_{CN} = 220\angle 120^o$$

Because the load is Star connected and balanced the total impedance per phase is then a simple series connection. The current in each phase is then;

$$I_{Aa} = I_{an} = \frac{V_{AN}}{Z_a} = \frac{220\angle 0^o}{(2+j6)+(2-j6)} = \frac{220\angle 0^o}{4\angle 0^o} = 55\angle 0^o\ Amps$$

$$I_{bn} = 55\angle{-120^o}\ Amps$$

$$I_{cn} = 55\angle{+120^o}\ Amps$$

The phasor diagram is shown in Figure 3.7

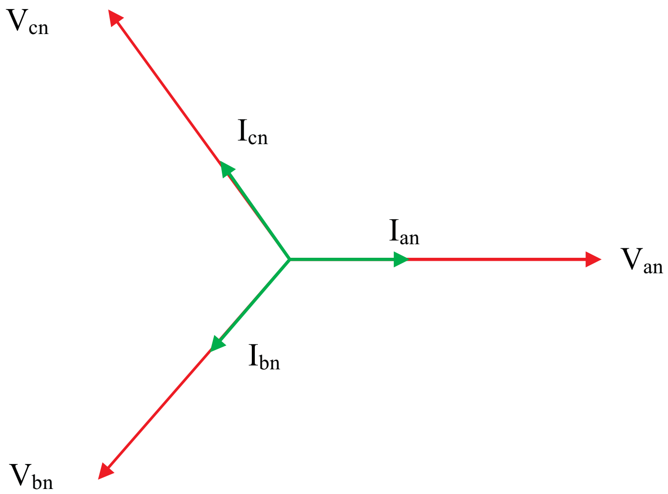

Figure 3.7:

The total power transferred from the source to the load is then;

$$P_{Total} = 3(220)(55)\cos 0^o = 36300\ W$$

By comparing the total real power transferred from the source to the load in examples 2 and 3 it can be seen that there is a significant difference, and this difference is due to the change in the reactance of the load. In example 2 the reactive power was being transferred back and forth between the source and the load, whereas in example 3 the reactive power was being transferred back and forth between the inductance of the line and the capacitance of the load.

3.4 EXERCISES:

3.4.1 A three phase 4 wire system with a line voltage of 208 volts is connected to a balanced Star connected load of $Z = 20\angle{-30^o}$. The phase rotation is CBA. Given that

$V_{AN} = 120\angle{-90^o}$, determine the line currents in the circuit.

3.4.2 Three identical impedances of $5\angle{-30^o}$ are connected to a three phase system with a line voltage of 150 V. The phase rotation is CBA. The neutral between the source and the load is not connected. Determine the total real power consumed by the circuit. It is known that $V_{AN} = \frac{150}{\sqrt{3}}\angle{-90^o}$

3.4.3 A balanced Star connected source has a phase voltage of 220 V per phase. The phase rotation is ABC. The source is connected to a balanced Star load with an impedance of 4 + j3 Ω per phase. The neutral points are connected. Using V_{AN} as the reference, determine the line currents, the neutral current and the total real power delivered by the source.

4 BALANCED DELTA CONNECTED THREE PHASE CIRCUITS

4.1 OBJECTIVE:

The aim of this chapter is to show how a Delta connected balanced three phase circuit is analysed. It will show the advantages of a Delta connection and explain the application of a Star-Delta transformation.

4.2 BALANCED DELTA CONNECTED LOAD:

A Delta connected load has the advantage of being a three wire system and so only three conductors are needed to connect the source to the load, which reduces the cost. Since no neutral connection is required the source could be either Star or Delta connected. The connection of the source has no effect on how the load should be analysed. It will therefore be shown simply as phase A, B, and C with the phase rotation being ABC. This is illustrated in Figure 4.1

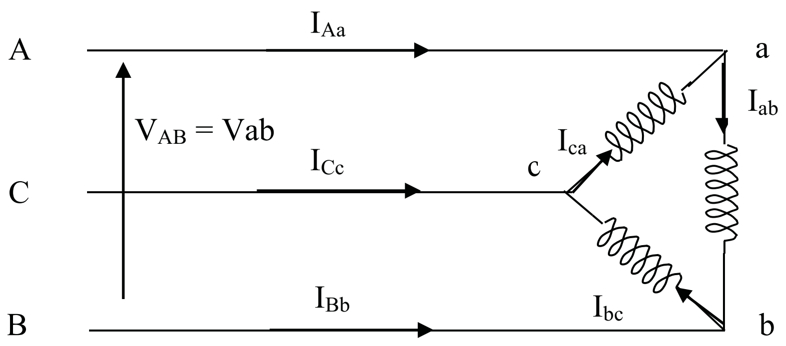

Figure 4.1:

The load in this case is represented by a pure inductance because a very common balanced Delta connected load is the stator winding of an ideal AC motor. The purpose of the winding is to convert the electrical current into a rotating magnetic field, which in turn causes the rotor to follow the rotating magnetic field.

It can be seen from the diagram that $V_{AB} = V_{ab}$. It can also be seen that the line currents and the phase currents are not equal, as was the case for a Star connected load. Not all balanced Delta loads are however purely inductive, which will be illustrated in Example 1, shown in figure 4.2.

Example 1:

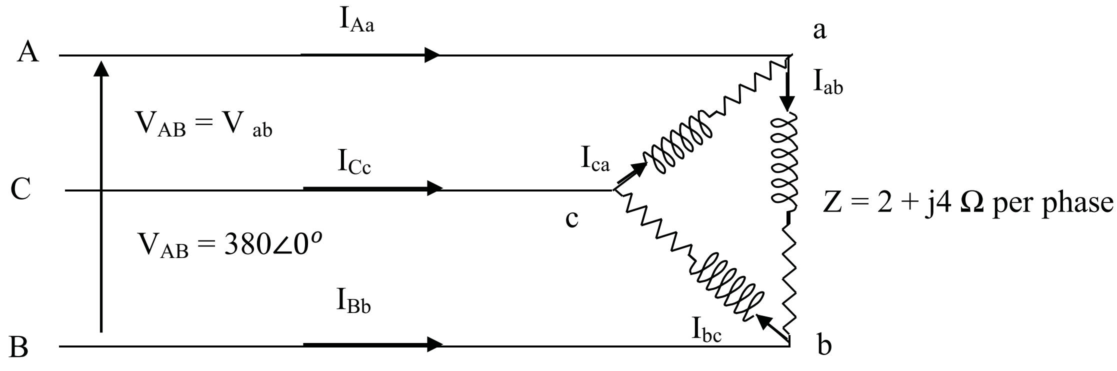

Figure 4.2

Using V_{AB} as the reference;

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{380\angle 0^o}{2+j4} = \frac{380\angle 0^o}{4{,}472\angle 63{,}4^o} = 84{,}973\angle{-63{,}4^o}$$

Knowing that the load is balanced, the magnitude of the currents I_{bc} and I_{ca} must be equal but displaced by 120^o from each other. Therefore

$$I_{bc} = 84{,}973\angle{-183{,}4^o}$$

$$I_{ca} = 84{,}973\angle 56{,}6^o$$

Using Kirchoff's current law, the line current I_{Aa} is then the sum of the currents at node "a", so

$$I_{Aa} = I_{ab} - I_{ca} = (84{,}973\angle{-63{,}4^o}) - (84{,}973\angle 56{,}6^o)$$

$$I_{Aa} = (38{,}047 - j75{,}979) - (46{,}776 + j70{,}939)$$

$$I_{Aa} = (38{,}047 - 46{,}776) - j(70{,}939 - 75{,}979)$$

$$I_{Aa} = -8{,}729 - j146{,}918$$

$$I_{Aa} = 147{,}177\angle{-93{,}4^o}$$

Because the load is balanced it is then clear that the magnitude of the currents I_{Bb} and I_{Cc} will be equal but displaced by 120^o. So;

$$I_{Bb} = 147{,}177\angle 146{,}6^o, \text{ and}$$

$$I_{Cc} = 147{,}177\angle 26{,}6^o$$

The phasor diagram shown in Figure 4.3 illustrates the rotating vectors.

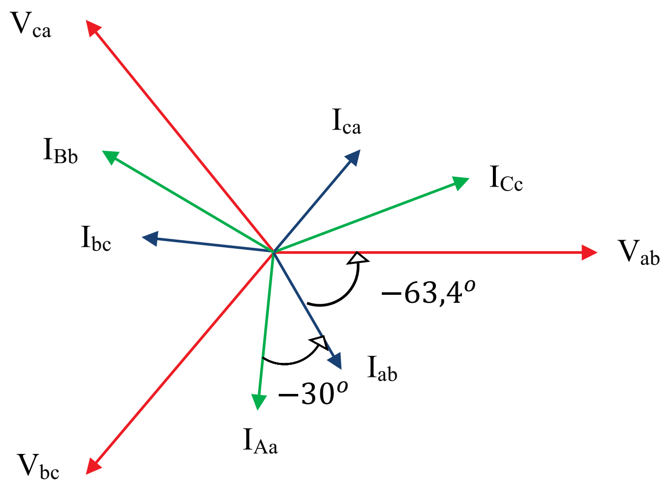

Figure 4.3

From this analysis it can be seen that the phase shift between the currents I_{ab} and I_{Aa} is 30^o and the ratio of the magnitudes of the line current to the phase current is $\frac{147,177}{84,973} = \sqrt{3}$. The reason for this was explained in Chapter 1. Refer to Figure 1.11

The total real power consumed is then;

$$P_{Total} = (3)(I_{Ph}^2)(R) = (3)(84,973^2)(2) = 43,2\ kW$$

Or using phase values;

$$P_{Total} = (3)(V_{Ph})(I_{Ph}) \cos\theta = (3)(380)(84,973)\cos(-63,4^o) = 43,4\ kW$$

Or using line values;

$$P_{Total} = \sqrt{3}(V_L)(I_L)\cos\theta = \sqrt{3}(380)(147,177)\cos(-63,4^o) = 43,4\ kW$$

The small difference in the answer is due to rounding errors in the trigonometry.

It should be noted that the phase shift of 30^o and the ratio of $\sqrt{3}$ is only applicable to a balanced load. For an unbalanced load the approach is different which will be explained in the next chapter of this book.

4.3 DELTA-STAR TRANSFORMATION

The previous example did not take the source impedance into account. In Chapter 3 it was shown that for a Star connected load the source impedance is in series with the load. For a Delta connected load it is not that simple. Example 2 will illustrate how this can be done for a Delta connected load. To simplify the analysis it will be assumed that the impedance is purely resistive. This simplifies the mathematics involved so the student can focus on the principle rather than the complex arithmetic involved. Figure 4.4 shows a Delta connected system and an equivalent Star connected system.

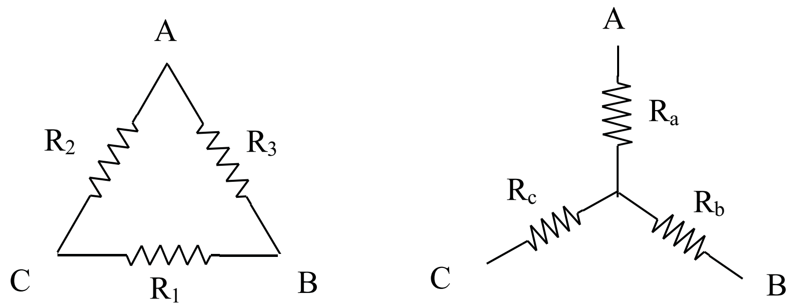

Figure 4.4:

For the Star connection to be the equivalent of the Delta connection, the resistance between A and B must be the same (Hughes 2002, p 81). In the Delta connection;

$$R_{AB} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

In the Star connection;

$$R_{AB} = R_a + R_b$$

So;

$$R_a + R_b = \frac{R_3R_1 + R_3R_2}{R_1 + R_2 + R_3}$$

Similarly;

$$R_b + R_c = \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$$

And;

$$R_c + R_a = \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

By solving these last three simultaneous equations we get;

$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$
$$R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$
$$R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

Example 2:

Consider a Delta connected balanced load with an impedance of 9 Ω per phase. The source impedance is 1 Ω. The supply is 380 V. V_{AB} will be used as the reference.
The circuit is shown in Figure 4.5

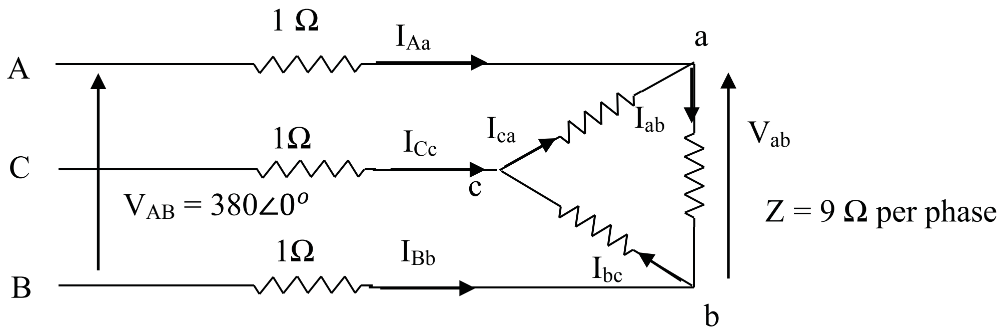

Figure 4.5:

In this case V_{AB} is not equal to V_{ab} due to the volt drop across the 1 Ω source impedance. To determine the volt drop across this resistance I_{Aa} needs to be known. Since $I_{Aa} = I_{ab} - I_{ca}$ and V_{ab} needs to be known to determine I_{ab}, and hence I_{Aa} , there are too many variables to determine the currents and voltages needed. An alternative method is required.

A method that can be used is the Delta-Star transformation, where the Delta connection is represented by an equivalent Star connection, and the source impedance is then is series with the Star equivalent impedance. This is shown in Figure 4.

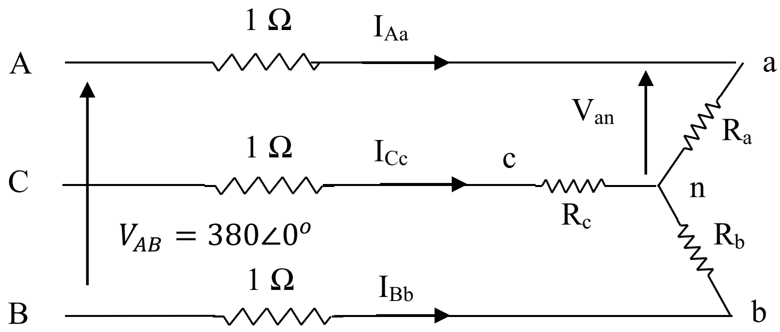

Figure 4.6:

From the Delta-Star transformation;

$$R_a = \frac{(9)(9)}{(9+9+9)} = 3\,\Omega$$

Similarly $R_b = R_c = 3\,\Omega$

The source impedance is now in series with the load impedance, and the analysis of the circuit is the same as was shown in Chapter 3.

4.4 EXERCISES

4.4.1 A balanced three phase supply is connected to a balanced three phase Delta connected load of $6 - j4\ \Omega$.The supply has a line voltage of 380 V. Using V_{AB} as the reference, determine the phase currents and the line currents.

4.4.2 A three wire, three phase, 100 V system supplies a balanced Delta connected load with an impedance of $20\angle 45^o\Omega$. The phase rotation is ABC. Using V_{BC} as the reference, determine the line currents and the total real power consumed.

4.4.3 Three identical impedances of $30\angle 30^o\Omega$ are connected in Delta to a three phase, three wire 208 V system. The conductors connecting the source to the load have an impedance of $0{,}8 + j0{,}6\ \Omega$. Calculate the magnitude of the line voltage at the load.

5 UNBALANCED THREE PHASE LOADS

5.1 OBJECTIVE:

The purpose of this chapter is to explain how an unbalanced three phase load should be analysed. An unbalanced Delta connected load will be analysed, as well as unbalanced Star connected loads, with and without a neutral connection. The use of mesh current analysis will also be demonstrated.

5.2 UNBALANCED DELTA CONNECTED CIRCUITS:

Unbalanced Delta connected loads are not common. An example of an unbalanced Delta load is in the motor industry where spot welding is done. In this case the higher line voltage is used because very high currents are needed for the welding process. Not all spot welding machines operate at the same time and this leads to an unbalanced load. Another cause of an unbalanced Delta load is when one of the supply phases to an electrical motor is lost due to a fault on the supply system. This is also known as "single phasing".

It should be noted that although the supply is balanced, the load is not so the ratio of the line current to the phase current is not $\sqrt{3}$ and the phase shift is not 30^o.

Example 1:

Figure 5.1 shows a balanced supply connected to an unbalanced Delta connected load. The phase rotation is ABC. The supply has a line voltage of 380 V and we will use V_{AB} as the reference. The assumed direction of the currents within the Delta connection is shown in the diagram. A negative answer in the calculation indicates that the assumed direction was wrong, but the magnitude is correct. The same applies to the line currents. The impedances in the three phases are;

$$Z_{ab} = 2 + j1 = 2{,}24\angle 26{,}6^o\ \Omega$$

$$Z_{bc} = 3 + j1 = 3{,}16\angle 18{,}4^o\ \Omega$$

$$Z_{ca} = 4 + j2 = 4{,}47\angle 26{,}6^o\ \Omega$$

Since the supply is balanced;

$$V_{AB} = V_{ab} = 380\angle 0^o$$

$$V_{BC} = V_{bc} = 380\angle -120^o$$

$$V_{CA} = V_{ca} = 380\angle +120^o$$

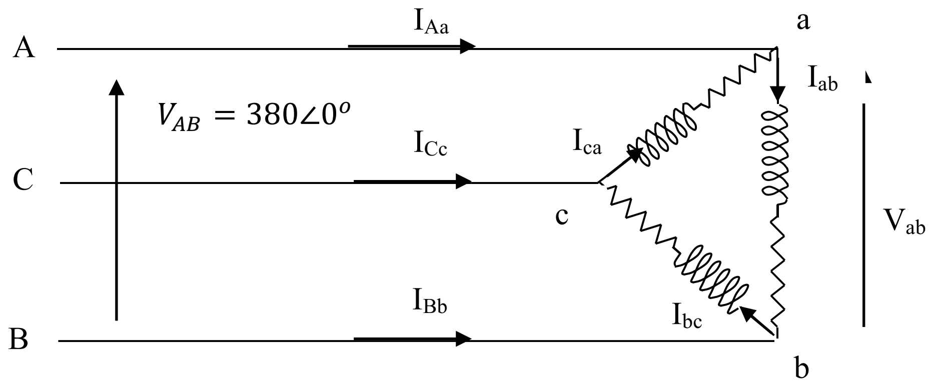

Figure 5.1

The phase currents are then;

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{380\angle 0^o}{2{,}24\angle 26{,}6^o} = 169{,}64\angle -26{,}6^o = 151{,}68 - j75{,}96\ Amps$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{380\angle -120^o}{3{,}16\angle 18{,}4^o} = 120{,}25\angle -138{,}4^o = -89{,}92 - j79{,}84\ Amps$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{380\angle 120^o}{4{,}47\angle 4{,}47^o} = 85{,}01\angle 115{,}5^o = -36{,}60 + j76{,}73\ Amps$$

To determine the line currents we then apply Kirchoff's current law at the nodes a,b and c

Then $I_{Aa} = I_{ab} - I_{ca} = (151{,}68 - j75{,}96) — (-36{,}60 + j76{,}73)$

$$I_{Aa} = 188{,}68 - j152{,}69 = 242{,}72\angle -38{,}98^o$$

$$I_{Bb} = I_{bc} - I_{ab} = (-89{,}92 - j76{,}73) - (151{,}68 - j75{,}96)$$

$$I_{Bb} = -241{,}60 - j0{,}77 = 241{,}60\angle -179{,}8^o$$

$$I_{Cc} = I_{ca} - I_{bc} = (-36{,}60 + j76{,}73) — (-89{,}92 - j79{,}84)$$

$$I_{Cc} = -126{,}52 + j156{,}57 = 201{,}30\angle 128{,}94^o$$

The phasor diagram is illustrated in figure 5.2

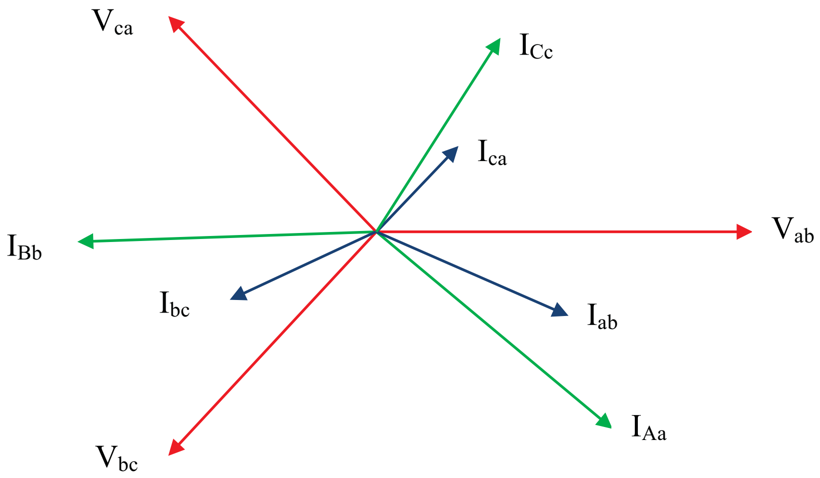

Figure 5.2:

It should be noted that although the supply voltage is balanced with the three vectors equal in magnitude and displaced from each other by 120°, the phase currents are not equal in magnitude, nor displaced from each other by 120°. The ratio of the line current to the phase current is also not $\sqrt{3}$. This is because the load is not balanced.

In this case the total power consumed needs to be calculated for each individual phase.

So;

$$P_{ab} = I_{ab}^2 R_{ab} = (169{,}64^2)(2) = 57{,}56\ kW$$

$$P_{bc} = I_{bc}^2 R_{bc} = (120{,}25^2)(3) = 43{,}38\ kW$$

$$P_{ca} = I_{ca}^2 R_{ca} = (85{,}01^2)(4) = 28{,}91\ kW$$

The total power consumed is then;

$$P_{Total} = P_{ab} + P_{bc} + P_{ca} = 129{,}85\ kW$$

5.3 UNBALANCED STAR CONNECTED LOADS:

A common unbalanced Star connected load is the electrical distribution to domestic houses. For safety reasons each house needs to have a live, neutral and earth connection so a Star connected supply is needed. Not all households consume the same amount of power at the same time, so the load is not balanced.

Figure 5.3 shows a typical supply to domestic premises;

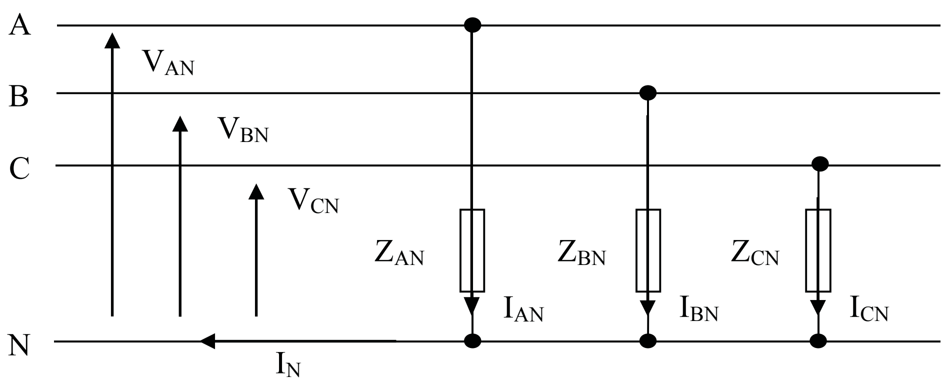

Figure 5.3:

Example 2:

Referring to figure 5.3, we will use V_{AN} as the reference and the following quantities to analyse the circuit. The supply is 220 V per phase. The impedance of each phase is;

$$Z_{AN} = 2 + j1 = 2{,}24\angle 26{,}6^o\ \Omega$$

$$Z_{BN} = 3 + j1 = 3{,}16\angle 18{,}4^o\ \Omega$$

$$Z_{CN} = 4 + j2 = 4{,}47\angle 26{,}6^o\ \Omega$$

Again we will assume that the supply is balanced, so;

$$V_{AN} = 220\angle 0^o\ Volts$$

$$V_{BN} = 220\angle -120^o\ Volts$$

$$V_{CN} = 220\angle 120^o\ Volts$$

The phase currents are then;

$$I_{AN} = \frac{V_{AN}}{Z_{AN}} = \frac{220\angle 0^o}{2{,}24\angle 26{,}6^o} = 98{,}21\angle -26{,}6^o\ Amps$$

$$I_{BN} = \frac{V_{BN}}{Z_{BN}} = \frac{220\angle -120^o}{3{,}16\angle 18{,}4^o} = 69{,}62\angle -138{,}4^o\ Amps$$

$$I_{CN} = \frac{V_{CN}}{Z_{CN}} = \frac{220\angle 120^o}{4{,}47\angle 26{,}6^o} = 49{,}22\angle 93{,}4^o\ Amps$$

The neutral current (I_N) is then the sum of the three phase currents.

$$I_N = I_{AN} + I_{BN} + I_{CN}$$

$$I_N = (98{,}21\angle -26{,}6^o) + (69{,}62\angle -138{,}4^o) + (49{,}22\angle 93{,}4^o)$$

$$I_N = (87{,}81 - j43{,}97) + (-52{,}06 - j46{,}22) + (-2{,}9 + j49{,}13)$$

$$I_N = 32{,}85 - j41{,}06 = 52{,}58\angle -51{,}3^o\ Amps$$

The total real power consumed is then;

$$P_{Total} = P_{AN} + P_{BN} + P_{CN}$$

$$P_{Total} = I_{AN}^2 R_{AN} + I_{BN}^2 R_{BN} + I_{CN}^2 R_{CN}$$

$$P_{Total} = (98{,}21^2)(2) + (69{,}62^2)(3) + (49{,}22^2)(4)$$

$$P_{Total} = 19290 + 14541 + 9690$$

$$P_{Total} = 43{,}521\ kW$$

5.4 UNBALANCED STAR CONNECTED LOAD WITHOUT A NEUTRAL CONNECTION:

In the case of a balanced Star connected load we have shown that the neutral current is zero, so even though there is no neutral connection,

$$V_{ab} = \frac{V_{AB}}{\sqrt{3}} \angle{-30^o}$$

However if the load is not balanced this equation does not apply. An alternative method should be used. In this case we will apply Kircoff's laws to the circuit. (Boylestad 2007, p. 1054)

We define the three circulating currents in the circuit to be I_1, I_2 and I_3 as shown in Figure 5.4.

It is assumed that the supply voltage is balanced and the magnitude is known. It is also assumed that the values of Z_1, Z_2 and Z_3 are known.

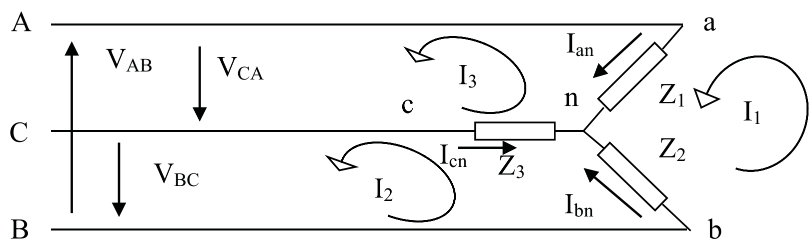

Figure 5.4

We then apply Kirchoff's voltage law to the three current loops;

Loop 1:

$$V_{AB} = (Z_1)(I_1 - I_3) + (Z_2)(I_1 - I_2)$$

From Figure 5.4 it can be seen that;

$$(I_1 - I_3) = I_{an}$$

$$(I_1 - I_2) = -I_{bn}$$

$$(I_2 - I_3) = -I_{cn}$$

And so;

$$V_{AB} = Z_1 I_{an} - Z_2 I_{bn} \qquad \text{Eqn (1)}$$

Loop 2:

$$V_{BC} = (Z_2)(I_2 - I_1) + (Z_3)(I_2 - I_3)$$

$$V_{BC} = Z_2 I_{bn} - Z_3 I_{cn} \qquad \text{Eqn (2)}$$

Loop 3:

$$V_{CA} = (Z_3)(I_3 - I_2) + (Z_1)(I_3 - I_1)$$

$$V_{CA} = Z_3 I_{cn} - Z_1 I_{an} \qquad \text{Eqn (3)}$$

In these three equations the unknowns are I_{an}, I_{bn} and I_{cn}. By solving these three simultaneous equations it can be shown that;

$$I_{an} = \frac{V_{AB}Z_3 - V_{CA}Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_{bn} = \frac{V_{BC}Z_1 - V_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_{cn} = \frac{V_{CA}Z_2 - V_{BC}Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Example 3:

In this example we will consider the case where the impedance is purely resistive. This simplifies the mathematics involved so the student can focus on the principle rather than the complex arithmetic involved.

Consider Figure 5.4. The supply line voltage is balanced at 380 V. The impedances per phase are $Z_1 = 2\ \Omega$; $Z_2 = 3\ \Omega$; $Z_3 = 5\ \Omega$. The phase rotation is ABC and we will use V_{AB} as the reference.

Determine the total power consumed.

$$V_{AB} = 380\angle 0^o\ Volts$$

$$V_{BC} = 380\angle{-120^o} Volts$$

$$V_{CA} = 380\angle{+120^o} Volts$$

$$I_{an} = \frac{V_{AB}Z_3 - V_{CA}Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} Amps$$

$$I_{an} = \frac{(380\angle 0^o)(5) - (380\angle{-120^o})(3)}{(2)(3) + (2)(5) + (3)(5)} Amps$$

$$I_{an} = \frac{(1900\angle 0^o) - (1140\angle{-120^o})}{31} Amps$$

$$I_{an} = (61{,}29\angle 0^o) - (36{,}77\angle{-120^o})Amps$$

$$I_{an} = (61{,}29 + j0) — (-18{,}39 - j31{,}84)Amps$$

$$I_{an} = 79{,}68 + j31{,}84 = \mathbf{85{,}81\angle 21{,}78^o}Amps$$

Similarly;

$$I_{bn} = \frac{V_{BC}Z_1 - V_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}Amps$$

$$I_{bn} = \frac{(380\angle{-120^o})(2) - (380\angle 0^o)(5)}{31}Amps$$

$$I_{bn} = (24{,}52\angle{-120^o}) - (61{,}29\angle 0^o)Amps$$

$$I_{bn} = (-12{,}26 - j21{,}23) - (61{,}29 + j0)$$

$$I_{bn} = -73{,}55 - j21{,}23 = \mathbf{76{,}55\angle{-163{,}9^o}}Amps$$

$$I_{cn} = \frac{V_{CA}Z_2 - V_{BC}Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_{cn} = \frac{(380\angle 120^o)(3) - (380\angle{-120^o})(2)}{31}$$

$$I_{cn} = (36{,}77\angle 120^o) - (24{,}52\angle{-120^o})$$

$$I_{cn} = (-18{,}39 + j31{,}84) — (12{,}26 - j21{,}23)$$

$$I_{cn} = (-30{,}65 + j10{,}61) = \mathbf{32{,}43\angle{-19{,}1^o}}Amps$$

The power consumed in each phase is then;

$$P_{an} = (I_{an}^2)R_1 = (85{,}81^2)(2) = 14\,727\;W$$

$$P_{bn} = (I_{bn}^2)(R_2) = (76{,}55^2)(3) = 17\,580\;W$$

$$P_{cn} = (I_{cn}^2)(R_3) = (32{,}43^2)(5) = 5\,259\;W$$

The total power consumed by the circuit is then;

$$P_{Total} = P_{an} + P_{bn} + P_{cn} = (14\,727 + 17\,580 + 5\,259) = \mathbf{37,566}\boldsymbol{kW}$$

5.5 EXERCISES:

5.5.1 A balanced three phase 380 V supply is connected to an unbalanced Delta connected load. The impedances of the load are $Z_{ab} = 6 + j4\Omega$, $Z_{bc} = 8 + j2\Omega$, and $Z_{ca} = 8 - j4\Omega$. Using V_{AB} as the reference, determine the line currents in the system.

5.5.2 A balanced three phase four wire supply is connected to an unbalanced three phase Star connected load. The neutral points are connected. The supply is 220 V per phase. The load impedances are $Z_{an} = 6 + j4\Omega$, $Z_{bn} = 8 + j2\Omega$, and $Z_{cn} = 8 - j4\Omega$. Using V_{AN} as the reference, determine the total power consumed.

5.5.3 For the same circuit described in exercise 5.5.2, calculate the neutral current in the circuit.

REFERENCES

1. Hughes, E 2002, Electrical and Electronic Technology, 8th edition, Prentice Hall, Essex.
2. Boylestad, R 2007, Introductory Circuit Analysis, 11th edition, Prentice Hall, New Jersey.
3. Stevenson, WD 1975, Elements of Power System Analysis, 3rd edition, McGraw Hill, Kogakusha
4. Edminister, JA 1972, Electric Circuits, 1st edition, McGraw Hill, New York

ANNEXURE 1

ANSWERS TO EXERCISES

Chapter 1:

1.4.1 60 Hz

1.4.2 110 V, 60 Hz

1.4.3 3 A, 36 W

1.4.4 $V_{CA} = 190\angle + 120^o\ Volts$, 60Hz, 16,67ms

Chapter 2:

2.5.1 $P_{Real} = VI\cos\Omega.$

From Ohm's law $V = IZ$.

So $P_{Real} = I^2 Z\cos\theta$

From the impedance diagram;

$Z\cos\theta = R$

So $P_{Real} = I^2 R$

2.5.2 $Z_{Eq} = 8{,}94\angle 63{,}4^o$

2.5.3 $Z_{eq} = 124{,}2\angle{-14{,}9^o}$

2.5.4 $P_{Real} = 100{,}4$ W; $P_{Reactive} = 13$ VAR; $P_{Apparent} = 101{,}2$ VA

2.5.5 $P_{Real} = 91{,}96$ W; $P_{Reactive} = 30{,}98$ VAR; $P_{Apparent} = 96{,}8$ VA

Chapter 3:

3.4.1 $I_{AN} = 6{,}0\angle{-60^o}$; $I_{BN} = 6{,}0\angle 60^o$; $I_{CN} = 6{,}0\angle 180^o$; $I_{Neutral} = 0$
(Edminister 1972, p. 198)

3.4.2 1299 W (Edminister 1972, p. 208)

3.4.3 $I_{an} = 44\angle{-36{,}87^o}$Amps; $I_{bn} = 44\angle{-156{,}87^o}Amps$; $I_{cn} = 44\angle{+83{,}13^o}Amps$; $I_N = 0$
$P_{Real} = 23\ 232\ W$

Chapter 4:

4.4.1 $I_{ab} = 52,7\angle 33,69^o\ Amps;\ I_{bc} = 52,7\angle -86,31^o\ Amps;\ I_{ca} = 52,7\angle 153,69^o\ Amps$
$I_{Aa} = 91,28\angle 3,7^o$ Amps; $I_{Bb} = 91,28\angle -116,3^o\ Amps;\ I_{Cc} = 91,28\angle 123,7^o\ Amps$

4.4.2 $I_A = 8,66\angle 45^o\ Amps;\ I_B = 8,66\angle -75^o\ Amps;\ I_C = 8,66\angle 165^o\ Amps;\ P_{Real} = 1061W$

4.4.3 $V_{Line} = 189\ Volts$ (Edminister 1972, p 215)

Chapter 5:

5.5.1 $I_{Aa} = 95,21\angle -33,57^o;\ I_{Bb} = 75,97\angle -177,007^o;\ I_{Cc} = 56,64\angle 93,5^o$

5.5.2 16 120 W

5.5.3 $I_N = 26,35\angle -121,23^o\ Amps$