The World of Modigliani and Miller

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Part One:
An Introduction

1 An Overview

Introduction

Financial analysis has never been an exact science. Occasionally, the theoretical models upon which it is based are even "bad" science. The root cause is that economic decisions undertaken in a *real* world of uncertainty are invariably characterised by *hypothetical* human behaviour, for which there is little *empirical* evidence. Thus, a financial model may satisfy a fundamental requirement of all theory construction. It is based on *logical* reasoning. But if the *objectives* are too divorced from *reality*, or underpinned by *simplifying assumptions* that rationalise *complex phenomena*, the *analytical conclusions* may be invalid.

Nevertheless, all theories, whether bad or good, still serve a useful role.

- At worst, they provide a benchmark for future development to overcome their deficiencies, which may require correction, or even a thorough revision of objectives.
- At best, they serve to remind us that the ultimate question is not whether a theory is an abstraction of the real world. But does it work?

The purpose of this study is to illustrate the development of basic financial theory and what it offers, with specific reference to the seminal work of two Nobel Prize economists who came to prominence in the 1950s and have dominated the world of finance ever since:

Franco Modigliani (1918–2003)
Merton H. Miller (1923–2000)

The text's inspiration is based on readership feedback from my *bookboon* series, which welcomed various explanations of Modigliani and Miller's controversial hypothesis that *identical* financial assets (for example, two companies, their individual shares, or capital projects) cannot be valued and traded at *different* prices.

Many readers also mentioned that this application of the economic "law of one price", which permeates the series, concerning the *irrelevance* of dividend policy, capital structure and its portfolio theory implications, should be published in a single volume to focus their studies.

I agree, whole-heartedly.

All too often, throughout my academic career, I have observed that Modigliani and Miller's body of work is a "wall of worry" that finance students must climb when revising for examinations. Consequently, it is frequently regarded as a topic best avoided (even though it crops up in different questions) and is soon forgotten when they enter the real world of work.

If you don't want to fall into this trap, let us therefore return to first principles and remind ourselves of some significant developments in modern finance theory, which predate Modigliani and Miller, concerning its objectives, assumptions and conclusions.

Having set the scene, we can then evaluate the positive theoretical contribution of Modigliani and Miller (MM henceforth) to the academic debate and what it offers as a springboard for sound financial analysis.

As we shall discover, no one should doubt that MM's original conclusions are logically conceived, given their rigorous theoretical assumptions. The question we can then address in this text's subsequent Exercise companion is the extent to which MM's theoretical conclusions still apply, once their basic assumptions are relaxed to introduce greater realism and subsequent empirical research.

1.1 The Foundations of Finance: An Overview

Today, most theorists still begin their analyses of corporate investment and financial behaviour with the following over-arching *normative* objective.

> The maximisation of shareholders' wealth, using ordinary share price (common stock) as a universal metric, based on a managerial interpretation of their "rational" and "risk-averse" expectations (by which we mean the receipt of more money rather than less, and more money earlier).

Management model shareholder expectations using the "time value of money" concept (the value of money over time, irrespective of inflation) determined by borrowing-lending rates. Using net present value (NPV) maximisation techniques, their strategy is to invest in a portfolio of capital projects that delivers the "highest *absolute* profit at minimum risk".

This model has a long-standing academic pedigree.

It begins with the "Separation Theorem" of Irving Fisher (1930) that assumes *perfect* capital markets, characterised by perfect knowledge, freedom of information and "no barriers to trade" (for example, innumerable investors, uniform borrowing-lending rates, tax neutrality and zero transaction costs).

Subject to the constraint that management's discount rate for project appraisal at least equals the shareholders' *opportunity* cost of capital (or desired return) to be earned elsewhere on comparable investments of equivalent risk:

- The wealth and consumption (dividend) preferences of all shareholders are satisfied by the managerial investment and financing policies of the company that they own.

By definition, because *perfect* markets are also *efficient*, whereby market participants (including management) respond *instantaneously* to events as they unfold, it follows, that:

- Shares should always be correctly priced at their *intrinsic* true value.
- All shareholders earn a return commensurate with the risk of their investment and so wealth is maximised.

Decades later, Fisher's analysis and specifically the importance of his investment constraint, were formalised by the "Agency Theory" of Jenson and Meckling (1976). They explained that even though corporate (shareholder) *ownership* is divorced from managerial *control*:

> The *agent* (management) motivated by self-preservation should always act in the best interests of the *principal* (shareholder). Otherwise, any failure to satisfy shareholder expectations may result in their replacement.

The Efficient Market Hypothesis (EMH) of the Nobel Prize winning Laureate Eugene Fama (1965) also lent further credence to Fisher's Separation Theorem. As he observed, history tells us that capital markets or not "perfect". For example, access to information may incur costs and there are barriers to trade. But if we assume that they are "reasonably efficient":

> The consequence of decisions undertaken by management on behalf of their shareholders (the *agency* principle) will eventually be communicated to market participants. So, share price adjusts quickly but not instantaneously to a new *equilibrium* value in response to "technical" and "fundamental" analyses of historical data, current events and trending media news.

1.2 The Development of Financial Analysis

As a convenient benchmark for subsequent analyses and critiques of modern finance theory, all the texts in my *bookboon* series begin with this *idealised* picture of market behaviour.

The *majority* of investors are rational and risk-averse, motivated by *self-interest*, operating in *reasonably* efficient capital markets characterised by a *relatively* free flow of information and *surmountable* barriers to trade.

If we also assume a world of certainty, where future events can be specified in advance, it follows that investors can formally analyse one course of action in relation to another for the purpose of wealth maximisation with confidence.

For an *all-equity* firm financed by ordinary shares (common stock) summarised in Figure 1.1 below, where the ownership of corporate assets is divorced from control (the *agency* principle), we can formally define and model the *normative* goal of strategic financial management under conditions of certainty as:

- The implementation of optimum investment and financing decisions using net present value (NPV) maximisation techniques to generate the highest money profits from all a firm's projects in the form of retentions and distributions. These should satisfy the firm's *existing* owners (a multiplicity of shareholders) and *prospective* equity investors who define the capital market, thereby maximising share price.

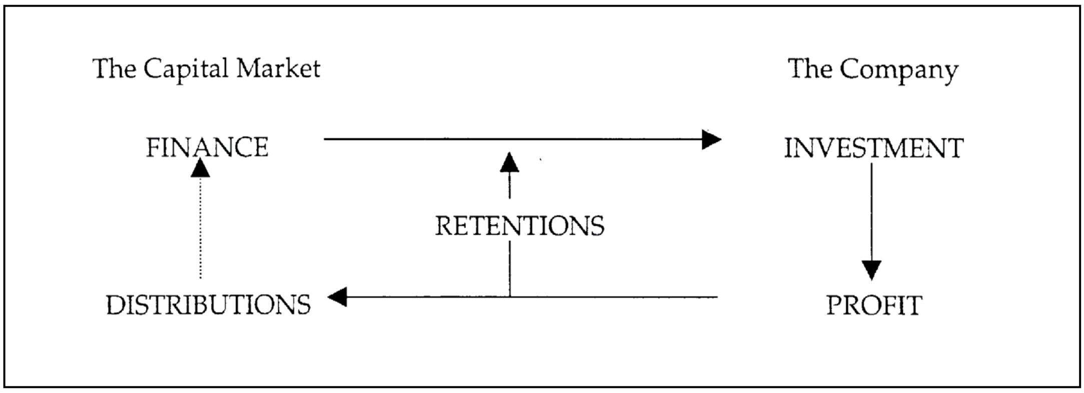

Figure 1.1: The Mixed Market Economy

Over their life, individual projects should eventually generate net cash flows that *exceed* their overall cost of funds to create wealth. This future *positive* net terminal value (NTV) is equivalent to a *positive* NPV, expressed in today's terms, defined by the project discount rate using the time value of money.

Even when modern financial theory moves from a risk-free world to one of uncertainty, where *more than one future outcome is possible*, this analysis remains the bedrock of rational investment behaviour. Providing markets are reasonably efficient, all news (good or bad) is soon absorbed by the market, such that:

- *Short-term*, you win some, you lose some.
- *Long-term*, the market provides returns commensurate with their risk.
- *Overall*, you cannot "beat" the market.

Without permanent access to "insider information" (which is illegal) investment strategies using "public" information, such as share price listings, corporate and analyst reports, plus press and media comment, represent a "fair" game for all (*i.e.* a *martingale*).

As I have also illustrated throughout my *bookboon* series with reference to volatile, historical events: from Dutch "tulip mania" (1637) to the 1929 and 1987 stock market crashes, the millennium dot.com bubble, global financial meltdown (2008), subsequent Euro crises and the 2015 Dow Jones and FTSE 100 (Footsie) record highs:

> Even the most sophisticated financial institutions and private investors, with the time, money, financial and fiscal expertise to analyse *all* public information, have failed spectacularly to identify trends.

So, the only way foreword for uncertain investors is to accept that knowledge of the past (or even current events) is no guide to future plans. It is already incorporated into the latest share price listings. And this is where Fama's EMH (*op.cit*.) provides a lifeline.

Taking his *linear* view of society, where "efficient markets have no memory" and participants lack perfect foresight, it is still possible to define *expected* investor returns for a given level of risk, using the techniques of "classical" statistical analysis (*Quants*).

Assuming a firm's project or stock market returns are linear, they are *random variables* that conform to a "normal" distribution. For every level of risk, there is an investment outcome with the highest expected return. For every expected return there is an investment outcome with the lowest expected risk. Using mean-variance analysis, the standard deviation calibrates these risk-return profiles and the likelihood of them occurring, based on probability analysis and confidence limits. Wealth maximisation equals the maximisation of investor *utility* using this trade-off, plotted as an *indifference* curve, which calibrates the *certainty equivalence* associated with the maximisation of an investment's *expected* NPV (ENPV).

According to Modern Portfolio Theory (MPT) and the pioneering work of Markowitz (1952), Tobin (1958) and Sharpe (1963), if numerous investments are then combined into an optimum portfolio, management (or any investor) can also plot an "efficiency frontier" using Quants and evaluate a new investment's inclusion into the mix, according to their risk-return profile (utility curve) relative to their existing corporate portfolio, or the market as a whole.

If we now relax our *all-equity* assumption to introduce an element of cheaper borrowing (debt) into the corporate financial mix, managerial policies designed to maximise shareholder wealth comprise two distinct but nevertheless *inter-related* functions.

- The *investment function*, which identifies and selects a portfolio of investment opportunities that *maximise expected net cash inflows* (ENPV) commensurate with risk.
- The *finance function*, which identifies potential fund sources (equity and debt, long or short) required to sustain investments.

Management's task now extends beyond satisfying shareholder expectations. They need to evaluate the *risk-adjusted* return for each capital source. Then select the optimum structure that will *minimise* their overall weighted average cost of capital (WACC) as a discount rate for project appraisal. However, the principles of investment still apply.

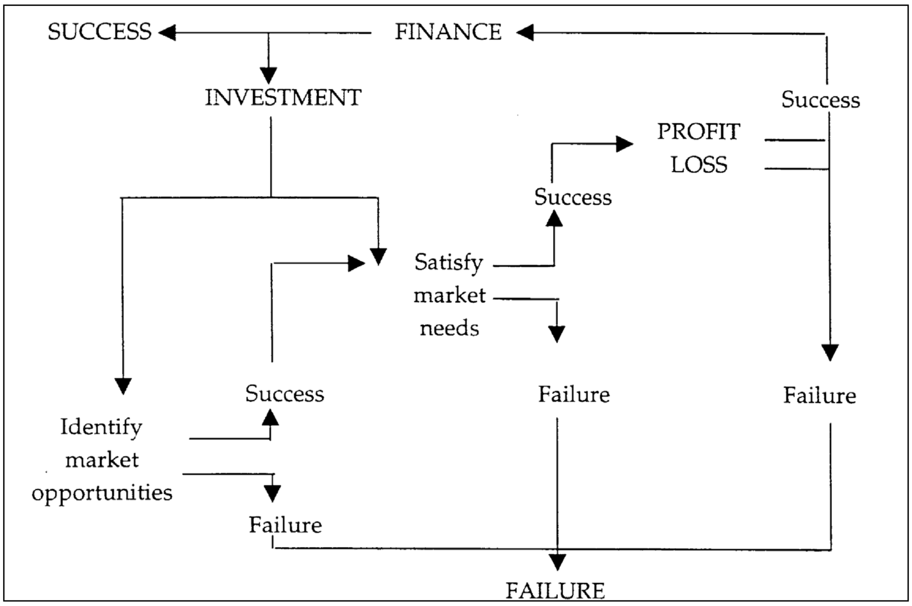

Figure 1:2: Corporate Economic Performance – Winners and Losers.

Figure 1.2 distinguishes the "winners" from the "losers" in their drive to create wealth by summarising in financial terms why some companies fail. These may then fall prey to take-over as share values plummet, or even become bankrupt and disappear altogether.

- Companies engaged in inefficient or irrelevant activities, which produce losses (negative ENPV) are gradually starved of finance because of reduced dividends, inadequate retentions and the capital market's unwillingness to replenish their borrowing, thereby producing a fall in share price.

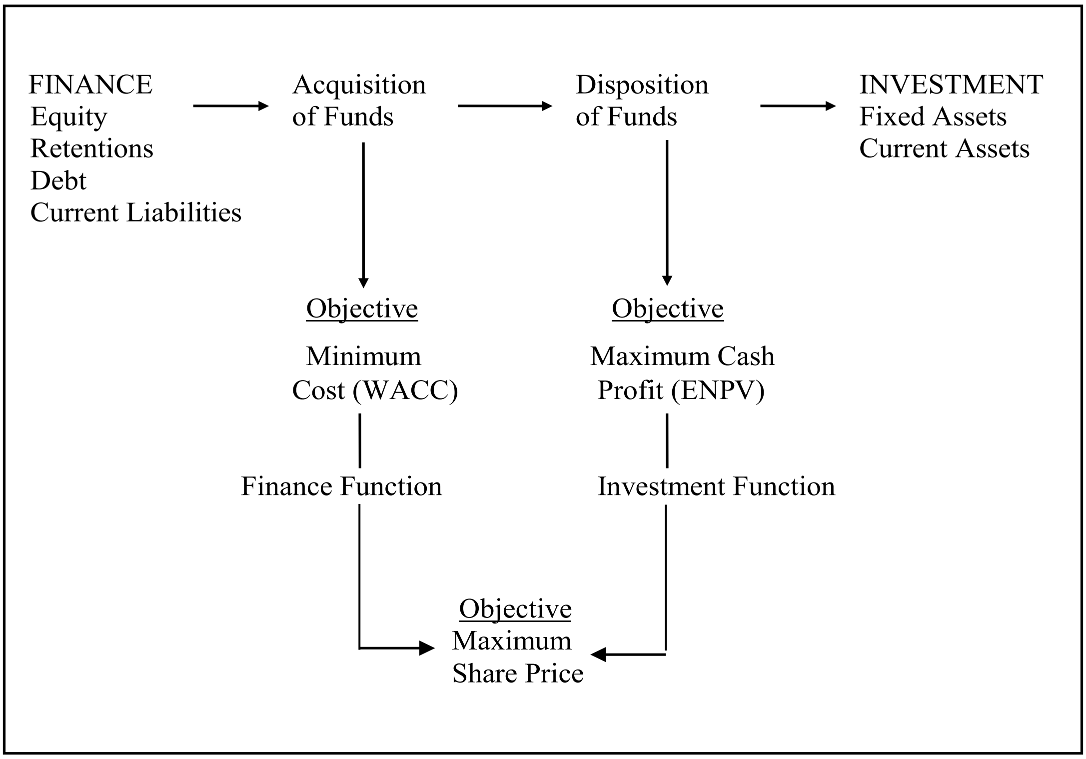

Figure 1.3: Corporate Financial Objectives

Figure 1.3 summarises the strategic objectives of financial management relative to the inter-relationship between *internal* investment and *external* finance decisions that enhance shareholder wealth (share price) based on the law of supply and demand to attract more rational-risk averse investors to the company.

The diagram reveals that a company wishing to maximise its wealth using share price as a *vehicle*, must create cash profits using ENPV as the *driver*. Management would not wish to invest funds in capital projects unless their *marginal* yield at least matched the rate of return prospective investors can earn elsewhere on comparable investments of equivalent risk.

In an ideal world, total cash profits from a portfolio of investments should exceed the overall cost of investment (WACC) producing a positive ENPV, which not only covers all interest on debt but also yields a residual that satisfies shareholder expectations, to be either distributed as a dividend, or retained to finance future profitable investments.

1.3 Questions to Consider

So far so good: but what if capital markets are *imperfect*?

Information is not freely available and there are barriers to trade. Moreover, if a significant number of market participants, including corporate management, financial institutions and private investors, pursue their own agenda, characterised by short-term goals at the expense of long-run shareholder wealth maximisation?

- Are shares still correctly priced?
- Are financial resources still allocated to the most profitable investment opportunities, irrespective of shareholder consumption preferences?

In other words, are markets *efficient* once the *agency* principle breaks down and short-termism takes hold?

As all other texts in my *bookboon* series suggest, based on historical real-world volatility mentioned earlier, perhaps they are not.

Post-modern theorists with cutting-edge mathematical expositions of "speculative" bubbles, "catastrophe" theory and market "incoherence", now hypothesise that classical statistical analyses (Quants) are discredited. Investment prices and returns may be *non-random* variables and markets *have a memory*. This "new finance" takes a *non-linear* view of society, which frequently dispenses with the assumption that we can *maximise* anything.

Unfortunately, none of these models are yet sufficiently refined to provide market participants with alternative guidance in their quest for greater wealth. This explains why the investment community still clings to the time-honoured objective of shareholder wealth maximisation, based on Quants as a framework for analysis.

Nevertheless, *post-modernism* serves a dual theoretical purpose mentioned at the outset.

- First, it reminds us that the foundations of traditional *modern* finance may sometimes be "bad science" by which we mean that theoretical investment and financing decisions are all too often based on simplifying assumptions without any empirical support.
- Second, it reveals why investors (sophisticated or otherwise) should always interpret conventional statistical analyses of wealth maximisation behaviour with caution and not be surprised if subsequent events invalidate their conclusions.

1.4 Fisher's Legacy and Modigliani-Miller

Once a company has made an issue of ordinary shares and received the proceeds, management is neither directly involved with their subsequent transactions on the capital market, nor the prices at which they are transacted. These are matters of negotiation between prevailing shareholders and prospective investors.

> In sophisticated, mixed market economies where *ownership is divorced from control*, the normative objective of modern financial management is therefore defined by the maximisation of shareholder wealth, based on ENPV maximisation using mean-variance analysis.

We examined these propositions by considering perfect (efficient) capital markets under conditions of certainty with *no barriers to trade*, characterised by freedom of information, no transaction costs and tax neutrality. According to Fisher's Separation Theorem, Jenson and Meckling's Agency Theory and the EMH of Fama (*op.cit*):

> An *all-equity* firm can justify retained earnings to finance future investments, rather than pay a current dividend, if their marginal return on new projects at least equals the market rate of interest that shareholders could obtain by using dividends to finance alternative investments of equivalent business risk elsewhere.

Even if markets are uncertain, providing they are still efficient, rational, risk-averse shareholders should support such behaviour. It cannot detract from their wealth, because at any point in time, retentions and dividends are perceived as *perfect economic substitutes*. What they lose through dividends foregone, they expect to receive through increased equity value (capital gains) generated by internally financed projects discounted at their required opportunity rate of return.

And this is where MM first contribute to our analysis.

According to their *dividend irrelevancy* hypothesis (1961) explained in Chapter Four, when shareholders need to replace a missing dividend to satisfy their consumption preferences, the solution is simple.

- Shareholders can create a *home-made* dividend by either borrowing an equivalent amount at the same rate as the company, or sell shares at a price that reflects their earnings and reap the capital gain.

Since the borrowing (discount) rate is entirely determined by the *business* risk of investment (the variability of future earnings) and not *financial* risk (the pattern of dividends), the firm's distribution policy is trivial.

- Dividend decisions are concerned with what is done with earnings but do not determine the risk originally associated with the quality of investment that produces them.

To set the scene for MM, let us therefore consider a simple example that clarifies the inter-relationship between shareholder wealth maximisation, the *supremacy* of investment policy and the *irrelevance* of dividend (financial) policy, given the assumptions of a perfect market.

Review Activity

Suppose a company has issued ordinary shares (common stock) which generate a net annual cashflow of £1 million in perpetuity to be paid out as dividends. The market rate of interest and corporate discount rate commensurate with the degree of risk is 10 percent.

The capitalisation of this constant dividend stream (a formula with which you should be familiar) defines a total equity value:

V_E = £1 million / 0.10 = <u>£10 million</u>

The company now intends to finance a new project of equivalent risk by retaining the next dividend to generate a net cash inflow of £2 million twelve months later, paid out as an additional dividend. Thereafter a full distribution policy will be adhered to.

Required:

Is management correct to retain earnings and would you invest in the company?

An Indicative Outline Solution

The data provides an opportunity to review your knowledge of the investment and financial criteria that underpin the normative objective of shareholder wealth maximisation, using NPV maximisation as a determinant of share price.

- The Optimum Dividend-Retention Policy

The first question we must ask ourselves is whether the incremental investment financed by the non-payment of a dividend affects the shareholders adversely?

We can present the managerial decision in terms of the revised dividend stream:

	t_0	t_1	t_2	t_3	….	t_∞
£ million	£	£	£	£		£
Existing dividends		1	1	1		1
Project cash flows		(1)	2	-		
Revised dividends		-	3	1		1

If we now compare total equity value using the *discounted value* of future dividends:

V_E (existing) = £1 million / 0.10 = £10 million

V_E (revised) = £3 million / $(1.1)^2$ + (£1 million / 0.10) / $(1.1)^2$ = £10.744 million

Once the project is accepted, the present value (PV) of the firm's equity capital will rise and the shareholders will be £744,000 better off with a revised dividend stream.

Perhaps you need to pause here, because the application of the discounted cashflow (DCF) formula to the new valuation of the dividends requires explanation. If so, take time out to revise your understanding of its rationale before we proceed.

- **Net Present Value (NPV) Maximisation**

If you are comfortable with DCF analysis, we can determine the same wealth maximisation decision without even considering the fact that the pattern of dividends has changed, thereby proving the veracity of Fisher's Separation Theorem and the MM dividend irrelevancy hypothesis quite independently.

The increase in total value is simply the new project's *net present value* (NPV). This is proven by implementing the corporate DCF capital budgeting model, with which you are familiar:

$$\text{NPV} = \frac{(£1\text{million})}{1.1} + \frac{£2\text{ million}}{(1.1)^2} = \underline{£744{,}000}$$

In our example, the shareholders simply relinquish their next dividend and gain an increase in the subsequent value of their ordinary shares from £10,000,000 to £10,744,000.

Conclusions

- In a perfect capital market, where the firm's investment decisions can be made independently of the consumption decisions of shareholders, NPV project maximisation represents shareholder wealth maximising behaviour.
- It is the *investment* decision that has determined the value of equity and *not* the *financing* (dividend) decision.

1.5 Summary and Conclusions

The remainder of this study is designed to complement and develop your understanding of the normative shareholder wealth maximisation objective, within the context of modern finance theory and MM's pivotal "law of one price".

It extends beyond all-equity firms, dominated by the *irrelevance* of dividend policy relative to corporate value, into a world of corporate borrowing (leverage) and a multiplicity (portfolio) of investments. And as we shall discover, MM's basic position is entirely consistent.

> The overall cut-off rate for investment and corporate value are *independent* of financial structure. Just like dividend-retention policies, companies agonising over whether to issue debt or equity are wasting their time.

Like my previous *bookboon* texts, some topics will focus on financial numeracy and mathematical modelling. Others will require a literary approach. The rationale is to vary the pace and style of the learning experience. It not only applies mathematics and accounting formulae through a series of Activities (with outline solutions) some of which are sequential, but also develops your own arguments and a critique of the subject as a guide to further study.

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Part Two:
The Dividend Decision

2 How to Value a Share

Introduction

Part One surveyed the development of modern finance theory, based on Fisher's Separation Theorem (1930) with specific reference to the *Investment Decision*, to illustrate why a preponderance of academics and analysts still support the *normative* objective of shareholder wealth maximisation. Based on management's expected NPV (ENPV) maximisation of all a firm's projects and its impact on the market price of equity, we explained how under certain conditions:

> An *all-equity* firm can justify retained earnings to finance future investments, rather than pay a current dividend, if their marginal return on new projects at least equals the market rate of interest that shareholders could obtain by using dividends to finance alternative investments of equivalent business risk elsewhere.

Even if markets are uncertain, providing they are still efficient, rational, risk-averse shareholders should support such behaviour. It cannot detract from their wealth, because at any point in time, retentions and dividends are perceived as *perfect economic substitutes*. What they lose through dividends foregone, they expect to receive through increased equity value (capital gains) generated by internally financed projects discounted at their required opportunity rate of return.

So far, so good: throughout Part One we accepted without question the fundamental assumption that dividends and earnings are *equally* valued by investors who model share price. However:

- If dividends and retentions are not *perfect economic substitutes*, a firm's distribution policy may determine an optimum share price and hence share price maximisation, which runs counter to the "dividend irrelevancy" hypothesis of Miller and Modigliani (1961).

Part Two now deals explicitly with MM and the *Dividend Decision*, namely its impact on current share price and the market capitalisation of equity (*i.e*. shareholders' wealth) determined by the consequence of managerial *financial* policies to distribute or retain profits, which stem from their *investment* decisions.

The key to understanding stock market performance, used by investors to analyse these inter-relationships, requires a theoretical appreciation of the relationship between a share's value and its return (dividend or earnings) using various models based on discounted revenue theory.

To set the scene, we shall keep this Chapter's analysis simple by outlining the theoretical determinants of share price, with particular reference to the *capitalisation of a perpetual annuity* using both dividend and earnings yield formulae.

Detailed consideration of the MM controversy as to whether dividends or earnings are a prime determinant of share value will then be covered in subsequent Chapters, with reference to their comprehensive critique of the case for dividends presented by Myron J. Gordon (1962).

- According MM's "law of one price" the current value of an all-equity firm is *dependent* upon its investment strategy and *independent* of its dividend policy.
- The variability of earnings, (*business* risk) rather than how they are packaged for distribution (*financial* risk) determines the shareholders' desired rate of return (cost of equity) and management's cut-off rate for investment (project discount rate) and hence its share price.

Part Three (the *Finance Decision*) then introduces MM's entirely consistent theory of capital structure by relaxing our *all-equity* assumption to introduce an element of cheaper borrowing (debt) into the corporate financial mix, premised on managerial policies designed to maximise shareholder wealth.

By reformulating the share valuation models of Part Two and introducing the pricing and return of loan stock and other sources of finance, a managerial cut-off rate for project appraisal using an overall weighted average cost of capital (WACC) will be derived. Given its assumptions and limitations, we shall then consider the vexed question as to whether capital gearing (leverage) is a *determinant* of WACC and total corporate value (the "traditional" view) or an *irrelevance* as MM hypothesise.

Based on their *arbitrage* concept (1958) we shall arrive at two conclusions, which conform to MM's dividend irrelevancy position.

- Total corporate value (debt plus equity) represented by the expected NPV of a firm's income stream discounted at a rate appropriate to its business risk, should be unaffected by financial risk associated with its mode of financing.
- Any rational debt-equity ratio should produce the same overall cut-off rate for investment (WACC) equivalent to the cost of equity in an all-equity firm.

Part Four (the *Portfolio Decision*) establishes a final mathematical connection between MM's "law of one price" and Modern Portfolio Theory (MPT), with specific reference to the general Capital Asset Pricing Model (CAPM) of William Sharpe (1963).

According to the CAPM and *beta factor* analysis, if different capital projects are combined into an optimum portfolio, management can plot an "efficiency frontier" using Quants analysis and then select further investments for inclusion into the existing asset mix, according to their desired risk-return profile (utility curve).

As we shall discover, without debt in it capital structure, a company's *asset* beta equals its *equity* beta for projects of equivalent business risk. However, according to MM's theory of capital structure and the *arbitrage* process:

- Companies that are identical in every respect apart from their gearing should also have identical asset beta factors because the variability of earnings is the same. These factors are not influenced by financial risk.
- So, just like WACC (relative to the cost of equity in an unlevered firm) the asset beta (equity beta) of an all-equity company can be used to evaluate geared projects in the same class of business risk without considering differences in financial structure.

2.1 The Capitalisation Concept

Discounted revenue theory defines an investment's present value (PV) as the sum of its relevant periodic cash flows (C_t) discounted at an appropriate opportunity cost of capital, or rate of return (r) on alternative investments of equivalent risk over time (n). Expressed algebraically:

1. $$PV_n = \sum_{t=1}^{n} C_t /(1+r)^t$$

The equation has a convenient property. If the investment's annual return (r) and cash receipts (C_t) are *constant and tend to infinity*, ($C_t = C_1 = C_2 = C_3 = C_\infty$) their PV simplifies to the formula for the *capitalisation of a constant perpetual annuity*:

2. $PV_\infty = C_t / r \; C_1 / r$

The return term (r) is called the *capitalisation* rate because the transformation of a cash flow series into a capital value (PV) is termed "capitalisation". With data on PV_∞ and r, or PV_∞ and C_t, we can also determine C_t and r respectively. Rearranging Equation (2) with one unknown:

3. $C_t = PV_\infty \text{ x } r$

4. $r = PV_\infty / C_t$

Activity 1

The previous PV equations are vital to your understanding of the various share valuation models that follow. If you are unsure of their theory and application, then I recommend that you download *Strategic Financial Management* (*SFM*) from the author's *bookboon* series and read Chapters Two and Five before you continue.

Having completed this reading, you will also appreciate that shares may be traded either *cum-div* or *ex-div*, which means they either include (cumulate) or exclude the latest dividend. For example, if you sell a share *cum-div* today for P_0 the investor also receives the current dividend D_0. Excluding any transaction costs, the investor therefore pays a total price of $(D_0 + P_0)$. Sold *ex -div* you would retain the dividend. So, the trade is only based on current price (P_0).

This distinction between *cum-div* and *ex-div* is important throughout the remainder of our study because unless specified otherwise, we shall adopt the time-honoured academic convention of defining the current price of a share using an *ex-div* valuation.

2.2 The Capitalisation of Dividends and Earnings

Irrespective of whether shares are traded *cum-div or ex-div*, their present values can be modelled in a *variety* of ways using discounted revenue theory. Each depends on a definition of future periodic income (either a dividend or earnings stream) and an appropriate discount rate (either a dividend or earnings yield) also termed the equity capitalisation rate.

For example, given a forecast of periodic future dividends (D_t) and a shareholder's desired rate of return (K_e) based on current dividend yields for similar companies of equivalent risk:

The present *ex-div* value (P_0) of a share held for a *given* number of years (n) should equal the discounted sum of future dividends (D_t) plus its eventual *ex-div* sale price (P_n) using the current dividend yield (K_e) as a capitalisation rate

Expressed algebraically:

5. $P_0 = [(D_1/1 + K_e) + (D_2/1 + K_e)^2 + \ldots + (D_n/1 + K_e)^n] + (P_n/1 + K_e)^n$

Rewritten and simplified, this defines the *finite-period dividend valuation model:*

6. $P_0 = \sum_{t=1}^{n} D_t/(1+K_e)^t + P_n/(1 + K_e)^n$

Likewise, given a forecast for periodic future earnings (E_t) and a desired return (K_e) based on current earnings yields of equivalent risk:

The present *ex-div* value (P_0) of a share held for a *given* number of years (n) equals the sum of future earnings (E_t) plus its eventual *ex-div* sale price (P_n) all discounted at the current earnings yield (K_e).

Algebraically, this defines the *finite-period earnings valuation model:*

7. $P_0 = \sum_{t=1}^{n} E_t/(1+K_e)^t + P_n/(1 + K_e)^n$

Activity 2

We observed in Part One that a logical approach to financial analysis is to make *simplifying* assumptions that rationalise its *complexity*. A classic example is the derivation of a series of dividend and earnings valuations, other than the *finite* model. Some are more sophisticated than others, but their common purpose is to enable investors to assess a share's performance under a variety of conditions.

To illustrate the point, briefly summarise the theoretical assumptions and definitions for the following models based on your reading of *SFM* (Chapter Five) or any other source material.

The *single-period* dividend valuation
The *general* dividend valuation
The *constant* dividend valuation

Then give some thought to which of these models underpins the data contained in stock exchange listings published by the financial press worldwide.

We know that the *finite-period* dividend valuation model assumes that a share is held for a given number of years (n). So, today's *ex div* value equals a series of expected year-end dividends (D_t) plus the expected *ex-div* price at the end of the entire period (P_n), all discounted at an appropriate dividend yield (K_e) for shares in that risk class. Adapting this formulation we can therefore define:

- The *single-period* dividend valuation model

  Assume you hold a share for one period (say a year) at the end of which a dividend is paid. Its current *ex div* value is given by the expected year-end dividend (D_1) plus an *ex-div* price (P_1) discounted at an appropriate dividend yield (K_e).

- The *general* dividend valuation model

  If a share is held indefinitely, its current *ex div* value is given by the summation of an infinite series of year-end dividends (D_t) discounted at an appropriate dividend yield (K_e). Because the share is never sold, there is no final *ex-div* term in the equation.

- The *constant* dividend valuation model

  If the annual dividend (D_t) not only tends to infinity but also remains constant, and the current yield (K_e) doesn't change, then the *general* dividend model further simplifies to the *capitalisation of a perpetual annuity*.

2.3 The Capitalisation of Current Maintainable Yield

Your answers to Activity 2 not only reveal the impact of different assumptions on a share's theoretical present value, but why basic price and yield data contained in stock exchanges listings published by the financial press and internet favour the *constant* valuation model, rather than any other.

Think about it. The derivation and analyses of current share prices based on future estimates of dividends, *ex-div* prices and appropriate discount rates for billions of market participants, even over a single period is an impossible task.

To avoid any forecasting weakness, characterised by uncertainty and to provide a *benchmark* valuation for the greatest possible number, stock exchange listings therefore assume that shares are held in *perpetuity* and the latest reported dividend per share will remain *constant* over time. This still allows individual investors with other preferences, or information to the contrary, to model more complex assumptions for comparison. There is also the added commercial advantage that by using simple metrics, newspaper and internet stock exchange listings should have universal appeal for the widest possible readership.

Turning to the mathematics, given your knowledge of discounted revenue theory and the *capitalisation of a perpetual annuity* (where $PV = C_t / r$) share price listings define a current *ex- div* price (P_0) using the *constant* dividend valuation model as follows:

8. $P_0 = D_1 / K_e$

Next year's dividend (D_1) and those thereafter are represented by the latest reported dividend (*i.e.* a constant). Rearranging terms, (K_e) the shareholders desired rate of return (equity capitalisation rate) is also a constant represented by the current yield, which is assumed to be *maintainable* indefinitely.

9. $K_e = D_1 / P_0$

2.4 The Capitalisation of Earnings

For the purpose of exposition, so far we have focussed on dividend income as a determinant of price and value, with only passing reference to earnings. But what about shareholders interested in their *total* periodic returns (dividends plus retentions) from corporate investment? They need to capitalise a post-tax earnings stream (E_t) such as *earnings per share* (EPS) and analyse its yield (K_e). No problem: the *structure* of the valuation models summarised in Activity 2 remains the same but E_t is substituted for D_t and K_e now represents an earnings yield, rather than a dividend yield. Thus, we can define a parallel series of equations using:

The *single-period*, earnings valuation model
The *finite-period*, earnings valuation model
The *general* earnings valuation model
The *constant* earnings valuation model

Turning to stock exchange listings, the financial press and internet, we also observe that for simplicity the publication of earnings data is still based on the *capitalisation of a perpetual annuity*.

10. $P_0 = E_1 / K_e$

Next year's earnings (E_1) and those thereafter are represented by the latest reported profit (*i.e.* a constant). Rearranging terms, (K_e) the shareholders desired rate of return (equity capitalisation rate) is also a constant represented by the current earnings yield, which is assumed to be *maintainable* indefinitely.

11. $K_e = E_1 / P_0$

Review Activity

Having downloaded this text and perhaps others in my *bookboon* series, it is reasonable to assume that you can already interpret a set of published financial accounts and share price data. To test your level of understanding for future reference, select a newspaper of your choice and a number of companies from its stock exchange listings. Then use the data:

1. To explain the mathematical relationship between a company's dividend and earnings yields and why the two may differ.
2. To define earnings yields published in the financial press.

An Indicative Outline Solution

1. The Mathematical Yield Relationship

Our discussion of efficient markets in Chapter One explained why a company's shares cannot sell for different prices at a particular point in time. So, it follows that:

12. $P_0 = D_1 / K_e = E_1 / K_e$

And if a company adopts a policy of full distribution (whereby $D_1 = E_1$) then the equity capitalisation rates for dividends and earnings, using a current maintainable yield (K_e) must also be identical.

13. $K_e = D_1 / P_0 = K_e = E_1 / P_0$

But what about the more usual situation, where a company retains a proportion of earnings for reinvestment? By definition, the respective equity capitalisation rates (K_e) must now differ because

14. $K_e = D_1 / P_0 < K_e = E_1 / P_0$

Given P_0 and $D_1 < E_1$

As we shall discover in Chapter Three, there is a *behavioural* explanation for the relationship between the two yields. For the moment, suffice it to say that there is also an underlying *mathematical* relationship. For example, if a company's current share price, latest reported dividend and earnings per share are \$100, \$10 and \$20 respectively, then because earnings *cover* dividends twice the dividend yield is half the earnings yield (10 and 20 percent respectively).

This difference in yields is not a problem for investors who know what they are looking for. Some will prefer their return as current income (dividends and perhaps the sale of shares). Some will look to earnings that incorporate retentions (future dividends plus capital gains). Most will hedge their bets by combining the two in share portfolios that minimise risk. So, their respective returns will differ according to their risk-return profile. Which is why share price listings in newspapers worldwide focus on dividends *and* earnings, as well as the *interrelationship* between the two measured by dividend cover.

2. The Yield and Price-Earnings (P/E) Ratio

Moving on to the second question posed by our Review Activity, if you are at all familiar with share price listings published in the financial press, you will be aware of a *convention* that also enables investors to avoid any confusion between dividend and earnings yields when analysing a share's performance.

Given the current earnings yield:

11. $K_e = E_1 / P_0$

The equation's terms can be rearranged to produce its *reciprocal*, the price-earnings (P/E) ratio.

15. $P/E = P_0 / E_1 = 1/K_e$

> Unlike the earnings yield, which is a *percentage* return, the P/E ratio is a *real* number that analyses price as a *multiple* of earnings. On the assumption that a firm's current post tax profits are maintainable indefinitely, the ratio therefore provides an alternative method whereby a company's distributable earnings can be capitalised to establish a share's value.

Because the two measures are *reciprocals* whose product always equals one, the interpretation of the P/E is that the *lower* the number, the *higher* the earnings yield and *vice versa*. And because investors are dealing with an *absolute* P/E value and not a *percentage* yield, there is no possibility of confusing a share's dividend and earnings performance when reading share price listings, articles or commentaries from the press, media, analyst reports, or internet downloads.

Finally, having noted that low valuation multipliers correspond to high returns and that a number multiplied by its reciprocal equal's one: use Table 2.1 to confirm a *perfect inverse* relationship between a share's P/E and its earnings yield. Not only will this exercise be useful for future reference throughout this text, but your future reading of the financial press should also fall into place.

$P/E = P_0 / E_1 = 1/K_e$	50	40	20	15	12	10	8	5	2
$Yield = E_1 / P_0 = K_e$	2	2.5	5	6.66	8.33	10	12.5	20	50

Table 2.1: The Relationship between the P/E Ratio and Earnings Yields

2.5 Summary and Conclusions

This Chapter has outlined the fundamental relationships between share valuation models and the derivation of the cost of equity capital for the purpose of analysing stock market returns.

We set the scene by explaining the derivation of share valuation models using discounted revenue theory, with reference to the capitalisation of a perpetual annuity. We noted that corresponding equity valuations based on current dividend and earnings should be financially equivalent.

The relationship between an *ex-div* dividend and earnings valuation revealed why a few select metrics (based on price, dividend yield, the P/E ratio and cover) published in the media encapsulate a company's stock market performance and provide a guide to future investment.

However, as we shall discover in later chapters, a share's intrinsic value (price) is only meaningful if we move beyond the mathematics and place it in a *behavioural* context. For example, given a company's latest reported dividend and profit figures, investors can use existing dividend yields and P/E ratios to place a comparative value on that company's shares. These can then be compared with its actual value (current market price) to establish whether the company is either undervalued, equitable, or overvalued, relative to the market for similar shares of equivalent risk. Needless to say, undervalued, rational investors buy, equitable they hold, overvalued they sell.

But what motivates their trading decisions: is it the dividend policy of the firm, or its earning potential?

2.6 Selected References

1. Fisher, I., *The Theory of Interest*, Macmillan, 1930.
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3 The Role of Dividend Policy

Introduction

For simplicity, we have assumed that if shares are held indefinitely and future dividends or earnings per share remain constant, their current *ex-div* price can be expressed using the *capitalisation of a perpetual annuity* based on current dividend or earnings yields. This Chapter refines these *constant* valuation models by considering two inter-related questions.

- What happens to current share price if forecast dividends or earnings are not constant in perpetuity?
- When valuing a company's shares, do investors value current dividends more highly than earnings retained for future investment?

3.1 The Gordon Growth Model

Chapter One began with a discussion of investment principles in perfect capital markets characterised by certainty. According to Fisher's Separation Theorem (1930), it is irrelevant whether a company's future earnings are paid as a dividend to match shareholders' consumption preferences at particular points in time. If a company decides to retain profits for reinvestment, shareholder wealth will not diminish, providing that:

- Management's *minimum* required return on a project financed by retention (the discount rate, r) matches the shareholders' *desired* rate of return (the yield, K_e) that they can expect to earn on alternative investments of comparable risk in the market place, i.e. their *opportunity* cost of capital.
- In the interim, shareholders can always borrow at the market rate of interest to satisfy their income requirements, leaving management to invest current unpaid dividends on their behalf to finance future investment, growth in earnings and future dividends.

From the late 1950s, Myron J. Gordon developed Fisher's theory that dividends and retentions are *perfect substitutes* by analysing the impact of different dividend and reinvestment policies (and their corresponding yields and returns) on the current share price for all-equity firms using the mathematical application of a *constant growth* formula.

What is now termed the *Gordon dividend-growth model* defines the current *ex-div* price of a share by capitalising next year's dividend at the amount by which the shareholders' desired rate of return exceeds the constant annual rate of growth in dividends.

Using Gordon's original notation where K_e represents the equity capitalisation rate; E_1 equals next year's post-tax earnings; b is the proportion retained; (1-b) E_1 is next year's dividend; r is the return on reinvestment and r multiplied by b equals the constant annual growth in dividends:

16. $P_0 = (1\text{-}b)\ E_1 / K_e - rb$ subject to the proviso that $K_e > rb$ for share price to be finite.

Today, the equation's notation is simplified in many Finance texts as follows, with D_1 and g representing the dividend term and growth rate respectively, subject to the constraint that $K_e > g$.

17. $P_0 = D_1 / K_e - g$

In a *certain world*, Gordon confirms Fisher's relationship between corporate reinvestment returns (r) and the shareholders' opportunity cost of capital (K_e). Share price only responds to profitable investment (business) opportunities and not changes in dividend (financial) policy because investors can always borrow to satisfy their income requirements. To summarise the dynamics:

Shareholder wealth (price) will stay the same if r equals K_e
Shareholder wealth (price) will increase if r is greater than K_e
Shareholder wealth (price) will decrease if r is lower than K_e

Activity 1

To confirm the impact of retention-financed investment on share price defined by Gordon under conditions of *certainty*, use the following data for Jovi plc with a full dividend distribution policy to establish its current share price.

EPS 10 pence Dividend Yield 2.5%

Now recalculate price, with the same EPS forecast of 10 pence, assuming that Jovi revises its distribution policy. The company reinvests 50 percent of earnings in projects with rates of return that equal its current dividend yield. Also comment on your findings.

- Full Distribution (Zero Growth)

  Without future injections of outside finance, a forecast EPS of 10 pence and a policy of *full distribution* (dividend per share also equals 10 pence) Jovi currently has a *zero growth rate*. Shareholders are satisfied with a 2.5 per cent yield on their investment. We can therefore define the current share price, using either *constant* dividend or earnings valuations for the capitalisation of a *perpetual annuity*, rather than a growth model, because they are all financially *equivalent*.

  $P_0 = E_1 / K_e = D_1 / K_e = 10 \text{ pence} / 0.025 = D_1 / K_e - g = 10 \text{ pence} / 0.025 - 0 = £4.00$

- Partial Distribution (Growth)

  Now we have the same EPS forecast of 10 pence but a reduced dividend per share. 50 percent of earnings are reinvested in projects with rates of return equal to the current equity capitalisation rate (yield) of 2.5 percent.

  According to Gordon, dividends will *grow at a constant rate in perpetuity*. Thus, Jovi's revised current *ex-div* share price is determined by capitalising next year's dividend at the amount by which the desired rate of return exceeds the constant annual growth rate of dividends.

  Using Equations (16) or (17):

  $P_0 = (1\text{-}b)\, E_1 / K_e - rb = P_0 = D_1 / K_e - g = 5 \text{ pence} / 0.025 - 0.0125 = £4.00$

- Commentary

  Despite abandoning a constant share valuation in favour of the growth model to accommodate a change in economic variables relating to dividends retention, reinvestment and growth, Jovi's share price remains the same.

According to Gordon, this is because movements in share price relate to the profitability of corporate investment opportunities and not alterations to dividend policy. So, if the company's rate of return on reinvestment (r) equals the shareholders' yield (K_e) price will not change. It therefore follows logically that:

Shareholder wealth (price) will only increase if r is greater than K_e
Shareholder wealth (price) will only decrease if r is lower than K_e

Activity 2

Can you confirm the Gordon model's prediction that if K_e = 2.5%, b = 0.5 but r moves from 2.5% to 4.0%, or down to 1.0%, then P_0 moves from £4.00 to £10.00 or £2.50 respectively?

3.2 Gordon's 'Bird in the Hand' Model

Gordon's initial analysis of share price determination depends absolutely on the assumption of *certainty*. For example, our previous Activity data initially defined a constant equity capitalisation rate (K_e) *equivalent* to a managerial assessment of a constant return (r) on new projects financed by a constant retention (b). This ensured that wealth remained constant (effectively Fisher's Separation Theorem). We then applied this mathematical logic to demonstrate that share price and hence shareholder wealth stays the same, rises or falls only when:

$$K_e = r,\ K_e > r,\ K_e < r$$

But what if the future is *uncertain*?

According to Gordon (1962) rational, risk averse investors should *prefer dividends earlier, rather than later* (a "bird in the hand" philosophy) even if retentions are more profitable than distributions (*i.e.* r > K_e). From period to period, they should also prefer *high dividends to low dividends.* Thus, shareholders will discount near dividends and higher payouts at a lower rate, which is dated (K_{et}). In other words, they require a higher overall *average* return on equity (K_e) from firms that retain a higher proportion of earnings, with obvious implications for share price. Expressed mathematically:

$$K_e = f\,(\ K_{e1} < K_{e\,2} < \dots\ K_{e\,n}\,)$$

The equity capitalisation rate is no longer a *constant* but an *increasing* function of the *timing* and *size* of a dividend payout. So, an *increased* retention ratio results in a *rise* in the discount rate (dividend yield) and a *fall* in the value of ordinary shares:

To summarise Gordon's plausible *uncertainty* hypothesis, where dividend (financial) policy, rather than investment (business) policy, determines share price:

*The lower the dividend, the higher the risk and the higher the yield, the lower the price***.**

Review Activity

According to Gordon, the theoretical policy prescription for an *all-equity* firm in a world of uncertainty is unambiguous.

- Maximise the dividend payout ratio and you minimise the equity capitalisation rate, which maximises share price and hence shareholder wealth.

But from 1959 to 1963 Gordon published a body of theoretical and empirical work using real world stock market data to prove his "bird in the hand philosophy" with conflicting statistical results.

To understand why, analyse the two data sets below for Jovi plc in a world of *uncertainty*. The first represents a full dividend policy distribution. The second reflects a rational managerial decision to retain funds, since the company's return on investment exceeds the shareholders' increased capitalisation rate (Fisher's theorem again).

1. Explain why the basic requirements of the Gordon growth model under conditions of uncertainty are satisfied.
2. Confirm whether the corresponding share prices are positively related to the dividend payout ratio, as Gordon predicts.

Dividend Policy, Growth and Uncertainty					
Forecast EPS	**Retention Rate**	**Dividend Payout**	**Return on Investment**	**Growth Rate**	**Overall Shareholder Returns**
E_1	(b)	(1-b)	(r)	rb = g	K_e
£0.10	0	1.0	-	-	0.025
£0.10	0.5	0.5	0.075	0.0375	0.050

An Indicative Outline Solution

1. The Basic Requirements

Under conditions of *certainty* Gordon asserts that movements in share price relate to the profitability of corporate investment and not dividend policy. However, in a world of *uncertainty* the equity capitalisation rate is no longer a constant but an increasing function of the timing of dividend payments. Moreover, an increase in the retention ratio results in a further rise in the periodic discount rate.

So far so good, since our data set satisfies these requirements. Moving from full distribution to partial distribution elicits a rise in K_e even though withholding dividends to finance investment accords with Fisher's wealth maximisation criterion ($r > K_e$) and also satisfies the mathematical constraint of the Gordon growth model ($K_e > rb$).

2. Has Share Price Fallen with Dividend Payout?

Rational, risk averse investors may prefer their returns in the form of dividends now, rather than later (a "bird in the hand" philosophy that values them more highly). But using the two data sets, which satisfy all the requirements of the Gordon model under conditions of uncertainty, reveals that despite a change in dividend policy, share price remains unchanged!

Uncertainty, *Differential* Dividend and Growth Rates with a *Uniform* Price: $P_0 = (D_1/K_e\text{-}g) = £4.00$					
Forecast EPS	**Retention Rate**	**Dividend Payout**	**Return on Investment**	**Growth Rate**	**Overall Shareholder Returns**
E_1	(b)	(1-b)	(r)	rb = g	K_e
£0.10	0	1.0	-	-	0.025
£0.10	0.5	0.5	0.075	0.0375	0.050

3.3 Summary and Conclusions

The series of variables in the previous table were deliberately chosen to ensure that share price remained unchanged. But the important point is that they all satisfy the requirements of Gordon's model, yet contradict his prediction that share price should fall.

Moreover, it would be just as easy to provide another data set that satisfies these requirements but produces a rise in share price. No wonder Gordon and subsequent empirical researchers have often been unable to prove with statistical significance that *real world equity values* are:

Positively related to the dividend payout ratio
Inversely related to the retention rate
Inversely related to the dividend growth rate

Explained simply, Gordon confuses dividend policy (*financial risk*) with investment policy (*business risk*). For example, an increase in the dividend payout ratio, without any additional finance, reduces a firm's operating capability and *vice versa*.

Using Equation (17)

$$P_0 = D_1 / K_e - g$$

The weakness of Gordon's hypothesis is obvious. Change D_1, then you change K_e and g. So, how do investors unscramble their differential effects on price (P_0) when all the variables on the *right hand side* of the equation are now affected? And in our example, cancel each other out!

For the moment, suffice it to say that Gordon encountered a very real world problem when testing his theoretical model empirically. What statisticians term *multicolinearity*. Fortunately, as we shall discover, two other academic researchers (Modigliani and Miller) were able to provide the investment community with a more plausible explanation of the determinants of share price behaviour.

3.4 Selected References

1. Fisher, I., *The Theory of Interest*, Macmillan (New York), 1930.
2. Gordon, M.J., *The Investment, Financing and Valuation of a Corporation,* Irwin, 1962.

4 MM and Dividends

Introduction

Under conditions of *certainty*, the Gordon growth model ($P_0 = D_1/K_e - g$) reveals why share price movements relate to the nature of a company's profitable investment opportunities (business risk) and not variations in its dividend policy (financial risk). In a world of *uncertainty*, Gordon (1962) then explains how price becomes a function of dividends. Rational, risk-averse investors prefer their returns in the form of dividends now, rather than later (a "bird in the hand" philosophy).

The purpose of this Chapter is to evaluate an alternative hypothesis developed by the joint Nobel Prize winning economists, Franco Modigliani and Merton H. Miller (MM henceforth). Since 1961, their views on the *irrelevance of dividend policy* when valuing shares based on the economic "law of one price" have dominated the subsequent development of modern finance.

4.1 The MM Dividend Hypothesis

MM criticise the Gordon growth model under conditions of uncertainty supported by a wealth of subsequent empiricism, notably the consultancy work of Joel M. Stern and G. Bennett Stewart 111 (Stern-Stewart) referenced by the author in Chapters Eight of *Strategic Financial Management* (2008) and its *Exercise* companion (2009). According to MM and their proponents, dividend policy is not a determinant of share price in reasonably efficient markets because dividends and retentions are *perfect economic substitutes*.

If *shareholders* forego a current dividend to benefit from a future retention-financed capital gain, they can still create their own *home made* dividends to match their consumption preferences by the sale of shares or personal borrowing and be no worse off.

If a *company* chooses to make a dividend distribution, it too, can still meet its investment requirements by a new issue of equity, rather than retained earnings. So, the effect on shareholders' wealth is also neutral.

Consequently, *business risk*, rather than *financial risk*, defines all investors and management need to know about corporate economic performance.

Theoretically and mathematically, MM have no problem with Gordon under conditions of *certainty*. Their equity capitalisation rate (K_e) conforms to the company's class of business risk. So, as Fisher predicted (1930) share price is a function of variations in profitable corporate investment and not dividend policy. But where MM depart company from Gordon is under conditions of *uncertainty*.

As we concluded in Chapter Three, Gordon confuses dividend policy with investment policy. For example, an increase in the dividend payout ratio, without any additional finance, reduces a firm's operating capability and *vice versa*. MM also assert that because uncertainty is *non-quantifiable*, it is logically impossible to capitalise a *multi-period* future stream of dividends, where $K_{e1} < K_{e2} < K_{e3}$ …*etc.* according to the investors' perception of the unknown, as Gordon recommends.

MM therefore define a current *ex-div* share price using the following *one period* model, where K_e equals the shareholders' desired rate of return (capitalisation rate) relative to the "quality" of a company's periodic earnings (class of business risk). The greater their variability, the higher the risk, the higher K_e, the lower the price and *vice versa*.

(18) $P_0 = D_1 + P_1 / 1 + K_e$

MM then proceed to prove that for a *given* investment policy of *equivalent* business risk (where K_e remains constant) a change in dividend policy cannot alter current share price (P_0) because:

The next ex-div price (P_1) increases by a corresponding reduction in (D_1) and vice versa.

Activity 1

To illustrate MM's dividend irrelevancy hypothesis, let us reinterpret the stock exchange data for Jovi plc, initially applied to Gordon's growth model in Chapter Three.

- With an EPS of 10 pence a full dividend distribution policy and yield of 2.5 per cent, establish Jovi's current *ex-div* share price using Equation (18).
- Now recalculate this price, with the same EPS forecast of 10 pence, assuming that Jovi revises its dividend policy to reinvest 100 percent of earnings in future projects with rates of return that equal its current yield.

With a policy of *full* dividend distribution, MM would define:

$$(18)\ P_0 = D_1 + P_1 / 1 + K_e = £0.10 + £4.00 / 1.025 = £4.00$$

Refer back to Chapter Three and you will discover that this *ex-div* price is *identica*l to that established using the Gordon growth model.

Turning to a policy of *nil* distribution (*maximum* retention) where profits are reinvested in projects of equivalent business risk (*i.e*. 2.5 per cent):

$$(18)\ P_0 = D_1 + P_1 / 1 + K_e = £0 + £4.10 / 1.025 = £4.00$$

According to MM, because the managerial cut-off rate for investment still equals K_e, the *ex-div* price rise matches the fall in dividend exactly, leaving P_0 unchanged.

You might care to confirm that using the Gordon growth model from the previous Chapter:

$$(17)\ P_0 = D_1 / K_e - g = 0$$

In other words, if a company does not pay a dividend, which is not unusual (particularly for high-tech, growth firms), it is impossible to determine a share price.

4.2 The MM Hypothesis and Shareholder Reaction

You will also recall from Chapter Three that even if Gordon's model is mathematically definable (K_e>g and D_1>0) he argues that a *fall* in dividends should produce a *rise* in the equity capitalisation rate, causing share price to *fall*. However, MM reject this argument.

If a company's reduction in dividends fails to match shareholders' expectations, they can always create *home-made* dividends by selling part of their holdings (or borrowing) to satisfy their consumption preferences, without affecting their overall wealth.

To understand MM's proposition, let us develop the data from Activity 1 using Equation (18) assuming that the number of shares currently owned by an individual shareholder is defined by (n) to represent their holding.

(19) $nP_0 = nD_1 + nP_1 / 1 + K_e$

Activity 2

Assume you own a number of shares (n = 10,000) in Jovi plc and expect an initial managerial policy of full dividend distribution. From the previous Activity and Equation (19) it follows that your current stock of wealth is worth:

$$nP_0 = nD_1 + nP_1 / 1 + K_e = £1{,}000 + £40{,}000 / 1.025 = £40{,}000$$

Now assume that the firm withholds all dividends for reinvestment. What would you do, if your income requirements (consumption preferences) equal the dividend foregone (£1,000)?

According to MM, the *ex-div* price should increase by the reduction in dividends. So, your holding is now valued as follows, with no overall change:

$$(19)\ nP_0 = nD_1 + nP_1 / 1 + K_e = £0 + £41{,}000 / 1.025 = £40{,}000$$

However, you still need to satisfy your income preference for £1,000 at time period one.

So, why not sell 250 shares for £41,000 / 10,000 at £4.10 each?

You now have £1,025, which means that you can take the income of £1,000 and reinvest the balance of £25 on the market at your desired rate of return (K_e=2.5%). And remember you still have 9,750 shares valued at £4.10.

To summarise your new stock of wealth:

Shareholding 9,750: Market value £39,975: Homemade Dividends £1,000: Cash £25

Have you lost out?

According to MM, *of course not*, since future income and value are unchanged:

	£
nP_1 = 9,750 x £4.10	39,975
Cash reinvested at 2.5%	25
Total Investment	40,000
Total annual return at 2.5%	1,000

> MM conclude that if *shareholders do not like the heat they can get out of the kitchen* by selling an appropriate proportion of their holdings, borrowing (or lending) to match their consumption (income) preferences.

4.3 The MM Hypothesis: A Corporate Perspective

Let us move from the shareholder to the company and what is regarded as the *proof* of MM's dividend irrelevancy hypothesis. Usually, it is lifted *verbatim* from the mathematics of their original article (1961) and relegated to an Appendix in the appropriate chapter of many financial texts, with little, if any, numerical exposition. As I mentioned in Chapter One, this explains why finance students are often left confused when revising for examinations and soon forget MM when they enter the real world of work.

To remedy this situation, let us it examine and apply the equations of MM's proof in detail (using their notation) with reference to the previous data for Jovi plc.

> According to MM, dividends and retentions are *perfect economic substitutes,* leaving shareholder wealth unaffected by changes in distribution policy. For its part too, a firm can resort to new issues of equity to finance any shortfall in its investment plans without compromising its current *ex-div* price.

To illustrate MM's *corporate* proposition, assume a firm's total number of shares currently in issue equals (n). We can define its *total market capitalisation of equity* as follows:

(19) $nP_0 = nD_1 + nP_1 / 1 + K_e$

Now assume the firm decides *to distribute all earnings as dividends.* If investment projects are still to be implemented, the company must therefore raise new equity capital equivalent to the proportion of investment that is no longer funded by retentions.

According to MM, the number of new shares (m) issued at an ex-*div* price (P_1) must therefore equal the total dividend per share retained (nD_1) defined by:

(20) $mP_1 = nD_1$

Based on all shares *outstanding* at time period one (nP_1+mP_1) MM then rewrite Equation (19) to represent the total market value of *original* shares in issue as follows:

(21) $nP_0 = 1/ K_e \ [\ nD_1 + (n + m) \ P_1 - mP_1]$

And because $mP_1 = nD_1$ this equation simplifies to:

(22) $nP_0 = 1/ K_e \ (n + m) \ P_1$

MM therefore conclude that because the dividend term disappears from their market capitalisation of equity, it is impossible to assert that share price is a function of dividend policy.

To illustrate the *corporate* dynamics of MM's argument, let us develop the data from Activity 2, using the preceding equations where the company's total number of shares in issue equals (n).

Activity 3

Assume Jovi plc begins the period with a *maximum* retention policy (*nil* distribution) and a given investment policy. Shares are therefore valued currently at £4.00 with an *ex-div* price of £4.10 at time period one as follows:

(18) $P_0 = D_1 + P_1 / 1 + K_e = £0 + £4.10 / 1.025 = £4.00$

If Jovi has one million shares in issue, we can also derive the company's *total market capitalisation of equity*:

(19) $nP_0 = nD_1 + nP_1 / 1 + K_e = £0 + £4.1m / 1.025 = £4m$

But now assume that the firm decides *to distribute all earnings as dividends* (10 pence per share on one million issued) without compromising investment (*i.e.* it is still a "given").

Confirm that this policy leaves Jovi's share price unchanged, just as MM hypothesise.

If investment projects are still to be implemented, the company must raise new equity capital equal to the proportion of investment that is no longer funded by retained earnings. According to MM, the number of new shares (m) issued *ex-div* at a price (P_1) must therefore equal the total dividend per share retained (nD_1) defined by the following equation.

(20) $mP_1 = nD_1 = £100{,}000$

Based on all shares *outstanding* at time period one (nP1+mP_1) we can rewrite Equation (19) representing the total market value of *original* shares in issue as follows:

(21) $nP_0 = 1/ K_e \; [\; nD_1 + (n + m)\; P_1 - mP_1]$

This simplifies to the following equation where *the dividend term disappears*.

(22) $nP_0 = 1/ K_e \; (n + m)\; P_1 = 1/ 1.025 \; (nP_1 + £100{,}000) = £4 \text{ million}$

Since there is also only one unknown in the equation (P_1) then dividing through by the number of shares originally in issue (n = one million) and rearranging terms, we revert to:

(18) $P_0 = D_1 + P_1 / 1 + K_e = P_1 + £0.10 / 1.025 = £4.00$

And simplifying, solving for P_1:

$P_1 = £4.00$

Thus, as MM hypothesise:

- The *ex-div* share price at the end of the period has fallen from its initial value of £4.10 to £4.00, which is exactly the same as the 10 pence rise in dividend per share, therefore leaving P_0 unchanged.
- Because the dividend term has disappeared from the equations, it is impossible to conclude that share price is a function of dividend policy.

Review Activity
To reaffirm the logic of the MM dividend irrelevancy hypothesis, revise the Jovi data set for a *nil* distribution to assess the implications for both the shareholders and the company if management now adopt a policy of *partial* dividend distribution, say 50 per cent?

4.4 Summary and Conclusions

MM criticise the Gordon growth model under conditions of uncertainty from both a *proprietary* (shareholder) and *entity* (corporate) perspective. The current value of a firm's equity is *independent* of its dividend distribution policy, or alternatively its retention policy, because they are *perfect economic substitutes:*

- The *quality* of earnings (business risk), rather than how they are *packaged* for distribution (financial risk), determines the shareholders' desired rate of return and management's cut-off rate for investment (project discount rate) and hence its share price.
- If a company *chooses* to make a dividend distribution it can always meet its investment requirements by a new equity, issue rather than use retained earnings, so that the effect on shareholders' wealth is neutral.
- As a corollary, dividend policy should therefore be regarded as a *passive residual*, whereby management return unused funds to shareholders because their search for new investment opportunities cannot maintain shareholder wealth, subsequently confirmed by the *agency* principle of Jensen and Meckling (1976).

It therefore seems reasonable to conclude Part Two with the following practical observations on our analyses of share valuation theories and their application by stock market participants, including management.

The P/E ratio (reciprocal of the earnings yield) associated with the profitability of investment (*business risk*), rather than a dividend yield associated with the periodic distribution of earnings (*financial risk*) published in the financial press and on the internet, should encapsulate all the investment community needs to know about corporate economic performance.

4.5 Selected References

1. Gordon, M.J., *The Investment, Financing and Valuation of a Corporation,* Irwin, 1962.
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Part Three:
The Finance Decision

5 Debt Valuation and the Cost of Capital

Introduction

Part Two detailed the MM-Gordon controversy as to whether earnings or dividends are a prime determinant of share value

- According MM's "law of one price", the current value of an all-equity firm is *dependent* upon its investment strategy and *independent* of its dividend policy.
- The variability of earnings (*business* risk), rather than how they are packaged for distribution (*financial* risk), determines the shareholders' desired rate of return (cost of equity) and management's cut-off rate for investment (project discount rate) and hence its share price.

Part Three (the *Finance Decision*) now introduces MM's earlier, consistent theory of capital structure (1958) by relaxing our *all-equity* assumption to introduce an element of cheaper borrowing (debt) into the corporate financial mix, premised on managerial policies designed to maximise shareholder wealth.

By reformulating the share valuation models of Part Two and introducing the pricing and return of loan stock and other sources of finance, a managerial cut-off rate for project appraisal using an overall weighted average cost of capital (WACC) will be derived. Given its assumptions and limitations, we shall then consider the vexed question as to whether capital gearing (leverage) is a determinant of WACC and total corporate value (the "traditional" view) or an irrelevance as MM hypothesise.

Based on their arbitrage concept, we shall arrive at two conclusions, which conform to MM's dividend irrelevancy position.

- Total corporate value (debt plus equity) represented by the expected NPV of a firm's income stream discounted at a rate appropriate to its business risk, should be unaffected by financial risk associated with its mode of financing.
- Any rational debt-equity ratio should produce the same overall cut-off rate for investment (WACC) equivalent to the cost of equity in an all-equity firm.

5.1 Capital Costs and Gearing (Leverage): An Overview

Firms rarely finance capital projects by equity alone. They utilise long and short term funds from a variety of sources at a variety of costs. No one source is free. As the following table reveals, some have an *explicit* cost but others have only an *implicit* or *opportunity* cost. For example, loan issues are sourced at an explicit market rate of interest, whereas the *marginal* cost of earnings retained for new investment is measured by the current return *foregone* by shareholders.

Source of Finance	Capital Cost
Share Issues: Ordinary Preference	Earning per share (EPS) or Dividends plus growth Fixed Dividend
Loan Issues: Secured and Unsecured Convertible	Interest payable plus any premium payable on repayment. Present interest, plus future EPS (with normal conversion price typically above current market price)
Retained earnings	Shareholder return: EPS or Dividends plus growth
Depreciation	Opportunity cost
Short-term borrowings	Market rate of interest
Deferred taxation	Opportunity cost
Deferred payments to creditors	Opportunity cost, plus any loss of goodwill and administrative costs
Reduction in stocks	Opportunity cost, plus any loss of goodwill and loss of sales
Reduction in debtors	As above
Debt factoring	Above base rate
Sale of excess or idle assets	Alternative yield
Sale of property and lease back	Leasing cost plus, any capital appreciation
Research and Development	Opportunity cost
Unallocated Overheads	Opportunity cost

Figure 5.1: Sources of Finance and Capital Costs

Explicit or not, management must first identify the current (marginal) costs of each type of capital employed (debt, as well as equity) to establish an *overall* cost of capital as a project discount rate for corporate investment. The component costs must then combine to derive a marginal, *weighted average cost of capital* (WACC).

To simplify the conceptual computation of WACC (considered in Chapter Six) we shall restrict our analyse to the impact of the value and cost (return) of the most significant alternative to equity as an external source of finance, namely corporate borrowing in the form of debentures (or corporate bonds and loan stock to use American parlance).

Like other sources of debt and creditor finance itemised in Table 5.1, corporate borrowing is attractive to global management.

- Interest rates are typically lower than the cost of equity.
- Debt-holders accept lower returns than shareholders, because their investment is less risky.
- Unlike dividends, interest is guaranteed and a prior claim on profits.
- Debt-holders are also paid before shareholders from the sale of assets in the event of liquidation.
- In many countries, interest payments on debt also qualify for corporate tax relief, which does not apply to dividends, thereby reducing their "real" cost to the firm.

Combine these factors and we can summarise the *traditional* approach to the corporate *financing* decision, which runs counter to MM's theory of capital cost, finance and investment outlined earlier.

According to traditionalists, the introduction of borrowing into a firm's financial structure, termed capital *gearing* or *leverage*, should lower the overall return (cut-off rate) that management need to earn on new investments relative to *all-equity* funding. Consequently, the expected NPV of geared projects should rise with a fall in their discount rates, producing a corresponding increase corporate wealth.

5.2 The Value of Debt Capital and Capital Cost

As marketable securities, the principles of loan valuation are similar to those for equity but less problematical. Stock is issued above, below or at *par* value depending on economic conditions. However, the annual cash return is known from the outset. It always equals a specific rate of interest relative to par value (termed the *coupon rate* or *nominal* yield). The stock's life might also be specified in advance with a guaranteed capital repayment (i.e. *redeemable* as opposed to *irredeemable* debt). Ignoring tax for the moment:

- The current price of any debenture (bond) is determined by a summation of future interest payments, plus the redemption price (if applicable) all discounted back to a present value (PV).
- The annual cost of corporate debt or yield (to redemption if applicable) is the discount rate that equates current price to these expected future cash flows, namely their Internal Rate of Return (IRR) using discounted cash flow (DCF) analysis.

In the case of irredeemable debentures, about to be issued or subsequently trading at par, the market price and IRR obviously equal the par value and *coupon rate* respectively. However, if *price differs from par value*, either at issue or when the debt is later traded, the *IRR no longer equals the coupon rate*. To see why, let us define the price of debt (P_0) at any point in time.

(1) $P_0 = I / 1+K_d + I / (1+K_d)^2 + \dots I / (1+K_d)^\infty$

Where: I = interest (the coupon rate expressed in money terms) received per annum in perpetuity
K_d = the company's annual cost of debt defined as an IRR percentage.

Since the annual interest payment is fixed in perpetuity, Equation (1) simplifies to the familiar valuation formula for a level annuity: interest divided by current market price:

(2) $P_0 = I / K_d$

If we rearrange terms, the cost of debt equals the investment's IRR defined as the annual money interest divided by current market price:

(3) $K_d = I / P_0$

And because interest (I) is constant year on year, it follows that if P_o rises (or falls) then K_d must fall (or rise) proportionately.

Turning to redeemable stock, the nominal return to debt-holders in the year of redemption will be uplifted by the redemption price payable. Thus, when debt is issued or whenever investors trade debentures, the current yield (K_d) is found by solving for the IRR in the following *finite* equation.

$$(4)\ P_0 = [(I / 1 + K_d) + (I / 1 + K_d)^2 \ldots.+\ldots. (I + P_n / 1 + K_d)^n]$$

Rewritten as follows:

$$(5)\ P_0 = \sum_{t=1}^{n} I / (1+K_d)^t + P_n / (1 + K_d)^n$$

Where: n = the number of periods to redemption,
P_n = the redemption value at time period n.

Irrespective of whether debt is redeemable, irredeemable, currently traded or about to be issued:

- The cost of capital (K_d) *always* equals an internal rate of return (IRR).
- The IRR equates current price to the discounted future cash receipts that the loan stock produces.
- Only if the current price and redemption value (if any) equal the par value, will the IRR equal the coupon rate (nominal yield).

If a debt issue has a coupon rate which is below the prevailing market rate of interest defined by its current IRR then by definition current market value (price) will be below par value and *vice versa*.

Activity 1

Manipulate the previous equations to calculate current debt yields if a company issued:

- £100 irredeemable debentures with a 10 percent coupon rate
- £100 debentures with the same coupon rate, redeemable at par ten years hence

You may assume that in both cases, similar debentures currently trade below par at £90.00 (conventionally termed as £90 per cent).

What do these calculations mean to investors and corporate management?

Given current market conditions both £100 issues must be priced at £90 to ensure full subscription.

If *irredeemable* debentures are issued at £90 percent with a *money* coupon rate of £10 per annum, it follows from Equation (3) that the current yield or cost of debt:

$$K_d = £10 / £90 = 11.1\%$$

If *redeemable* ten year debt was issued at the same price with the same coupon rate, we must derive the current yield by solving for IRR using Equation (5).

$$P_0 = \sum_{t=1}^{10} £10 / (1+K_d)^t + £100/ (1+K_d)^n = £90$$

Now the annual cost of debentures K_d is approximately 11.8%.

For the investor, both debenture formulae perform the same functions as the equity models presented in Chapter Two. Even though interest is fixed and a redemption date may be specified, debentures can be traded at either a premium or a discount throughout their life. Thus, the current rate of interest, like an equity yield, is only a guide to the *true* return on life-time investment. In a world of uncertainty it can only be determined by incorporating the capital gain or loss *retrospectively* when the security is sold. In the case of redeemable debentures, held from issue through to redemption, this *ex-post return* calculation is termed the *yield to maturity* or *redemption yield*.

The current yield on debentures K_d therefore represents the return from holding the investment, rather than selling at its current market price. It is an implicit *opportunity cost of capital*, because it is the minimum return below which debenture holders could transfer their funds elsewhere for a market rate of interest of equivalent risk, (Fisher's Separation Theorem, *op. cit.*).

For the company, a successful debenture issue must therefore match the risk-return profile (yield) of loan stock currently trading on the market. In an untaxed economy (more of which later) this rate of interest required by investors represents the company's *marginal* cost of capital for this fund source. As such, K_d is the relevant measure for assessing any new project financed by loan stock.

Returning to our previous Activity, if management wish to maximise corporate wealth using expected (ENPV) criteria then the 10 per cent coupon rate (nominal yield) is irrelevant. To be more precise, new projects should be financed by irredeemable debt at a "real" cost of 11.1 per cent discount rate, rather than redeemable debt with a cost of 11.8 per cent.

- The lower the discount rate, the higher the ENPV and vice versa. So at one extreme, a project discounted at the coupon rate might be accepted, whilst at the other, the redeemable rate signals rejection. Either way, corporate wealth is compromised; with a worst case scenario where the cash flows for a project's acceptance using the coupon rate as a discount rate will not service debt, forcing the firm into liquidation.

To conclude, projects financed by debt (just like equity) should always be evaluated using a *marginal* cost of capital and not the *nominal* yield Only if the incremental return equals the current yield will the marginal cost of raising additional finance equal the current cost of capital in issue and attract investors.

5.3 The Tax-Deductibility of Debt

Whilst tax regimes differ throughout the world, one policy many governments have in common that we need to consider is the treatment of debenture interest as an allowable deduction against a firm's tax liability. As mentioned earlier, not only does this lower the "true" cost of corporate borrowing but also widens the gap between yields on debt and equity.

Providing management can generate sufficient taxable profits to claim the tax relief on debt interest, the higher the rate of corporation tax, the greater the fiscal benefit conferred on the company through issuing debt, rather than equity to finance its investments.

In the preceding valuation models K_d represents the *gross* return received by investors *before* satisfying their *personal* tax liability. What is important to the company, however, is the project discount rate defined by this gross return *after corporation tax*.

To prove the point, let us first consider *irredeemable* debt (i.e. with no redemption value) with a level interest stream in perpetuity. The valuation model *incorporating* tax is given by:

(6) $P_0 = I\ (1\text{-}t)\ /\ K_{d\ t}$

Where: P_0 = the current market price of debt,
I = annual interest payments
t = rate of corporation tax
$K_d\,t$ = post-tax cost of debt

So, if we rearrange terms, the "real" cost of debt to the company after tax is

$$(7)\ K_{d\,t} = I\,(1\text{-}t) / P_0$$

And because the investors' *gross* return (K_d) equals the company's cost of debt before tax, it follows that with a tax rate (t) we can also rewrite Equation (7) as follows;

$$(8)\ K_{d\,t} = K_d\,(1\text{-}t)$$

In a world of corporate taxation, the capital budgeting implications for management are clear.

$$(9)\ K_{d\,t} < K_d$$

To maximise corporate wealth, the post-tax cost of debt should be incorporated into any overall discount rate as a cut-off rate for investment.

Equation (6) onwards might seem strange, since P_0 is still the market value of the debentures held by investors represented by the future cash flows which they expect to receive. But it is important to remember that we are now modelling income-value relationships from the *company's* perspective.

The interest cash flows capitalised on the right-hand side of Equation (6) are therefore *net* of corporation tax, which do not concern investors directly. So, if a company pays £100,000 a year interest on irredeemable debentures with a market price of £1 million and the rate of corporation tax is 25 percent, its effective cost of debt defined by Equation (7):

$$K_{d\,t} = [£100,000\ (1\text{-}0.25)] / £1\ \text{million} = 7.5\%$$

Turning to *redeemable* debt, the company still receives tax relief on interest. But often the redemption payment is not allowable for tax. To calculate the post-tax cost of capital, it is necessary to derive an IRR that incorporates tax relief on interest alone by solving for $K_{d\,t}$ in the following *finite* equation:

$$(10)\quad P_0 = \sum_{t=1}^{n} I\,(1\text{-}t) / (1 + K_{d\,t})^t + (P_n / 1 + K_{d\,t})^n$$

Consider five-year debt with a 15 percent coupon rate, redeemable at £100 par, issued at £90 percent. If the annual rate of corporation tax is 33 percent, we can determine the post-tax cost of debt by solving for K_{dt} in the following equation.

$$P_0 = £90 = \sum_{t=1}^{n=5} £15\,(1 - 0.33) / (1 + K_{d\,t})^{t} + (£110 / 1 + K_{d\,t})^{n}$$

$$K_{d\,t} = 13\%$$

Activity 2

A company's irredeemable debt has a coupon rate of 8 percent and a market value of £76 percent.
Corporation tax is 30 percent and the firm's has sufficient tax liability to set off against its interest.

Calculate the investor's gross return and the company's effective cost of debt.
Comment on the disparity between the two and the capital budgeting implications for management.

Investors receive the following gross return before personal taxation:

$$K_d = £8 / £76 = 10.53\%$$

The post-tax cost to the company for providing this return is:

K_{dt} = £8(1-0.30) / £76 = 7.36%

Loan interest reduces the corporate tax bill. For every £8 distributed to investors as interest, the company effectively pays:

I (1-t) = £8 (1-0.30) = £5.60

The £2.40 difference represents tax relief contributed by the tax authorities.

Turning to capital budgeting, if management finance new investment by issuing debt, this must reflect current post-tax yields of equivalent risk. Each £100 block will be priced at £76. The post-tax cost of debt capital (K_{dt} =7.36%) represents the discount rate that equates the amount raised to the PV of future cash flow required to service this new issue (interest less tax relief).

The tax adjusted cost of debt (K_{dt}) is the IRR that represents the true corporate cost of new debt issues. If the ENPV of a prospective debt-financed project discounted at this IRR is positive, then its return will exceed the cost of servicing that debt and management should accept it.

5.4 The Impact of Issue Costs on Equity and Debt

The introduction of a tax bias into our analysis of the cost of debt is our first example of a barrier to trade that runs counter to Irving Fisher's world of perfect competition outlined in Chapter One. But in the "real" world there are others, one of which we must now consider, namely issue costs.

In Chapter Four we hypothesised that dividends and earnings are *perfect economic substitutes*. At the beginning of this chapter we also stated that the cost of retained earnings is best measured by an opportunity cost, namely the shareholders' return foregone. But even if we ignore the MM dividend-earnings debate, how do we measure this?

In imperfect markets, a fundamental difference between a new issue of ordinary shares (like any other financial security) and retained earnings are the *issue costs* associated with the former. As a consequence, the marginal cost of equity issues is more expensive than retentions, which explains why management hold back earnings for reinvestment

To prove the point, using previous notation and our knowledge of equity valuation for a constant dividend stream (D) in perpetuity, let us introduce issue costs (C) into the *constant dividend valuation model.*

The *marginal* cost of an ordinary share P_o issued by a company is now given by:

(11) $K_e = D / P_o (1 – C)$

By definition, this is higher than the cost of retained earnings, since the latter do not incur issue costs. The cost of retained earnings is simply equivalent to the current dividend yield forgone by *existing* shareholders, namely their opportunity cost of capital:

(12) $K_e = D / P_o$

Note that also, if we substituted earnings (E) for dividends into both of the previous equations; management's preference for retentions, rather than dividend distributions, would still prevail in the presence of transaction costs.

Returning to the cost of loan stock, issue costs also increase the marginal cost of capital. This is best understood if we first substitute issue costs (C) into the cost of *irredeemable* debt in a *taxless* world. Like the equity model, the denominator of Equation (13) is reduced by issue costs.

(13) $K_d = I / P_0 (1\text{-}C)$

If we now assume that debt interest is tax deductible, the post-tax cost of debt originally given by Equation (7) also rises.

(14) $K_{d\,t} = I (1\text{-}t) / P_0 (1\text{-}C)$

Review Activity

In preparation for Chapter Six and the information required to derive a weighted average cost of capital (WACC) as a marginal cut-off rate for investment, use the data below for B.Ferry plc to calculate:

1. The *total* market value of the company's equity plus debt.
2. The *marginal* cost of each fund source.

The Data Set

- 5 million ordinary £1 shares (common stock) currently quoted at £1.20,
- £6 million in retained earnings,
- 4 million preference shares currently quoted at 60 pence,
- £2 million debentures (loan stock) currently trading below par at £80,
- Ordinary and preference shares currently yielding 20 and 10 per cent, respectively,
- Ordinary dividend growth of 5 per cent per annum,
- New issues costs of 20 pence per share for ordinary and preference shares,
- A pre-tax debt yield of 10 per cent,
- A 20 per cent rate of corporation tax.

An Indicative Outline Solution

1. Total Market Value

The overall market value for B.Ferry plc equals the summation of ordinary shares, retained earnings, preference shares and debentures. With the exception of retained earnings (£6m) which are derived from *historical* cost based accounts, all capital issues are valued at their *market* price as follows:

$$(5\text{m} \times £1.20) + £6\text{m} + (4\text{m} \times £0.60) + (£2\text{m} \times 0.80) = £16\text{m}$$

2. Marginal Component Costs

The capital cost of each fund source is based on *market* value, not *book* (nominal or par) value because management require today's yields to vet new projects. Component costs should therefore be underpinned by current returns for each category of investor who may finance projects.

However, Ferry's ultimate concern, (rather than investors) is its own *break-even* income stream that may differ from the multiplicity of views held by proprietors and creditors. Consequently, the firm's component costs not only incorporate any tax effects, but also the costs of capital issues as follows:

Issue of ordinary shares = Dividend / Net proceeds of issue, plus the growth rate

= [(£0.24 / £1.00] + 5% = 29%

Retained earnings = Dividend yield, plus the growth rate

= 20% + 5% = 25%

Preference share issue = Dividend / Net proceeds of issue

= £0.06 / £0.40 = 15%

Debentures (after tax) = (interest /net proceeds of issue) multiplied by (1-tax rate)

= (£10.00 / £80.00) × (1 – 0.20) = 10%

5.5 Summary and Conclusions

In Chapter One, our study of strategic financial management began with a hypothetical explanation of a company's overall cost of capital as an investment criterion, designed to maximise shareholder wealth. By Chapter Four, we demonstrated that an *all equity* company should accept capital projects using the marginal cost of equity as a discount rate, because the market value of ordinary shares will increase by the project's NPV.

In this Chapter we considered the implications for project discount rates if funds were obtained from a variety of sources other than the equity market, each of which requires a rate of return that may be unique.

For the purpose of exposition, we analysed the most significant alternative to ordinary shares as an external source of funding, namely redeemable and irredeemable loan stock. We observed that corporate borrowing is attractive to management because interest rates on debt are typically lower than equity yields. The impact of corporate tax relief on debenture interest widens the gap further, although the tax-deductibility of debt is partially offset by the costs of issuing new capital, which are common to all financial securities.

In this newly leveraged situation, the company's overall cost of capital measured by a weighted average cost of capital (WACC) would seem to be a more appropriate investment criterion, rather than its cost of equity.

So, given the solution to your latest Review Activity, let us move on to Chapter Six and formally analyse how management can combine the component capital costs from various fund sources to derive a WACC as an overall discount rate for project appraisal. Thereafter, we shall explain MM's startling contributions to the subject.

5.6 Selected References

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6 Capital Gearing and the Cost of Capital

Introduction

If an *all-equity* company undertakes a capital project using the marginal cost of equity as its discount rate, the total market value of ordinary shares should increase by the project's expected NPV (ENPV). However, most firms use a *mix* of ownership capital and borrowed funds for new investments. The relationship between the two is termed *capital gearing* or *leverage*. A company is highly geared (levered) when it has a significant proportion of borrowing relative to shares in its capital structure. It is low geared when the ratio of debt to equity is small.

In Chapter Five we observed that corporate borrowing is attractive to management because interest rates on debt are typically lower than equity. This arises because capital market providers perceive debt as a less risky investment than equity. Interest is paid before shareholders' dividends and creditors also have a prior claim on a firm's assets in the event of liquidation. Moreover, interest on debt often qualifies for corporate tax relief. As a consequence:

- A judicious amount of debt introduced into a firm's capital structure should lower the overall *weighted average cost of capital* (WACC) employed as a cut-off rate for the appraisal of new projects, thereby increasing their ENPV and total corporate value.

You will also recall from Chapter Five that a company's component capital costs are derived by identifying the *opportunity cost* of each fund source using an appropriate valuation model that determines debt and equity yields. Given the normative assumption that management should maximise profit at minimum cost, we shall now extend our analysis to answer three questions.

- How do individual capital costs combine to define WACC for use in investment appraisal?
- How valid are the theoretical assumptions that underpin WACC computations?
- What are the real-world problems associated with WACC estimations?

6.1 The Weighted Average Cost of Capital (WACC)

Let us begin our analysis by first defining an overall cost of capital in a *taxless* world where management has access to only two sources of finance: equity and debt.

A general formula for WACC is given by the formula for a *simple weighted average*:

$$(15)\ K = K_e (V_E / V_E + V_D) + K_d (V_D / V_E + V_D)$$

Where: K = WACC,
K_e = cost of equity,
K_d = cost of debt,
V_E = market value of equity,
V_D = market value of debt.

If we now introduce corporate taxation (at a rate t) the after tax cost of debt $K_{d\,t}$ should be substituted into the preceding equation using the appropriate debt formulae from Chapter Six as follows.

$$(16)\ K = K_e (V_E / V_E + V_D) + K_{d\,t} (V_D / V_E + V_D)$$

This is equivalent to:

$$(17)\ K = K_e (V_E / V_E + V_D) + K_d (1\text{-}t) [(V_D / V_E + V_D)]$$

Equations (16) and (17) may be rewritten using simpler notation. For example, with tax:

$$(18)\ K = K_e (W_E) + K_{dt} (W_D)$$

Where: W_E = the weighting applied to equity (V_E / V_E + V_D)
W_D = the weighting applied to debt (V_D / V_E + V_D)

Thus, a firm financed equally by equity and debt yielding 10 percent and 5 percent, respectively, would calculate its WACC using Equation (18) as follows:

$$K = 10\% (0.5) + 5\% (0.5) = 7.5\%$$

Activity 1

Given the following company data:

K_e = 12%, K_d = 8%, V_E = £6 million, V_D = 4 million

Calculate the WACC and write down your thoughts on any assumptions that might validate its use as a discount rate for project appraisal before reading the next section.

The individual costs of equity and debt capital are weighted by their proportion of the company's total market value. Using Equation (18) and simplifying:

$$K = [(0.12 \times 0.6) + (0.08 \times 0.4)] / 1.0 = 0.104$$

Thus, the WACC applied by management as a discount rate for new project appraisal is 10.4 percent.

6.2 WACC Assumptions

The use of WACC as a corporate discount rate depends upon three fundamental assumptions.

- New projects must have the same homogenous risk-return profile as existing activities.
- Each project is marginal to the scale of the firm's existing operations.
- The company will retain its existing capital structure, leaving financial risk unchanged.

The reason for the first assumption is obvious. A company's component capital costs reflect the variability of future expected dividend and interest flows. Thus, it follows, that WACC must also reflect the overall risk of these combined flows. So, if we use this figure as a discount rate in project appraisal, the new investment's risk-return characteristics must satisfy the company's existing expected dividend and interest payments.

The second assumption is also common sense. When firms consider new investment, the relevant costs refer to the returns that the company must earn on relatively small incremental additions to its total capital base. From an economic viewpoint, they are *marginal* costs of capital and are only applicable to the appraisal of marginal investments: projects that are small relative to the size of the company.

Finally, the third assumption is necessary because WACC can only provide an appropriate discount rate if new projects are financed in the *same proportion* as existing assets. This arises for two reasons.

> If a company alters its capital structure, the weights applied to the component costs in the WACC calculation would also change, leading to a new discount rate.
>
> A change in the capital mix (gearing) might also affect the investors' perception of the *financial* risk associated with their investment in the firm. They may then react by buying or selling (as opposed to holding) their securities, thereby affecting the respective yields which determine the WACC.

For example, a new debt issue could increase the uncertainly experienced by the shareholders when they recognise that debt-holders will receive their claim to earnings (interest) before any dividend distribution. With increased risk, they sell their holding and equity prices may fall because the market requires a higher return as compensation. For the firm, what seems a simple change in the debt-equity ratio is, therefore, a complex decision. Quite apart from revised weightings at new market prices, it must also consider the explicit *marginal* cost of issuing debt *and* the *implicit* cost to the shareholders of their increased financial risk. All three may combine to produce a drastic change in WACC.

> **Activity 2**
>
> Changes in the financial mix (gearing) of a company and the impact of risk on its overall cost of capital and value do not necessarily invalidate the use of WACC as an investment criterion.
>
> Can you think of any reasons for this?

Whilst corporate investment decisions should determine a firm's overall cost of capital, management should avoid the mistake of always associating the explicit marginal costs of new capital issues with a specific project. Often it will be difficult, if not impossible to assign a particular project to a particular source of finance. A company's funds should therefore be viewed *collectively*.

In as much as finance is withdrawn from a *pool* of funds to invest in new projects, the pool is replenished as fresh capital is raised from outside, or profits are retained. Thus, the cost of capital used for any particular project is not the cost of a *specific* source of funds, but the *overall* cost of the company's pool: namely its WACC.

In the short run, it is frequently the case that certain funds might also be secured at advantageous rates depending upon prevailing market conditions. This will encourage firms to depart briefly from their long-run capital structure. Under such circumstances, however, WACC still represents an appropriate discount rate for long-term investment, providing the projects exhibit a similar risk-return profile.

Even if funds are raised explicitly from one source to finance an incremental investment, there are sound reasons for using the WACC as a discount rate, particularly if the change in the capital structure represents a short-run deviation from the desired capital mix.

First, a rational choice of funds is a *financial* decision taken in relation to the firm's long-term capital structure and not the *investment* decision. Second, there are substantial economies of scale to be gained in terms of reduced issue costs by raising large amounts of capital from one source and then another.

6.3 The Real-World Problems of WACC Estimation

Given the assumptions of *homogenous* risk, *marginal* investment and a *stable* capital structure, WACC seems an appropriate *minimum* return criterion for new projects that will hopefully *maximise* wealth. However, a company's overall cost of capital is a complex concept, which may include far more than shareholder dividend-growth expectations and fixed rates of debt interest. Moreover, the WACC model assumes that once they are determined, the variables selected for inclusion in the model are correctly defined and will not change. But think about it?

WACC applied to investment projects extends over numerous time periods. Thus, its value is likely to change with economic circumstances, thereby invalidating original NPV calculations. A simple problem concerns the estimation of after-tax loan costs determined by an existing tax regime that government revises. More complex, is stock market volatility. The 2008 global financial meltdown with falling interest rates, equity yields and security values was characterised by the market's aversion to financing even "blue chip" firms. Yet by 2015, equity markets had recovered to an all-time high.

Even if we ignore recent dramatic events, it is important to realise that at any point in *normal* economic cycles, the cost of capital and financial mix for individual companies can vary considerably, even within the same sector. Some firms are naturally more risky than others. Different companies may have different capital structures, by accident if not design. As we shall discover, differences in WACC have important consequences for the relative economic performance of companies and wealth creation.

Review Activity

You are asked to evaluate a marginal investment costing £100,000 and yielding £11,500 per annum for the foreseeable future, subject to the constraints that its acceptance will not alter the firm's existing risk-return profile and capital structure:

1. Derive and explain WACC as a discount rate if the corporate tax rate is 25 percent.
2. Evaluate the project's viability by applying the NPV decision rule.
3. Outline the implications for shareholder wealth maximisation.

The following information is available:

(i) Existing Capital Structure (£000 at cost)

Ordinary shares (12 million)	12,000
Retained earnings	4,000
6% Preference shares	2,000
6% Irredeemable debt	6,000

Review Activity (cont)

(ii) Ordinary Shares
The current market price (ex div) is £7.00. Forecast total dividends are £6 million, which represent 75 percent of earnings. Dividends have been growing at an annual compound rate of 5 percent. If new ordinary shares were issued now the costs incurred would represent 25 pence per share and a reduction below market value of 50 pence per share would also be required to ensure full subscription.

(iii) Preference Shares
Despite a par value of £1.00, current trades are only at 43 pence, with new issues at 40 pence.

(iv) Debentures
£100 loan stock currently priced at £92 would need to be issued at £90 percent

An Indicative Outline Solution

1. The WACC Discount Rate

The derivation of WACC is straightforward. Using the appropriate capitalisation formulae, explained in previous Chapters, including the Gordon growth model, tax and issue costs where appropriate:

- *Marginal* component costs are defined as follows:

Issue of ordinary shares = (dividend per share / net proceeds of issue) + growth rate
= (0.50 / 6.25) + 0.05 = 13%

Retained earnings = dividend yield + growth rate
= (0.50 / 7.00) + 0.05 = 12.1%

Preference Shares = dividend per share / net proceeds of issue
= 0.06 / 0.40 = 15%

Debentures (post-tax) = [interest per debenture (1 – tax rate)] / net proceeds of issue
= 6.00(1 – 0.25) / 90.00 = 5.0%

- *WACC* is defined by weighting these individual costs by their proportion in the company's existing capital structure and summating the products to arrive at their WACC. The simplest method is to use balance sheet data as follows:

Weighted Average Cost of Capital: Book Value

	Capital Structure (£ million)	Weight	Component Cost (%)	Weighted Cost (%)
Ordinary shares	12	0.50	13.0	6.50
Retained Earnings	4	0.17	12.1	2.06
Preference Shares	2	0.08	15.0	1.20
Debentures	6	0.25	5.0	1.25
Totals	24	1.00		11.01

However, this approach invites criticism. Although the capital mix will not change, *book* weights have been applied to component costs when clearly *market values* relating to current additions to the capital structure are more appropriate. What is required for *new* investment is a weighted average of its *marginal* costs of capital and not *historical* costs.

Weighted Average Cost of Capital: Market Value

	Capital Structure (£ million)	Weight	Component Cost (%)	Weighted Cost (%)
Ordinary Shares	84.0	0.89	13.0	11.57
Retained Earnings	4	0.04	12.1	0.48
Preference Shares	0.8	0.01	15.0	0.15
Debentures	5.4	0.06	5.0	0.30
Totals	94.2	1.00		12.50

The substitution of market values for book values in our WACC calculation raises the company's discount rate from 11.01 percent to 12.5 percent.

2. Project Evaluation

Project viability is established by applying the NPV decision rule to the project data, using the12.5 per cent WACC based on market values as the cut-off rate. The NPV of the £100k investment yielding £11.5k in perpetuity is given by:

NPV = [(11,500 / 0.125) = £92,000] – £100,000 = (£8,000)

So, the project *under-recovers* and should be *rejected*. However, it is worth noting that if we had applied book values to WACC the project would appear acceptable.

NPV = [(11,500 / 0.1101) = £104,450] – £100,000 = £4,450

Even so, you will be in no doubt as to which decision is correct. If wealth is to be maximised, projects must always be evaluated in terms of current investment opportunities foregone. Hence, the market value of capital employed and its corresponding incremental yield are the correct factors to determine a firm's WACC as an overall cut-off rate for investment.

3. Shareholder Wealth Maximisation

The shareholder wealth implications of the correct accept-reject decision using WACC as a discount rate can be confirmed by analysing the investment's impact on the equity yield. Using market weights from the previous table, let us first calculate the proportion of equity applied to the investment:

£100,000 (0.890) = £89,000

Next calculate the annual cash return available to the new ordinary shareholders.

	Capital Investment £	Capital Cost %	Investor Return £	
Annual Cash Inflow				11,500
Retained Earnings £100,000 x 0.04	4,000	12.1	484	
Preference Shares £100,000 x 0.01	1,000	15.0	150	
Debentures £100,000 x 0.06	6,000	5.0	300	
	11,000			934
Ordinary Shares				10,566

Finally, let us reformulate this cash return as the investment's yield on the ordinary share issue.

Project equity yield = £10,566 / £89,000 = 11.87%

Because this is less than the 13 per cent marginal cost of new issues calculated at the outset of our analysis, we can confirm that the investment proposal should be rejected.

You may also care to verify that even if the 12.1 per cent cost of retained earnings were incorporated into the yield calculation to provide a more comprehensive measure of the equity rate (i.e. dividends plus retentions) the overall return would only be 11.88 percent.

Since this too, is lower than the 12.1 percent yield on shares currently in issue, the project should still be ignored.

6.4 Summary and Conclusions

The previous Activity serves as a timely reminder that efficient financial management (based on agency theory) should comprise two distinct but inter-related functions as shareholder wealth maximisation criteria.

- The *investment decision*, which identifies and selects opportunities to maximise ENPV.
- The *finance decision*, which identifies potential fund sources required to sustain investment, evaluates the return expected by each, then selects the optimum mix that minimises their overall combined cost (WACC).

However, as we mentioned in Chapter One, the derivation of an optimal capital structure and minimum WACC is a controversial subject. So far, we have observed that the issue of lower-cost debt (which might also incorporate tax relief) rather than equity, should reduce WACC and increase overall corporate value. But as we shall explain in Chapter Seven, this *may only* be *true up to a point*.

What is now termed a "traditional" view of finance states that when debt is introduced into a firm's capital structure, leverage may initially reduce WACC and increase total corporate value. However, when shareholders and debt financiers perceive that gearing levels are excessive, the WACC will increase and value fall. This *saucer-shaped* WACC plotted against increasing leverage is caused by a combination of higher returns required for existing equity issues and higher interest rates on new increments of debt. It compensates both capital providers for the higher *financial risk* associated with their respective corporate investment.

Because companies have a contractual obligation to pay interest on debt, any variability in earnings arising from business risk is transferred to the shareholders. As "lenders of last resort" they bear the inconsistency of investment returns, which may result in a reduction, or even non-payment, of dividends. This "pecking order" phenomenon is amplified as the gearing ratio rises.

To compensate for higher levels of financial risk, rational shareholders require higher yields on their holdings, thereby producing a lower capitalised value of expected earnings available for distribution and. a lower share price. At extremely high levels of gearing, the situation may be further aggravated by debt holders. They too, may require ever-higher rates of interest as their investment takes on the characteristics of equity. It no longer represents a *prior* claim, but perhaps the *only* claim, on either the firm's income or assets.

To summarise this "traditional" approach to theories of capital structure, WACC and corporate value:

> Beyond some minimum point, incremental borrowing will not reduce WACC. It increases because of the detrimental effect on existing equity prices, thereby increasing shares yields. In turn, this leads to higher marginal costs of debt and lower prices for further increments of borrowing, resulting in a dramatic decline in the overall value of the firm.

So far so good, but a contrary view originally published by Modigliani and Miller (MM) in 1958 hypothesised that WACC and total corporate value remain constant, irrespective of the level of gearing (leverage).

> Equity and debt returns (like dividends and retained earnings) are *perfect economic substitutes*. With the introduction of cheaper debt finance, any rational change in the gearing ratio immediately elicits a compensatory change in the cost of equity. Specifically, it offsets changes in the level of financial risk, thereby leaving WACC and overall corporate value the same.

If you are perplexed, or feel we are moving too fast, don't worry. From 1958 through to the late 1970s the traditional academic community and financial analysts' perception of leverage dynamics were thrown into disarray by the application of MM's "law of one price", creating a flurry of new research.

But as we shall discover in Chapter Seven, their basic hypothesis is "logical if you think about it".

- In terms of the investment decision, WACC still occupies a pivotal position as an opportunity cost (return) criterion which justifies the finance decision.
- A company wishing to maximise shareholders' wealth would still only deploy funds if their marginal yield at least satisfies the rates of return that its investors can earn elsewhere at commensurate risk.

Therefore, the two final questions we must answer in Part Three are "why" and "how" MM could theoretically discredit traditional approaches to capital structure, WACC and corporate value?

6.5 Selected Reference

Modigliani, F. and Miller, M.H., "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review,* Vol. XLVIII, No. 3, June 1958.

7 MM and Capital Structure

Introduction

For the purpose of exposition, the derivation of a company's weighted average cost of capital (WACC) in Chapter Six was kept simple. Given financial management's strategic objective is to maximise the market value of ordinary shares (common stock) our analysis assumed that:

- The value attributed by the market to any class of financial security (debt or equity) is the expected PV of its cash returns, discounted at an opportunity rate that reflects the financial risk associated with those returns.
- The ENPV of a project, discounted at a company's WACC (based on debt plus equity) is the amount by which the market value of the company will increase if the project is accepted; subject to the constraint that acceptance does not change WACC.

We specified *three* necessary conditions that underpin this constraint and justify the use of WACC as a cut-off rate for investment.

- The new project has the same business risk as the company's existing investment portfolio.
- The project is small, relative to the scale of its existing operations.
- The company intends to retain its existing capital structure (i.e. financial risk is constant).

Yet, we know that even if business risk is *homogenous* and projects are *marginal*, capital structures and the financial risk of future investments are rarely *stable*. The component costs of corporate finance (and hence WACC) are susceptible to periodic change as *external* forces unfold. The availability of new external funding may also be a *limiting factor*, as evidenced by the market's aversion to provide debt or equity after the 2007 global meltdown.

So, let us develop a *dynamic* critique of capital structure and the overall cost of capital (WACC) and ask ourselves whether management can increase the value of the firm, not simply by selecting an optimal *investment*, but also by manipulating its *finance*. If so, there may be an optimal capital structure arising from a debt-equity *trade-off*, which elicits a least-cost combination of financial resources that minimises the firm's WACC and maximises its total value.

In the summary to Chapter Six, we touched on the case for and against an optimal capital structure and WACC based on "traditional" theory and the MM economic "law of one price" respectively. Later in this Chapter we will pick up on these conflicting analyses in detail. Specifically, we shall examine the MM *arbitrage* proof (1958) whereby investors can profitably trade securities with different prices between companies with different leverage until their WACC and overall value are in *equilibrium*.

Unlike the traditionalists, MM maintain that the equilibrium value of any company is *independent* of its capital structure and derived by capitalising expected project returns at a *constant* WACC appropriate to their class of business risk. Yet both theories begin with a common assumption. Because of higher financial risk, the cost of equity is higher than the cost of debt and rises with increased leverage (gearing).

So, before we analyse why the two theories part company, let us initially explain how increased gearing affects shareholder returns by graphing the relationship between earnings yields and net operating income (NOI) *i.e.* earnings before interest and tax (EBIT) when firms incorporate cheaper debt into their capital structure.

Like our approach to the MM dividend debate in Chapter Four, we shall underpin their theoretical exposition with appropriate Activities (and outline solutions). And because of the subject's complexity, we shall (again) develop a data set using a Review Activity to summarise and critique our analysis as a guide to further study.

7.1 Capital Structure, Equity Return and Leverage

To assess the impact of a changing capital structure on capital costs and corporate values, let us begin with a fundamental assumption of capital market theory, which you first encountered in Chapter One, namely that investors are *rational* and *risk averse*. Companies must offer them a return, which is inversely related to the probability of its occurrence. Thus, the crucial question for financial management is whether a combination of stakeholder funds, related to the earnings capability of the firm, can minimise the risk that confronts each class of investor. If so, a firm should be able to minimise its own discount rate (WACC) and hence, maximise total corporate value for the mutual benefit of all.

We know from previous Chapters that *total risk* comprises two inter-related components with which you are familiar, *business risk* and *financial risk*. So, even when a firm is financed by equity alone, the pattern of shareholder returns not only depends upon periodic post-tax profits (business risk). It also arises from managerial decisions to withhold dividends and retain earnings for reinvestment (financial risk).

As we explained in Chapter Three, using the Gordon growth model (1962) if rational, risk averse investors prefer dividends now, rather than later, the question arises as to whether their equity capitalisation rate is a positive function of a firm's retention ratio. In otherwords, despite the prospect of a capital gain, does a "bird in the hand" philosophy require a premium for the financial risk associated with any diminution in the dividend stream? If so, for a given *investment* policy, corporate *financial* policy must also affect the overall discount rate that management applies to NPV project analyses and therefore the market value of ordinary shares.

When a firm issues debt, we can apply the same logic to arrive at similar conclusions. Financial policies *matter* because the degree of leverage arising from the debt-equity ratio (like the dividend payout ratio) determines the level of financial risk that confronts the investor.

The Theoretical Background

Initially, when a firm borrows, shareholder wealth (dividends, plus capital gains) can be increased if the effective cost of debt is lower than the original earnings yield. In efficient capital markets such an assumption is not unrealistic:

- Debt holders receive a guaranteed return and in the unlikely event of liquidation are usually given security in the form of a prior charge over the assets.
- From an entity viewpoint, debt interest qualifies for tax relief.

You should note that the productivity of the firm's resources is unchanged. Irrespective of the financing source, the same overall income is characterised by the same degree of business risk. What has changed is the mode of financing, which increases the investors' return in the form of earning per share (EPS) at minimum financial risk. So, if this creates demand for equity and its market price rises proportionately, the equity capitalisation rate should remain *constant*. For the company, the beneficial effects of cheaper financing therefore outweigh the costs and as a consequence, its overall cost of capital (WACC) falls and total market value rises.

Of course, the net benefits of gearing cannot be maintained indefinitely. As a firm introduces more debt into its capital structure, shareholders soon become exposed to greater financial risk (irrespective of dividend policy and EPS), even if there is no likelihood of liquidation. So much so, that the demand for equity tails off and its price begins to fall, taking total corporate value with it. At this point, WACC begins to rise.

The increased financial risk of higher gearing arises because the returns to debt and equity holders are *interdependent* stemming from the same investment. Because of the contractual obligation to pay interest, any variability in operating income (EBIT) caused by business risk is therefore transferred to the shareholders who must bear the inconsistency of returns. This is amplified as the gearing ratio rises. To compensate for a higher level of financial risk, shareholders require a higher yield on their investment, thereby producing a lower capitalised value of earnings available for distribution (*i.e.* lower share price). At extremely high levels of gearing, the situation may be further aggravated by debt holders. They too, may require ever-higher rates of interest as their investment takes on the characteristics of equity and no longer represents a prior claim on either the firm's income or assets.

Even without increasing the interest rate on debt, the impact of leverage on shareholder yields can be illustrated quite simply. Consider the following data (£ million):

Company	**Ulrich plc**			**Hammett plc**		
MARKET VALUES						
Equity	100			60		
Debt	_			40		
Total	100			100		
NET OPERATING INCOME						
	Norm	Deviations		Norm	Deviations	
EBIT	8.0	10.0	12.0	8.0	10.0	12.0
Interest (10%)				4.0	4.0	4.0
EBT	8.0	10.0	12.0	4.0	6.0	8.0
Corporation Tax (25%)	2.0	2.5	3.0	1.0	1.5	2.0
EAT	6.0	7 .5	9.0	3.0	4.5	6.0
Earnings Yield (%)	6%	7.5%	9%	5%	7.5%	10%

The two companies (Ulrich and Hammett) are identical in every respect except for their methods of financing. Ulrich is an all-equity firm. Hammett has £40 million of 10 percent debt in its capital structure. A comparison of net operating income (EBIT) and shareholder return (earnings yield) is also shown, if business conditions deviate 20 percent either side of the norm.

What the table reveals is that the returns to ordinary shareholders in the all-equity company only fluctuate between 6 percent and 9 percent as EBIT (business risk) fluctuates between £8 million and £12 million. However, the existence of a fixed interest component for the geared company amplifies business risk in terms of the total risk borne by the ordinary shareholder. Despite the benefits conferred on Hammett and its shareholders by the tax deductibility of debt, the greater range of equity returns (5–10 per cent) implies greater financial risk.

Thus, if shareholders act rationally and business prospects are poor, they may well sell their holdings in the geared company, thereby depressing its share price and buy into the all-equity firm causing its price to rise.

Our preceding discussion suggests that for a given level of earnings, a company might be able to trade the costs and benefits of debt by a combination of fund sources, which achieves a lower WACC and hence a higher value for equity. To implement this strategy, however, management obviously need to be aware of shareholder attitudes to its existing financial policy and those of competitors under prevailing economic conditions. Even "blue chip "companies with little chance of liquidation are not immune to financial risk.

Activity 1

Use the previous data for Ulrich and Hammett to:

1. Graph the relationship between their respective earnings yield (vertical axis) and EBIT (horizontal axis) and establish the indifference point between their shareholder clienteles.
2. Summarise your graph's illustrations concerning shareholder preferences.

From the raw data you should have observed that if shareholders require a 7.5 per cent return and the EBIT (NOI) of both companies equals £10 million, they would be indifferent to investing in either, irrespective of current financial policies. By plotting a graph, however, you can also see that the relationship between earnings yield and EBIT is *positive* and *linear* for both companies but *different*. For the all-equity firm it is less severe, with a shareholder's return of zero corresponding to an EBIT figure of zero that passes through the origin in Figure 7.1. For the geared company, the EBIT figure equivalent to a zero earnings yield intersects the horizontal axis at the value of 10 percent debenture interest payable (£4 million) and rises more steeply.

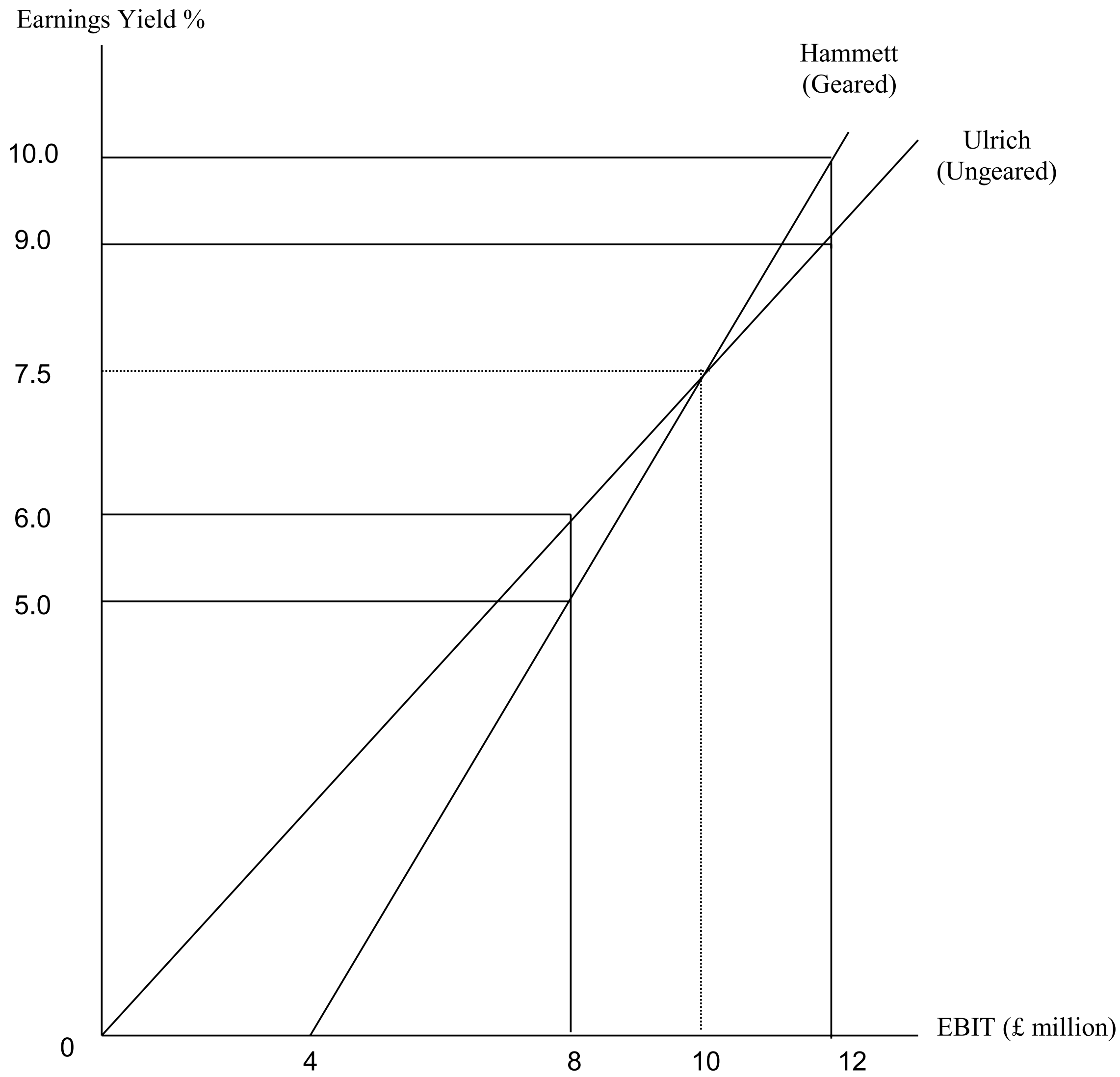

Figure 7.1: Capital Gearing and the Relationship between EBIT and Earnings Yield

The intersection of the two straight lines represents the point of *indifference* between the two companies. To the left of this point, shareholders would prefer to invest in Ulrich (ungeared) since they receive a better return for a lower level of EBIT. To the right, they would prefer Hammett (geared) for the same reasons. What we are observing is that leverage, which here means the incorporation of 10 percent loan stock into a firm's capital structure, increases shareholders' *sensitivity* to changes in EBIT (business risk) and therefore the financial risk associated with equity; hence the steepness of the line.

7.2 Capital Structure and the Law of One Price

The previous Activity illustrates why rational, risk adverse investors prefer the ordinary shares of higher geared companies when economic conditions are good or improving but switch to lower geared firms when recession looms. Both strategies represent an *optimum* risk-return trade off because:

- Ordinary shares represent a more speculative investment when there is an increasing contractual obligation on the part of the company to pay periodic interest on debt.
- The higher the gearing and more uncertain a firm's overall profitability (EBIT) the greater the fluctuation in dividends plus reserves.

As mentioned earlier, the returns to debt and equity holders are *interdependent*. They stem from the same resources. So, what we have observed is the transfer of business risk to shareholders who must bear the inconsistency of returns as the firm gears up. Thus, it would seem that management should finance corporate investments so that their shareholders (to whom they are ultimately responsible) receive the highest return for a given level of earnings and risk. And this is where MM disagree with traditional theorists.

The Traditional Theory of Capital Gearing and WACC

Traditionalists believe that if a firm substitute's lower-cost debt for equity into its capital structure, WACC will fall and value rise to a point of indebtedness where both classes of investor will require higher returns to compensate for increasing financial risk. Thereafter, WACC rises and value falls, suggesting an optimum level of gearing that minimises WACC and maximises value. Figure 7.2 sketches these phenomena.

The debt-equity ratio (V_D/V_E) is plotted along the horizontal axes of both diagrams. The costs of both types of capital (setting $K_d < K_e$) are given on the vertical axis of the upper graph. The vertical axis on the lower graph plots total market value ($V=V_E+V_D$). To keep the analysis simple K_d is held constant and its tax deductibility is ignored. Our aim is not to develop a real world model (more of which later) but to illustrate the basic relationships between capital costs, corporate value and leverage.

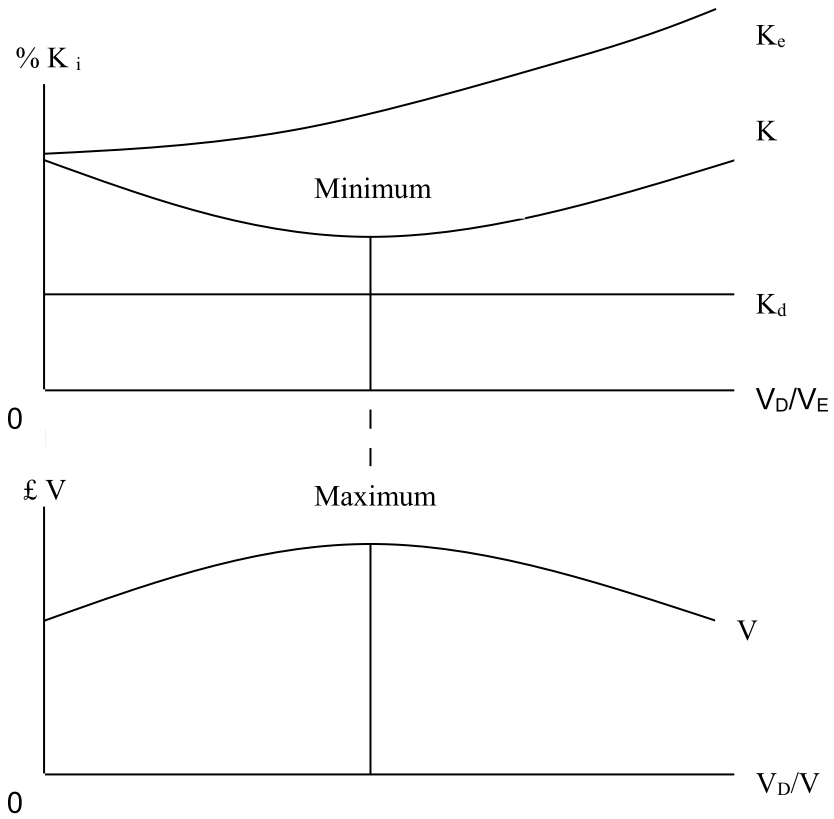

Figure 7.2: Traditional Theory with a Constant Cost of Debt in a Taxless World

Figure 7.2 confirms the traditional view that WACC is characterised by a U-shaped average cost curve K (familiar to economists).This is because the benefits of cheaper debt finance ($K_d<K_e$) are eventually offset by the cost of equity, which increases *exponentially* as the firm gears up.

Turning to total market value V, (equity plus debt) if we define the relationship:

(19) $V = I / K$

Where: I = NOI = net operating income (earnings before interest)
: $V = V_E + V_D$ = total market value
V_E = market value of equity
V_D = market value of debt
K = WACC,
$= K_e (V_E / V_E + V_D) + K_d (V_D / V_E + V_D)$
$= K_e (V_E / V) + K_d (V_D / V)$
K_e = cost of equity
K_d = cost of debt

We now observe that an *inverse* relationship exists between V and K, given I (NOI). As one rises, the other falls and *vice versa.* Thus, the lower graph of Figure 7.2 illustrates that the degree of leverage, relative to the total market value of the firm, has an *inverted* U-shaped function. As K (WACC) responds to changes in the gearing ratio and the rising cost of equity, V presents us with a mirror image. So, according to traditional theory, if companies borrow at an interest rate lower than their returns to equity, the implications for financial management are clear.

For a given investment policy, there exists an optimal financial policy (debt-equity ratio) which defines a least-cost combination of financial resources.

At the point where overall cost of capital (WACC) is minimised, total corporate value is maximised and so too, is the market value of ordinary shares.

The MM Cost of Capital Hypothesis

Like much else in finance, the traditional case for an optimal capital structure did not arise from empirical evidence: merely a plausible assumption concerning the cost of equity at different levels of gearing (initially constant, then rising with greater rapidity). But what if the relationship between the two is mistaken? Would an optimal WACC and corporate value still emerge?

To answer both these questions, MM (*op.cit.*) developed an alternative hypothesis, which produced two conclusions that confounded both traditional theorists and financial analysts.

- The total value of a firm represented by the NPV of an income stream discounted at a rate appropriate to its business risk, should be unaffected by shifts in financial structure.
- Any rational debt-equity ratio should produce the same cut-off rate for investment (WACC).

Unlike many of their contemporaries, MM based their conclusions on *partial equilibrium* analysis (not anecdotal evidence) prefaced by a number of rigorous assumptions, which they later relaxed to incorporate subsequent empirical research. These should be familiar, since they are based on *perfect markets* and Fisher's Separation Theorem (1930) outlined in Chapter One

- Investors are rational.
- Information is freely available.
- Transaction costs are zero.
- Debt is risk-free (the return is constant)
- Investors are indifferent between corporate and personal borrowing.
- The tax system is neutral.

MM also based their analysis on the traditional equation for total market value:

(19) $V = I / K$

However, where they disagree with traditional theory relates to their definition of WACC, which hinges on the behaviour of the equity capitalisation rate as a firm gears up.

MM reason that WACC reflects the business risk associated with the variability of total earnings, rather than their financial risk, *i.e.* how they are packaged for distribution in the form of interest and dividends. They maintain that irrespective of the debt-equity ratio (V_D/V_E) if expected earnings (I) remain the same, then WACC (K) and hence total value (V) must also be constant.

Based on their "economic law of one price" MM further reasoned that irrespective of leverage, close *financial substitutes*, such as similar companies cannot sell at different prices. Two companies with the same business risk and identical total income will have the same total market value and WACC, even if their gearing ratios differ.

Expressed algebraically, if:

$V_1 = V_2 =$ the value for two companies.
$I_1 = I_2 =$ average NOI represented by the expected value of its probability distribution

Then the WACC for any company in the same risk class:

(20) $K = I_1 / V_1 = I_2 / V_2$

And because K =K_e in the unlevered firm, the WACC for the geared company must also equal the cost of equity capital K_e of the all equity firm.

Thus, if the cost of debt K_d is constant (an assumption that MM later relax) all that needs to be resolved is the precise relationship between the rising cost of equity K_e and the debt-equity ratio V_D/V_E when a firm gears up. Is it *exponential*, as the traditionalists suggest (Figure 7.2) or not.

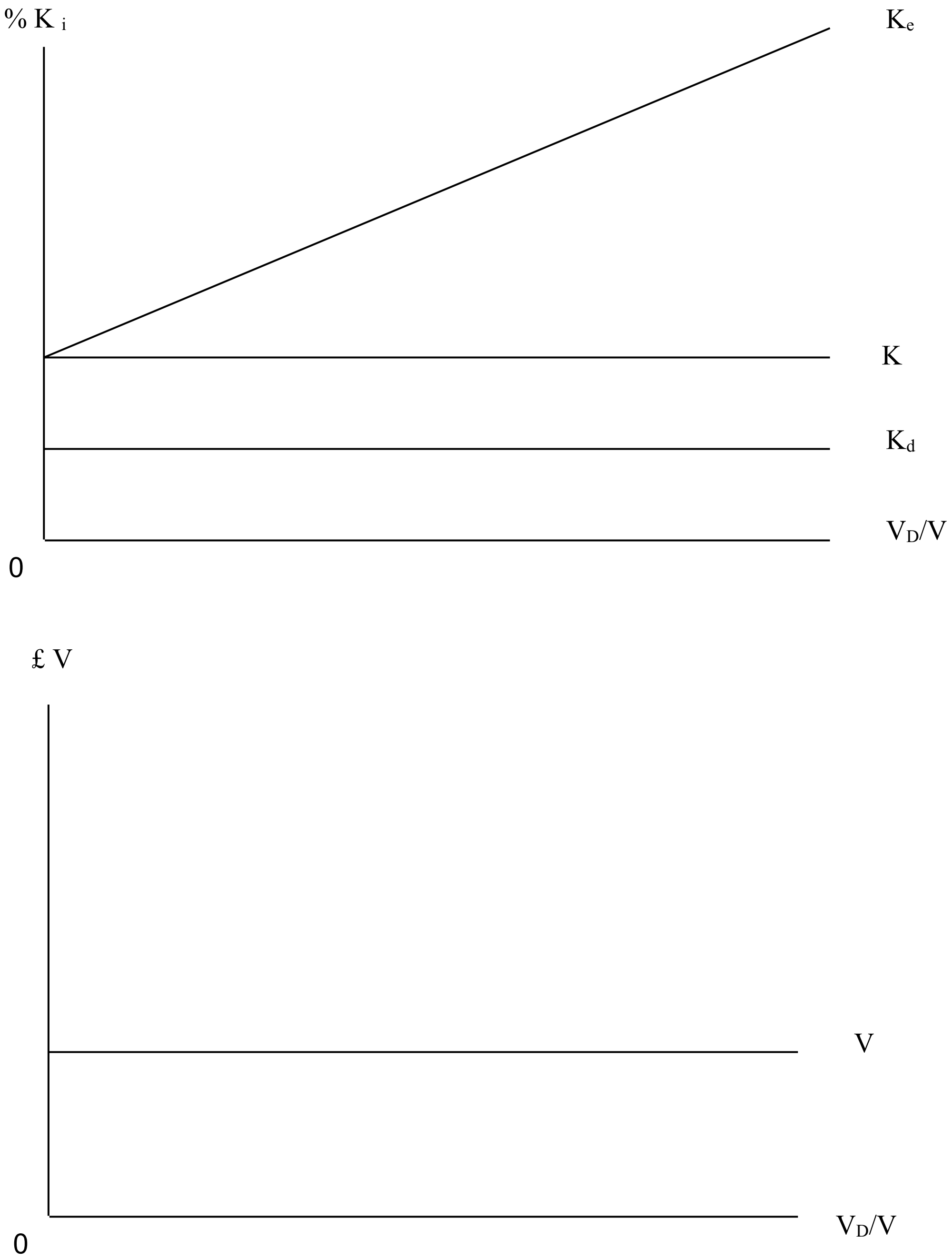

Figure 7.3: The MM Theory with a Constant Cost of Debt in a Taxless World

According to MM, if we ignore corporate taxation and tax relief on interest, the equity capitalisation rate K_e will still increase but not exponentially as the traditionalists believe. The rise *exactly offsets* the benefits of increasing the proportion of cheaper loan stock in a firm's capital structure leaving WACC unchanged. This *linear* relationship is sketched in the upper graph of Figure 7.3, which translates into the following equation.

(21) $K_{eg} = K_{eu} + [(V_D / V_E) (K_{eu} – K_d)]$

Where: K_{eg} = the cost of equity in a geared company
K_{eu} = the cost of equity in an ungeared company
K_d = the cost of debt capital
V_D = the market value of debt in the geared company
V_E = the market value of equity in the geared company

- K_{eg} (leveraged) is equivalent to K_{eu}, the capitalisation rate for an all-equity stream of the same class of business risk, plus a premium related to financial risk. This is measured by the debt-equity ratio (V_D/V_E) multiplied by the spread between K_{eu} and K_d.
- The financial risk premium (the second term on the right of the equation) causes equity yields to rise at a *constant* rate as compensation for financial risk when the firm gears up.

Since the WACC in companies of equivalent business risk is the same, irrespective of leverage, their total market value (V) will also be the same if the companies are identical in every respect except their gearing ratio. Thus:

(22) $V_U = V_G = V_E + V_D$

Where: V_U = the market value of an ungeared all equity company
V_G = market value of an identical geared company (equity plus debt)

The lower graph of Figure 7.3 plots constant value (V) against an increasing debt-equity ratio (V_D/V_E).

If WACC and overall corporate value are unaffected by leverage as MM hypothesise, the implication for strategic financial management are profound. As we first mentioned in Chapter One, financial decisions (which include the dividend policy, as well as gearing) are irrelevant to investment decisions (the valuation of capital projects and their selection).

Activity 2

In a subject still dominated by the work of Modigliani and Miller it is important that you refer to their original articles, if only to confirm what you read elsewhere.

MM's 1958 paper "The Cost of Capital, Corporation Finance and the Theory of Investment" (referenced at the end of this Chapter) sets out their original case for the irrelevance of financial structure to corporate valuation and capital cost (WACC) in a perfect capital market. Search for it and skim through, to get the broad thrust of their arguments (even if you find the mathematics complex). Then produce brief answers to the following questions.

a. There are three *propositions* advanced by MM. What are they and how are they proved?
b. How do MM's conclusions differ from a traditional view of capital structure in a taxless world where the cost of debt is constant?
c. Within the context of investment appraisal, what are the implications of MM's hypothesis for financial management?

a) The Propositions

Using our own notation, the three propositions advanced by MM are:

Proposition I: Overall market value (V) is *independent* of the debt-equity ratio (V_D/V_E).

Proposition II: To offset financial risk, the equity capitalisation rate (K_{eg}) increases at a *constant* rate as V_D/V_E rises, with two corollaries:

- K is unaffected by V_D/V_E ,
- K = K_{eu} for an unlevered firm.

Proposition III: Shareholder wealth is maximised by *substituting* an equity capitalisation rate of an unlevered firm ($K_{e\,u}$), for the cut-off rate (K) of a levered firm.

MM then explain how:

i. Proposition I can be proved by *arbitrage* (more of which later).
ii. Proposition I can be used to prove Proposition II, which states that K is unaffected by V_D/V_E.
iii. Proposition III follows logically from Propositions II and III, since market value equals equity value ($V = V_E$) and therefore $K = K_e$ in an unlevered firm.

b) MM's Conclusions

Even in a world of zero taxation with a constant cost of debt, a comparison of Figure 7.2 with Figure 7.3 reveals that MM's conclusions contrast sharply to a traditional view.

WACC does not vary with gearing. There is no optimal debt-equity ratio and the market value of the firm remains constant. According to MM, the cost of equity capital is no longer an *exponential* function of increasing leverage. Given MM's contention that K is constant, K_e rises *linearly* as V_D/V_E increases.

c) The Investment Implications

If MM's hypothesis is correct, the "traditional" financial decisions that confront management when investment decisions include debt are eliminated. The net result is that WACC (the cut-off rate for investment) and total corporate value remain the same. Gearing is therefore irrelevant to project evaluation and shareholder wealth maximisation.

7.3 MM and Proposition I (the Arbitrage Process)

Your reading for Activity 2 should confirm how the logic of MM's cost of capital hypothesis stems from their first proposition that corporate value is independent of capital structure based on *arbitrage* and *partial equilibrium* analysis.

Arbitrage occurs when investors sell financial securities to buy cheaper perfect substitutes, thereby depressing the price of the former and increasing the price of the latter, until their market prices are in equilibrium.

In perfect markets, MM maintain that if a traditional view of capital structure were to exist, it should only be a short-run *dis-equilibrium* phenomenon. Rational (risk-averse) *arbitrageurs* will respond quickly to prevent the existence of two firms with identical business risk and the same expected NOI from selling at different prices.

- Shareholders in an over-valued company (what traditionalists define as highly geared) will change its total value by selling shares in that company and buying shares in an under-valued (*i.e.*ungeared) company. To implement these transactions, shareholders will even undertake personal borrowing to maximise their stake in the ungeared company, up to a point where their personal investment portfolios have the same degree of leverage as the overvalued firm.
- As a result of what MM term *home-made leverage* (personal borrowing), investor income is increased at no greater financial risk. Eventually, through supply and demand, the price of shares in the overvalued company will fall, while that of the undervalued company will rise until no further financial advantage is gained. At this point of *equilibrium*, overall market value and the overall cost of capital (WACC) for the two companies will also be the same.

The Mathematics of Arbitrage

Before illustrating the arbitrage process with a numerical Activity, let us express it algebraically.

Given what MM term *disequilibrium*: a temporary phenomenon where the total market value of an ungeared company (V_1) is lower than that of a geared company (V_2), we can define:

$$(23)\ V_1 = V_{E1} < V_2 = V_{E2} + V_{D2}$$

Now assume a shareholder holds a proportion $A(V_{E2})$ of the total equity in the geared company. The investment will yield:

$$(24)\ Y_2 = A(I - K_d \,.\, V_{D2})$$

Where:

Y_2 = the income available to the shareholder
$I = I_1 = I_2$ = NOI
K_d = the interest on corporate debt.

The investor now sells his shares in Company 2 for $A(V_{E2})$ and also borrows an amount equal to $A(V_{D2})$ at the same rate of interest K_d to invest $A(V_{E2} + V_{D2})$ in the ungeared Company 1.

He is therefore substituting *personal* leverage for *corporate* leverage by borrowing the same proportion (AV_{D2} / AV_{E2}) as that of the geared company's debt–equity ratio (V_{D2} / V_{E2}).

This new holding in Company 1 will yield Y_1 (net of interest on personal borrowing):

$$(25)\quad Y_1 = \frac{\{A(V_{E2} + V_{D2})I\}}{V_1} - K_d \,.V_{D2}$$

$$= A\left(\frac{V_2}{V_1} .I - K_d \,.\, V_{D2}\right)$$

And further simplifying:

$$(26)\ Y_1 = A\,(V_2 \,.\, Y_2) / V_1$$

Thus, we observe that if $V_1 < V_2$, as the traditionalists advocate, then $Y_1 > Y_2$ and shareholders' income can be increased by arbitrage.

However, as MM suggest, switching from the geared (overvalued) company to the all-share firm will eventually depress the equity value of the former, while raising the price of the latter, until they are in equilibrium.

At this point, (where $V_1 = V_2$ and $Y_1 = Y_2$) the arbitrage process elicits no further gains and shareholders will be indifferent to levels of gearing. Moreover, because we have assumed that NOI (I) is identical for both companies, it also follows that their WACC (K) must be identical.

To summarise MM's basic theoretical proposition.

Given: $I = I_1 = I_2$

- when $V_1 < V_2 : Y_1 > Y_2$ and $K_1 > K_2$
- when $V_1 = V_2 : Y_1 = Y_2$ and $K_1 = K_2$

Thus, they conclude that in equilibrium under certain conditions, changes in capital structure can have no effect on overall shareholder income, corporate value, or cost of capital.

7.4 MM and Real World Considerations

We could now move on from Proposition I to prove Propositions II and III and then extend MM's analysis into a real world of differential corporate and personal taxation, where the cost of debt is tax deductible and no longer constant. However, we shall save all this for the Activities in our companion Exercise text.

For the moment, we simply need to appreciate that the MM arbitrage proof confounded the traditional academic and investment community, who argued that their assumption of perfect markets, particularly a neutral tax system without tax relief on debt interest invalidated their conclusions. However, MM (1963) were the first to concede that an allowance for tax relief will reduce the cost of loan stock, lower WACC and increase total value as a firm gears up. But this is a result of fiscal policy and not business or financial policy. The whole point of their hypothesis was to provide a *benchmark* to assess the impact of introducing more realistic assumptions as a basis for more complex analyses (which we shall evaluate in our Exercise text) such as:

- Do personal as well as corporate fiscal policies affect capital structure?
- Are corporate borrowing and investment rates equal?
- How do investor returns (debt and equity) behave with extreme leverage?
- Are management better informed than stock market participants?
- Do managerial objectives conflict with those of investors?
- And if so, do management prefer different sources of finance?

Review Activity

	Elbow (ungeared)		Dimebag (geared)
Distributable Earnings (No Tax)			
NOI (I)	100	=	100
Debt Interest (K_d = 5%)	-		10
Shareholder Income	100	>	90
Market Values			
Equity (V_E)	1000	>	900
Debt (V_D)	-		200
Total Value (V)	1000	<	1,100
Capital Costs			
Equity Yield (K_e)	10%	=	10%
Cost of Debt (K_d)	-		5%
WACC (K = I / V)	10%	>	9.09%

The previous table presents a series of *traditional* financial relationships between two firms (Elbow and Dimebag) that are identical in every respect, except for their capital structure (€ 000).

Required:

1. Use the data set to illustrate the benefits of arbitrage for an investor who currently owns 10 per cent of Dimebag's shares.
2. Summarise the effects of arbitrage as more investors enter the process.

An Indicative Outline Solution

From the data you should have observed what MM term *disequilibrium*. The total market value and WACC of two equivalent companies differ. So, arbitrage is a profitable strategy for all investors in the geared firm.

1. The Arbitrage Process

Now let us consider a series of arbitrage transactions for a single investor who holds 10 percent of the equity in Dimebag (the higher valued geared firm) whose annual income is therefore €9,000 (€90,000 × 0.10).

1. She sells her total shareholding for €90,000 (10 percent of €900,000) which reduces the financial risk of investing in the geared company to zero.
2. She now buys shares in Elbow (the ungeared, all-equity firm) but how much should she spend?

3. In order to compare like with like, it is important to hold the investor's exposure to financial risk at the same level as her original investment in Dimebag. With a €90,000 equity stake in that company, management presumably used this as collateral to borrow €20,000 of corporate debt on her behalf (i.e.10 per cent of €200,000). So, in a perfect capital market where private investors can borrow on the same terms as the company, she can substitute *homemade* leverage for *corporate* leverage to finance her new investment in the all-equity firm.
4. She borrows €20,000 at 5 percent per annum, an amount equal to 10 percent of the firm's debt.
5. As a result, the investor now has a total of €110,000 (€90,000 cash, plus €20,000 of personal borrowing) with which to purchase the ungeared shares in Elbow.
6. Because Elbow's yield is 10 percent, the investor will receive an annual return of €11,000 (€110,000 × 0.10). However, she must pay annual interest on her personal loan (€20,000 × 0.05 = €1000).Therefore, her annual net income will be €10,000 (€11,000 – €1000).

So, to conclude: is our investor better off?

We can measure her change in income as follows:

The Arbitrage Process

	€
Shareholder income in Elbow (ungeared)	11,000
Shareholder income in Dimebag (geared)	9,000
Change in income	2,000
Interest on borrowing (5%)	1,000
Net Gain from *Arbitrage*	1,000

Thus, shareholder income has increased with no change in financial risk. The reason the investor has benefited is because the leveraged shares of Dimebag are overvalued relative to those of Elbow. If proof be needed, you should be able to confirm that the equity capitalisation rates for both firms originally equalled 10 per cent, despite differences in their total shareholder income.

2. Summary

As more investors enter the arbitrage process (trading shares to profit from disequilibrium) the equity value of geared firms will fall, whilst those of their ungeared counterparts will rise. To similar but opposite effect, their equity capitalisation rates will rise and fall respectively, until their overall cost of capital (WACC) is equal. Thus, MM's message to "traditionalists" is clear.

In *equilibrium*, shareholders will be *indifferent* to the degree of leverage and the arbitrage process becomes *irrelevant* to management's strategic evaluation of project investment and its wealth maximisation implications.

7.5 Summary and Conclusions

We have considered whether companies can implement *optimum* financial policies concerning their capital structure, which *minimise* the weighted average cost of capital (WACC) and *maximise* total corporate value.

Given your knowledge of capital market efficiency (Part One), equity valuation (Part Two) and the cost of debt (Part Three: Chapters' Five and Six) we have also focussed upon the controversial question as to whether optimal financial decisions contribute to optimum investment decisions.

The traditional perception is that by trading lower-cost debt for equity, WACC will fall and value rise to a point of indebtedness where both classes of investor will require higher returns to compensate for increasing financial risk. Thereafter, WACC rises and value falls, suggesting an optimum capital structure.

In 1958, Modigliani and Miller (MM) theoretically discredited this view, given the assumptions of perfect markets with no barriers to trade and tax neutrality, proving that WACC and total value are *independent* of financial policy. Based on the economic *law of one price*, they used *arbitrage* to demonstrate that close *financial substitutes*, such as two firms in the same class of business risk with identical net operating income, cannot sell at different prices; thereby negating financial risk.

Since MM published their original hypothesis in 1958, the capital structure debate has ebbed and flowed with a surprising lack of consensus among academics, researchers and practitioners. To complicate matters, subsequent empirical evidence has inevitably focussed on modest (rational) debt equity ratios, which are the norm, rather than occasional, extreme (irrational) leverage that creates financial distress and bankruptcy, such as the 2007 global meltdown.

To learn the lessons of the recent past, perhaps the academic debate must take a new turn. Real world investors (including corporate management) could also evaluate their past mistakes by reviewing MM's basic propositions under extreme conditions. They provide a sturdy framework for analysis. Moreover, their cost of capital hypothesis is entirely consistent with their 1961 dividend irrelevancy hypothesis covered in Chapter Four, for which there is considerable empirical support, (as we shall also discover in our Exercise companion).

So, for future reference, here is a graph and summary of the basic arbitrage process.

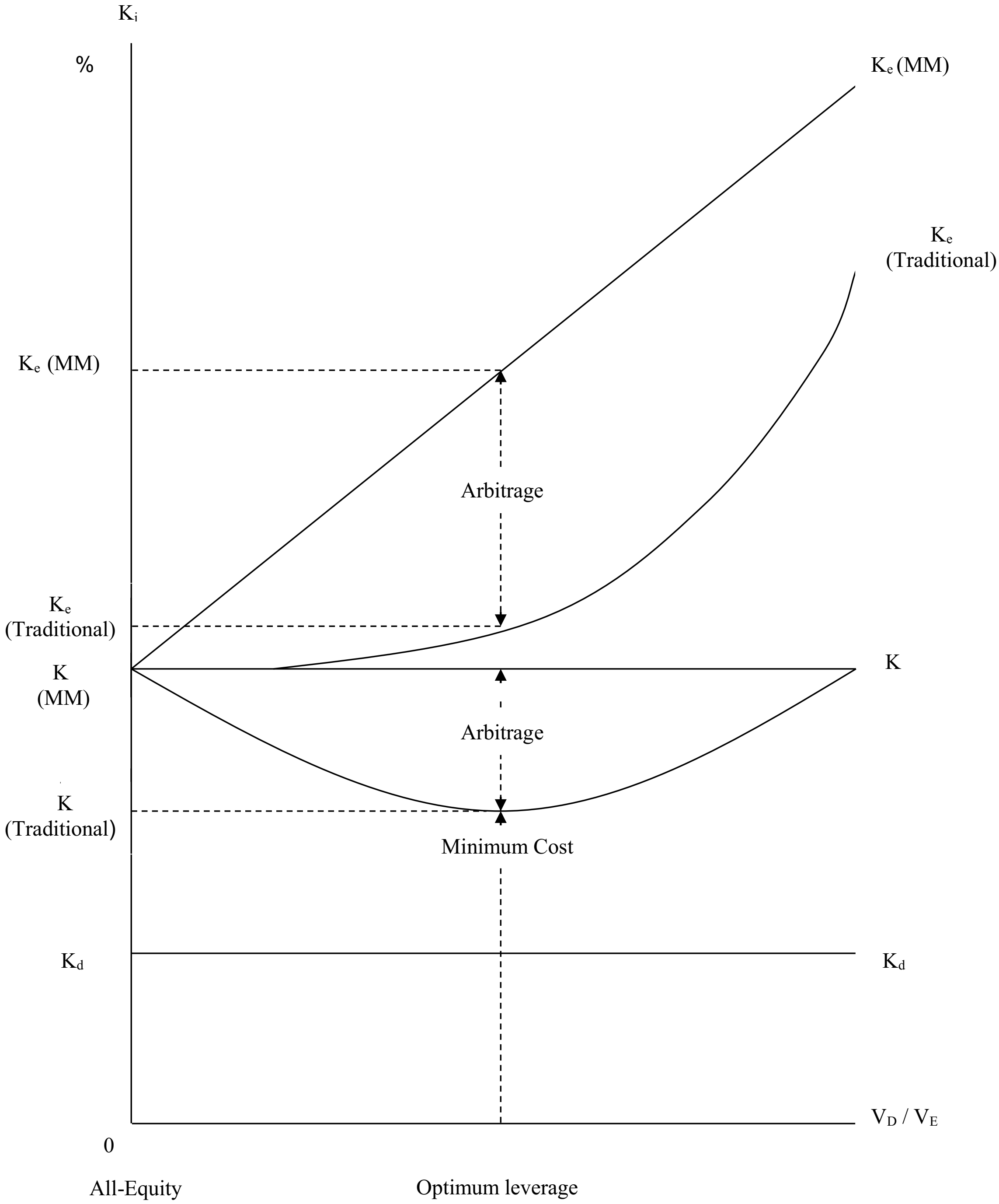

Figure 7.4: A Traditional View of Optimal Capital Costs and MM Equilibrium Proposition I: The Arbitrage Effects

According to MM, a traditional view of capital structure (characterised by perfect markets where companies in the same class of business risk have a different WACC and overall value) can only be a temporary phenomenon. *Arbitrageurs* will begin trading and force the two variables into equilibrium.

Figure 7.4 illustrates the effect. Diagrammatically, arbitrage causes both the traditional *U shaped* K curve (WACC) and the *exponential* K_e curve (cost of equity) to straighten out into *linear* functions.

To summarise MM's basic theoretical position:

- Corporate value should depend on the *agency principle* (Chapter One) defined by investor-managerial agreement on the average level of future earnings and their *variability* (business risk), rather than the *proportion* distributed (financial risk).
- Dividend and retention decisions should be *irrelevant* to the market price of a share (Chapter Four).
- As a determinant of WACC and total corporate value, the division of returns between debt and equity should also be *perfect substitutes*.

7.6 Selected References

1. Modigliani, F. and Miller, M.H., "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review,* Vol. XLVIII, No. 3, June, 1958.
2. Modigliani, F. and Miller, M.H., "Corporate Income Taxes and the Cost of Capital: A Correction", *American Economic Review,* Vol. LIII, No. 3, June, 1963.

Part Four:
The Portfolio Decision

8 Portfolio Selection and Risk

Introduction

Optimum investment, dividend and financing decisions implemented by corporate management on behalf of a multiplicity of shareholders, to whom it is ultimately responsible, ideally require knowledge of their disparate attitudes toward project risk-return and consumption-wealth preferences. But when ownership is divorced from control, direct communication between a company and its owners, as well as other stakeholders and prospective investors, concerning these motivational factors is impractical.

For real world companies with a stock exchange listing in markets characterised by uncertainty, what management does and what providers of capital desire are ultimately determined by the law of supply and demand for equity and other financial securities, measured by movements in their respective prices and returns. Unfortunately, there is still no uniformity of opinion as to what drives the market. For example, is it dividends; is it earnings, perhaps pure speculation, or some fluctuating combination? Whatever, management must still allocate the firm's resources efficiently between profitable investments for the mutual benefit of all stakeholders, which ultimately maximises shareholder wealth, using the market price of equity as a convenient proxy.

As a benchmark for analyses, this text (like all others in my *bookboon* series) therefore began with an *idealised* picture of market behaviour.

The *majority* of investors are rational and risk-averse, motivated by financial *self-interest*, operating in *reasonably* efficient capital markets characterised by a *relatively* free flow of information and *surmountable* barriers to trade.

In Part One we observed that in a world of certainty, where future events can be specified in advance, such investors can confidently analyse one course of action relative to another for the purpose of wealth maximisation.

For an *all-equity* firm financed by ordinary shares (common stock) where the ownership of corporate assets is divorced from control (the *agency* principle), we defined the *normative* objective of strategic financial management under conditions of certainty as:

- The implementation of optimum investment and financing decisions using net present value (NPV) maximisation techniques to generate the highest post-tax money profits from all a firm's projects in the form of retentions and distributions. These should satisfy *existing* owners (a multiplicity of shareholders) and attract *prospective* equity investors who define the firm's clientele, thereby maximising share price.

Over their life, individual projects should eventually generate net cash flows that *exceed* their overall cost of funds to create wealth. This future *positive* net terminal value (NTV) is equivalent to a *positive* NPV, expressed in today's terms, defined by the project discount rate using the time value of money.

Even when financial theory moves from a risk-free world to one of uncertainty, where *more than one future outcome is possible*, present value (PV) analysis remains the bedrock of rational investment behaviour. Providing markets are reasonably efficient, where all news (good or bad) is soon absorbed by its participants, investors expect to receive returns discounted at a rate commensurate with their risk.

Taking this *linear* view of society, where "markets have no memory and its participants lack perfect foresight", we observed that it is possible to define *expected* investor returns for a given level of risk, using the techniques of "classical" statistical analysis (*Quants*).

Assuming a firm's project cashflows (or its stock market returns) are linear, they are *random variables* that conform to a "normal" distribution. For every level of risk, there is an investment outcome with the highest expected return. For every expected return there is an investment outcome with the lowest expected risk. Using mean-variance analysis, the standard deviation calibrates these risk-return profiles and the likelihood of them occurring, based on probability analysis and confidence limits. Wealth maximisation equals the maximisation of investor *utility* using this trade-off, plotted as an *indifference* curve, which calibrates the *certainty equivalence* associated with the maximisation of an investment's *expected* NPV (ENPV).

So far so good, but once an all-equity company has made an issue of ordinary shares and invested the proceeds, the question management must then address is what to do with the returns. The crucial issue is whether the firm can increase its total value by the manipulation of financial risk associated with how new projects are funded, using retentions at the expense of dividends, rather than a further issue of shares.

And this is where Modigliani and Miller (MM) first contributed to our analysis.

In Part Two we presented a popular argument for wealth maximisation based on the investor's preference for current dividends, using the Gordon Dividend Growth Model (1962), which proved to be mathematically flawed. Using the economic "law of one price", MM (1961) also criticised Gordon, explaining why the shareholders' desired rate of return (and hence the firm's cut-off rate for investment and total value) are unaffected by dividend policy.

An *all-equity* firm can justify the retention of earnings to finance future investments, rather than pay a current dividend, if their marginal return on new projects at least equals the market rate of interest that shareholders could obtain by using dividends to finance alternative investments of equivalent business risk elsewhere.

MM believed shareholders should support such managerial behaviour. It cannot detract from their wealth or consumption preferences, if at any point in time, retentions and dividends are perceived as *perfect economic substitutes*. What they lose through dividends foregone (current income) they expect to receive through increased equity value (future capital gains) generated by internally financed projects discounted at their required opportunity rate of return.

According to MM's *dividend irrelevancy* hypothesis, if investors periodically need to replace a missing dividend to satisfy their consumption preferences, the solution is simple.

- Shareholders can create a *home-made* dividend by either borrowing an equivalent amount at the same rate as the company, or sell shares at a price that reflects their earnings and reap the capital gain.

Since the borrowing (discount) rate is entirely determined by the *business* risk of investment (variability of future earnings) and not the *financial* risk (pattern of dividends), the firm's distribution policy is trivial.

- Dividend decisions are concerned with what is done with earnings but do not determine the risk originally associated with the quality of investment that produces them.

In Part Three we extended our analysis beyond an *all-equity* firm to introduce cheaper borrowing (debt) into the corporate financial mix. Although the principles of investment using "Quants" still applied, we observed that managerial policies designed to maximise shareholder wealth now extend beyond satisfying shareholder expectations to other stakeholders and comprise the following *inter-related* functions.

- The *investment function*, which identifies and selects investment opportunities that *maximise expected net cash inflows* (ENPV) commensurate with risk.
- The *finance function*, which identifies potential fund sources (equity and debt, long or short) required to sustain investments.

Management therefore need to evaluate the *risk-adjusted* return for each mode of financing (not just equity) and. then select an *optimum* mix that will *minimise* the company's overall weighted average cost of capital (WACC) as a discount rate for project appraisal.

We then examined the case for and against, an optimum capital structure, which maximises total corporate value in the presence of gearing (leverage). It became apparent that if a company's current WACC is to be employed as a discount rate in ENPV calculations it must satisfy two conditions:

- The earnings of new projects selected by management must conform to the pattern (business risk) of existing corporate activities.
- The company's mode of finance must conform to the composition (financial risk) of its existing capital structure.

If not, the impact of leverage upon the company's division of investor returns (overall cost of capital) and value must also be considered.

Assuming business risk is held constant, the question therefore arose as to whether management can determine an optimum capital structure that minimises financial risk and hence the cost of capital as it gears up with cheaper debt finance. If so, ultimately the firm's overall value and share price can be maximised by manipulating financial policy.

Based on this assumption, what is now termed the "traditional" approach, models an *optimum* capital structure (financial mix) that will *minimise* a firm's overall weighted average cost of capital (WACC) as a discount rate for project appraisal and *maximise* total corporate value. However, according to MM's cost of capital hypothesis (1958) this strategy can only be a temporary phenomenon.

In this newly leveraged situation, where the financial returns to debt and equity result from a common investment decision, MM invoke the "law of one price" to prove that using debt capital to finance new investment (just like retentions at the expense of dividends) does not matter.

> If two companies in the same class of business risk exhibit a different WACC and overall value, *arbitrageurs* will begin trading their investments in the higher valued (geared) company to purchase a cheaper but otherwise perfect substitute in the lower valued firm (all-equity say) thereby depressing the price of the former and increasing the price of the latter, until the two are in equilibrium.

As a consequence of *arbitrage*:

- The total value of a firm represented by the NPV of an income stream discounted at a rate appropriate to its business risk, should be unaffected by shifts in capital structure (financial risk).
- Any rational debt-equity ratio should produce the same cut-off rate for investment (WACC).

In the presence of *arbitrage*, WACC and total corporate value are unaffected by the debt-equity ratio (financial risk) but the rather the quality of earnings (business risk) that stems from initial investment.

Today there is considerable empirical support for the MM hypotheses. However, when their papers on capital structure and dividend policy were first published in 1958 and 1961 they created controversy because no theory had been fully developed to explain the pricing of total risk and the relative impact of its components (business risk and financial risk) on a *diverse* portfolio of investments. This had to wait until the publication of the Capital Asset Pricing Model (CAPM) by William Sharpe (1963) based on Harry Markowitz' pioneering work on portfolio selection (1952) and the subsequent development of Modern Portfolio Theory (MPT).

Part Four of our text therefore considers the evolution of MPT and the contribution of MM's "law of one price" to the CAPM and its practical applications. However, what follows assumes that you are familiar with the body of work by Markowitz and Sharpe and others. If not, you should refer to my *bookboon* texts at the end of this Chapter. If so, here is summary of what they contain as a reminder, with the maths kept to a minimum!

8.1 Modern Portfolio Theory and Markowitz

Using the following rational investment decision rules based on optimum mean-variance efficiency criteria (Quants):

Maximise expected return for a given level of risk
Minimise risk for a given expected return

The statistical objective of efficient portfolio diversification is to achieve an overall *standard deviation*, which is lower than its component parts without compromising overall return.

For example, suppose there is a *perfect positive correlation* between two securities that comprise the stock market, or two projects that define a firm's total investment. In other words, high and low returns always move sympathetically. It would pay an investor, or a company, to place all their funds in whichever investment yields the highest return at the time. However, if there is *perfect inverse correlation*, where high returns on one investment are always associated with low returns on the other and *vice versa*, or there is *random (zero) correlation* between the returns, then it can be shown statistically that overall risk reduction can be achieved through diversification.

According to Markowitz (*op.cit.*) if the *correlation coefficient* between any number of investments is less then one (perfect positive), the total risk of a portfolio measured by its standard deviation is lower than the weighted average of its constituent parts, with the greatest reduction reserved for a correlation coefficient of minus one (perfect inverse).

Thus, if the standard deviation of an individual investment is higher than that for a portfolio in which it is held, it would appear that some of the standard deviation must have been diversified away through correlation with other portfolio constituents, leaving a residual risk component correlated with the economy as a whole. Measured by the *covariances* of each investment with the total portfolio (such as the stock market) the latter is undiversifiable. Consequently, the contribution of an individual investment to the variance of a well-diversified portfolio (its covariance) is the only risk that investors will pay a premium to avoid.

Indeed, as we shall discover later, the reduction in *total* risk only relates to the *specific* risk associated with *micro-economic* factors, which are unique to individual sectors, companies, or projects. A proportion of *total* risk, termed *market* risk, based on *macro-economic* factors correlated with the market is inescapable.

The distinguishing features of specific and market risk had important consequences for the development of Markowitz efficiency and the emergence of Modern portfolio Theory (MPT) during the 1960s. For the moment, suffice it to say that whilst market risk is not diversifiable, specific risk can be statistically eliminated entirely if all rational investors diversify until they hold the *market portfolio*, which reflects the risk-return characteristics for every available financial security. In practice, this strategy is obviously unrealistic. But as we shall discover later, studies have shown that with less than thirty diversified constituents it is feasible to reach a position where a portfolio's standard deviation is close to that for the market portfolio.

Unfortunately, throughout the1950s (without today's computer technology and sophisticated software) the derivation of the *covariance* terms in the Markowitz model was so unwieldy for most investors seeking a well-diversified risky portfolio drawn from a global capital market that it was untenable. Even by substituting the *correlation coefficient* into the covariance of the portfolio variance, the mathematical complexity of the variance-covariance matrix calculations for a risky multi-asset portfolio still limited its applicability. So, what was the alternative?

8.2 Modern Portfolio Theory and the Beta Factor

In an ideal world:

- Portfolio theory should offer management a practical tool for measuring the extent to which the pattern of returns from a new project affects the risk of a firm's existing operations.
- For those playing the stock market, portfolio analysis should also calibrate the effects of adding new securities to their existing spread.

To circumvent the complexity of the Markowitz variance-covariance matrix, various academics sought alternative ways to measure risk. This began with the realisation that the *total risk* of an investment (the standard deviation of its returns) within a diversified portfolio can be divided into *systematic* and *unsystematic* risk. You will recall that the latter (also termed *specific* risk) can be eliminated entirely by efficient diversification. The other (also termed *market* risk) cannot. It therefore affects the overall risk of the portfolio in which the investment is included.

Since all rational investors (including management) interested in wealth maximisation should be concerned with individual security (or project) risk relative to the stock market as a whole, and not simply their own asset portfolio, analysts were quick to appreciate the importance of systematic (market) risk. According to the Separation Theorem of John Tobin (1958) it represents the only risk that they will pay a premium to avoid.

By introducing market opportunities for risk-free investment and borrowing or lending at the risk-free rate to establish an optimum portfolio, Tobin defined the investor's required return on a risky investment as the risk-free return, plus a premium for risk (determined not by the total risk of the investment, but only by its systematic (market) risk).

Of course, the systematic risk of an individual financial security (a company's share, say) might be higher or lower than the overall risk of the market within which it is listed. Likewise, the systematic risk for some capital projects may differ from others within an individual company's portfolio. And this is where the theoretical development of a *relative* measure of an investments systematic risk fits into portfolio analysis.

Termed the *beta* factor (or beta coefficient) it calibrates the volatility of say a share's performance to market movements (rather than individual securities) defined by the ratio of the expected change in the stock's performance to the market itself.

Expressed statistically:

- The beta factor equals the *covariance* of the *j*th share's rate of return with the market portfolio divided by the *variance* of the market portfolio.

1) $$\beta_j = \frac{COV(j,m)}{VAR(m)}$$

For those searching for a computationally simple proxy for the covariance (relative risk) terms in a Markowitz portfolio, beta neatly solved their problem. Instead of generating numerous new covariance terms, all they required was the covariance of the rate of return on the additional share with the overall rate of return on the efficient market portfolio.

Activity 1

Since the1960s, innumerable empirical studies have shown that beta values are invaluable for portfolio selection. But do investors know what they mean?

If you are up to speed with MPT, can you interpret beta factors of 1.15, 1.0 and 0?

Investors can tailor a portfolio to their specific risk-return (utility) requirements, aiming to hold *aggressive* stocks with beta factors in excess of 1.0 while the market is rising (a "bull" market), and less than 1.0 (*defensive*) when the market is falling (a "bear" market).

A beta of 1.15 implies that if the underlying market with a beta factor of one were to rise by 10% then the stock may be expected to rise by 11.5%. Conversely, a security with a beta of less than one would not be as responsive to market movements. In this situation, smaller systematic risk would mean that investors would be satisfied with a return that is below the market average.

The market portfolio has a beta of one precisely because the covariance of the market portfolio with itself is identical to the variance of the market portfolio.

A risk-free investment has a beta of zero because its covariance with the market is zero.

8.3 Modern Portfolio Theory and the CAPM

By the 1960s academic research revealed that although a *linear* relationship between total portfolio risk and expected returns does not hold for individual risky investments using the Markowitz model, all the characteristics of systematic beta risk apply to portfolios *and* individual securities. The beta of an overall portfolio is simply the *weighted average* of all it beta factor constituents.

This opened the door to comprehensively pricing market risk and the key for developing the Capital Asset Pricing Model (CAPM), notably Sharpe's single period, single index model (*op.cit*). For a given level of systematic risk, the CAPM determines the equilibrium, mean rate of return for any investment using its beta value.

- The expected return is equal to the risk-free rate of interest, *plus* the product of a market risk premium (measured by the difference between the market return and the risk free rate) and the investment's beta coefficient.

For any risky investment (j) with a beta of β_j, the expected return (r_j) which provides adequate compensation for holding the investment is the value obtained by incorporating the beta factor into the *CAPM equation*:

2) $r_j = r_f + (r_m - r_f)\ \beta_j$

Where r_f and r_m equal the risk-free rate and market return respectively.

And because all the characteristics of systematic betas apply to a *portfolio*, as well as an *individual* security, any portfolio's return (r_p) with a portfolio beta (β_p) can be defined as

3) $r_p = r_f + (r_m - r_f)\ \beta_p$

Consequently, proponents of the CAPM concluded that all investors are capable of *eliminating unsystematic risk entirely* by expanding their investment portfolios until they reflect their market portfolio (such as the Dow Jones or Footsie).

Academics also realised the CAPM's utility, not only within a global stock market framework but also its relevance to corporate capital budgeting decisions, where an individual project's beta does not necessarily equal a firm's equity beta, let alone the risk measured by its WACC.

Perhaps you recall from Part Three that new projects with different risk-return trade-offs may not conform to the overall WACC valuation profile of a company. The latter reflects no more than the total *average* risk of all its existing investments, which may not even satisfy the aspirations of existing stakeholders, let alone potential investors. So, if a firm's WACC is only a *benchmark*, management need to evaluate the risk of any new asset investment, relative to its existing activities (which may already be diversified) as well as the performance of other companies in the same class of business risk

And this is where we shall pick up on our earlier discussion of MM's hypotheses and the CAPM in the following Chapter, by analysing the impact of gearing ratios on beta values (measured by their equity, asset and project coefficients respectively) and the possibility of eliminating their effect on the total value of a company.

Review Activity

The objective of portfolio diversification is the selection of investment opportunities that reduce *total* portfolio risk without compromising *overall* return.

If the standard deviation (risk) of an individual investment is higher than that of the portfolio in which it is held, then part of the standard deviation must have been diversified away through correlation with other portfolio constituents.

A high level of diversification results in rational investors holding the market portfolio, which they will do in combination with lending or borrowing at the risk-free rate. This only leaves an element of risk that is correlated with the market as a whole. In other words portfolio risk equals market risk, which is undiversifiable

To clarify these points for future reference, research and summarise the relationship between *total* risk and its *component* parts.

An Indicative Outline Solution

Total risk is divisible between:

- *Systematic* or *market risk*, so called because it is endemic throughout the system (market) and is undiversifiable. It relates to general economic factors that affect all firms and financial securities, and explains why share prices tend to move in sympathy.
- *Unsystematic risk,* sometimes termed *specific*, *residual*, or *unique risk*, relates to specific (unique) economic factors, which impact upon individual industries, companies, securities and projects. It can be eliminated entirely through efficient diversification.

In terms of our earlier analysis, systematic risk measures the extent to which an investment's return moves sympathetically (systematically) with all the financial securities that comprise the market portfolio (the *system*). It describes a particular portfolio's inherent sensitivity to global political and macro-economic volatility. The best recent example, of course, is the 2007 financial meltdown and subsequent economic recession. Because individual companies or investors have no control over such events, they require a rate of return commensurate with their relative systematic risk. The greater this risk, the higher the rate of return required by those with widely diversified portfolios that reflect movements in the market as a whole.

In contrast, unsystematic risk relates to the individual investment and is independent of market risk. Applied to a company, it is caused by micro-economic factors such as profitability, product innovation and the quality of management. Because it is completely diversifiable (variations in returns cancel out over time) unsystematic risk carries no market premium. Thus, all the risk in a fully diversified portfolio is market or systematic risk.

We have encountered systematic risk earlier in this study under other names. Figure 8.1 reveals that systematic risk comprises a company's *business risk* and *financial risk.* You will recall that business risk reflects the unavoidable variability of project returns defined by the nature of a firm's investment (*investment policy*). This may be higher or lower than that for other projects, or the market as a whole. Systematic risk may also reflect a premium for financial risk, which arises from the proportion of debt to equity in a firm's capital structure (gearing) and the amount of dividends paid in relation to the level of retained earnings, (*financial policy*).

Of course, there is empirical support for a contrary view that financial risk is irrelevant based on the seminal work of MM (1958 and 1961) explained earlier. Irrespective of whether financial policies matter, for the moment all we need say is that for all-equity firms with full dividend distribution policies, there is an academic consensus that business risk equals systematic (market) risk and is not diversifiable.

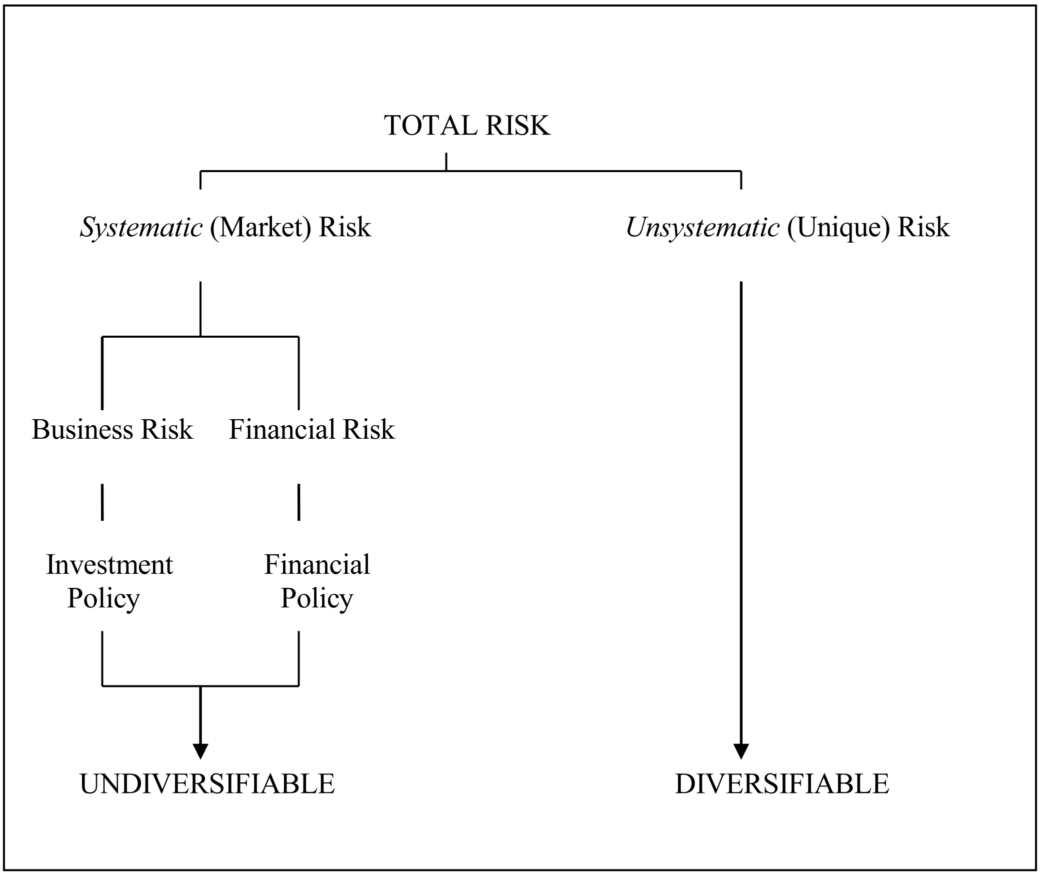

Figure 8.1: The Inter-relationship between Risk Concepts

8.4 Summary and Conclusions

At the beginning of this study we outlined theoretically how rational, risk-averse individuals and companies operating in reasonably efficient markets with few "barriers to trade" should rank *individual* investments. They interpret expected returns and standard deviations using the concept of expected utility to calibrate their risk-return attitudes. In this Chapter we began with the same mean-variance efficiency criteria to explain how an optimum *portfolio* of investments can reduce total risk (the standard deviation) without impairing overall return.

Markowitz, explains how individuals or companies can reduce risk but maintain their return by holding more than one investment, providing their returns are not positively correlated. This implies that all rational investors should diversify risky investments into an efficient portfolio. Unfortunately, as its constituents rise the model not only becomes statistically unwieldy, but also fails to eliminate risk entirely.

The CAPM fortunately offers investors a statistical lifeline, by discriminating between diversifiable, non-systematic and non-diversifiable, systematic risk. The latter is defined by a beta factor that measures relative (systematic) risk, which explains how rational investors with different utility (risk-return) requirements can choose an optimum portfolio by borrowing or lending at the risk-free rate. Consequently, they are capable of completely eliminating unsystematic risk by expanding their portfolios until they reflect the market portfolio.

By way of conclusion, however, it is worth noting that without the research expertise and financial resources of a global financial institution required to achieve such extreme diversification, all is not lost for private investors with modest funds.

An oft-forgotten fact (based on numerous studies) is that up to *95 per cent* of unsystematic risk can be diversified away by randomly increasing the number of investments in a portfolio to *about thirty*. With one investment, portfolio risk is represented by the sum of unsystematic and systematic risk. In other words, the investment's *total risk* as measured by its standard deviation. When the portfolio constituents reach double figures, increasingly all the risk associated with holding that portfolio becomes systematic or market risk.

See Fisher and Lorie (1970) for the earliest and best review of this phenomenon, which is graphed in Figure 8.2.

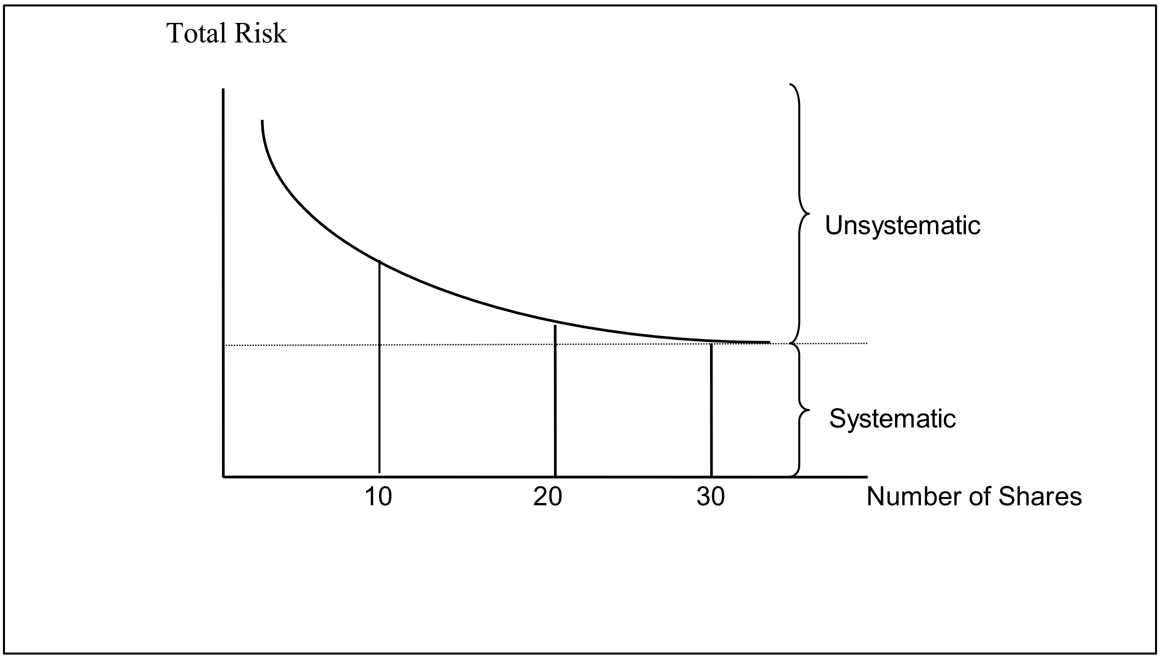

Figure 8.2: Portfolio Risk and Diversification

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9 MM and the CAPM

Introduction

So far, our study of portfolio efficiency, beta factors and the CAPM has concentrated on the stock market's analyses of security prices and expected returns by financial institutions and private individuals. This is logical because it reflects the rationale behind the chronological development of Modern Portfolio Theory (MPT). But what about the impact of MPT on individual companies and their appraisal of capital projects upon which all investors absolutely depend? If management wish to maximise shareholder wealth, then surely a new project's expected return and systematic risk relative to the company's existing investment portfolio and stock market behaviour, like that for any financial security, is a fundamental consideration.

Given your general knowledge of MPT, in this Chapter we shall explore specific corporate applications of the Sharpe CAPM by strategic financial management, namely:

- The derivation of a discount rate for the appraisal of capital investment projects on the basis of their systematic risk.
- How the CAPM can be used to match discount rates to the systematic risk of projects that differ from the current business risk of a firm.

Because the model can be applied to projects financed by debt as well as equity, we shall then conclude our analyses by establishing a mathematical connection between the CAPM and the Modigliani-Miller (MM) theory of capital gearing (1958) based on their "law of one price".

9.1 Capital Budgeting and the CAPM

As an alternative to calculating a firm's weighted average cost of capital (WACC) explained in Part Three, the theoretical derivation of a project discount rate using the CAPM and its application to NPV maximisation is quite straightforward. A risk-adjusted discount rate for the *j*th project is simply the risk-free rate added to the product of the market premium and the *project* beta. Using Chapter Eight's earlier notation for the *CAPM equation*:

4) $r_j = r_f + (r_m - r_f)\ \beta_j$

The project beta (β_j) measures the *systematic* risk of a specific project (more of which later). For the moment, suffice it to say that in many textbooks the project beta is also termed an *asset* beta denoted by β_A.

Using a mathematical formulation, with which you should be familiar, management can then derive a project's expected NPV by subtracting the initial cost of investment (I_0) from its periodic, average net annual cash flows (C_t) discounted at r_j the risk-adjusted rate (rather than WACC) over its useful life (n).

$$5)\quad ENPV = \sum_{t=1}^{n} C_t / (1+r_j)^t - I_0$$

Individual projects are acceptable if:

$$ENPV \geq 0$$

Collectively, if finance is a limiting factor (capital rationing) projects that satisfy this acceptance criterion can also be ranked for selection according to the size of their ENPV. Given:

$ENPV_A > ENPV_B > \ldots ENPV_N$ we prefer project A

So far, so good; but remember that CAPM project discount rates are still based on a number of simplifying assumptions. Apart from adhering to the traditional concept of perfect capital markets (Fisher's Separation Theorem) and mean-variance analysis (Markowitz efficiency) the Sharpe CAPM is only a *single-period* model, whereas most projects are *multi-period*.

> According to the CAPM, all investors face the same set of investment opportunities, have the same expectations about the future and make decisions within *one* time horizon. Any new investment made *now* will be realised *then*, next year (say) and a new decision made.

Given the assumptions of perfect markets characterised by random cash flow distributions, there is no theoretical objection to using a *single-period* model to generate an NPV discount rate for the evaluation of a firm's *multi-period* investment plans. The only constraints are that the risk-free rate of interest, the average market rate of return and the beta factor associated with a particular investment are *constant* throughout its life.

Unfortunately, in reality the risk-free rate, the market rate and beta are rarely constant. However the problem is not insoluble, as Fama and French observed. (1992). We just substitute *periodic* risk-adjusted discount rates (now dated $r_{j\,t}$) for a constant r_j into Equation (5) for each future "state of the world", even if only one of the variables in Equation (4) changes. It should also be noted that the phenomenon of multiple discount rates combined with different economic circumstances is not unique to the CAPM. It is common throughout NPV analyses, as well as other valuation theories (remember the Gordon Growth Model?).

On first acquaintance, it would therefore appear that the application of a CAPM return to capital budgeting decisions provides corporate financial management with a practical alternative to the WACC approach.

A particular weakness of WACC is that it defines a single discount rate applicable to *all* projects, based on the assumptions that their acceptance doesn't change the company's risk or capital structure and is *marginal* to existing activities.

In contrast, the CAPM rate varies from project to project, according to the systematic risk of each investment proposal. However, the CAPM still poses a number of practical problems that must be resolved if it is to be applied successfully, notably how to derive an appropriate *project* beta factor and how to measure the impact of *capital gearing* on its calculation.

9.2 The Estimation of Project Betas

So far, we have only used a *general* beta factor (β) applicable to the *overall* systemic risk of portfolios, securities and projects. But now our analysis is becoming more focussed, *precise* notation and definitions are necessary to *discriminate* between systemic *business* and *financial* risk. Table 9.1 summarises the beta measures that we shall be using for future reference. It also introduces a number of problems with their application.

β = total *systematic* risk, which relates portfolio, security and project risk to *market* risk.
β_j = the *business* risk of a specific project (*project* risk) for investment appraisal.
β_E = the *published* equity beta for a company that incorporates business risk and systematic *financial* risk if the firm is geared.
β_A = the overall business risk of a firm's *assets* (projects). It also equals a company's *deleveraged* published beta (β_E) which measures business risk *free* of financial risk.
β_D = the beta value of debt (which obviously equals zero if it is risk-free).
β_{EU} and β_{EG} are the respective equity betas for *similar* all-share and geared companies

Table 9.1: Beta Factor Definitions

When an all-equity company is considering a new project with the same level of risk as its current portfolio of investments, total systematic risk *equals* business risk, such that:

$$\beta = \beta_j = \beta_E = \beta_A = \beta_{EU}$$

So, if a company is funded by a combination of debt and equity, this series of equalities must be modified to incorporate a *premium* for systematic *financial* risk. As we shall discover, the equity beta will be a *geared* beta reflecting business risk *plus* financial risk, which measures shareholder exposure to debt in their firm's capital structure. Thus, the equity beta of an all-share company is always lower than that for a geared firm with the same business risk.

$$\beta_{EU} < \beta_{EG}$$

Table 9.1 reveals a further idiosyncrasy of the CAPM. A company's asset beta (β_A) represents a discount rate that is appropriate for evaluating projects with the same overall risk as the company itself. But what if a new project does not reflect the average risk of the company's assets?

You will recall from Part Three that irrespective of gearing, WACC poses a dilemma for management. It should only be used as a project discount rate if the risk of new investment equals the opportunity cost of its existing operations. So too, with the CAPM:

- The company's asset beta (β_A) produces a discount rate that is only appropriate for evaluating projects with the same overall risk as the company itself.
- Where a new project does not reflect the average risk of the company's assets, the use of an asset beta is no more likely to produce a correct investment decision than the use of a WACC calculation.

To illustrate the point, Figure 9.1 graphs the *Security Market Line* (SML) explained in my *bookboon* Portfolio and CAPM series. This shows the required return on a project for different beta factors, relative to a company's overall cost of capital (WACC). The use of WACC to evaluate projects whose risk differs from the company's average will be *sub-optimal* where the Internal Rate of Return (IRR) of a project is in either of the two shaded sections. To calculate the correct CAPM discount rate using Equation (4) we must determine the *project* beta.

The company's average beta, shown in the diagram, provides a measure of risk for the firm's overall returns compared with that of the *market*. However, management's investment decision is whether or not to invest in a *project*. So, like the WACC, if the project involves diversification away from the firm's core activities, we must use a beta coefficient appropriate to that class of investment. The situation is similar to a stock market investor considering whether to purchase the shares of the *company*. The individual would need to evaluate the share's return by using the *market* beta in the CAPM.

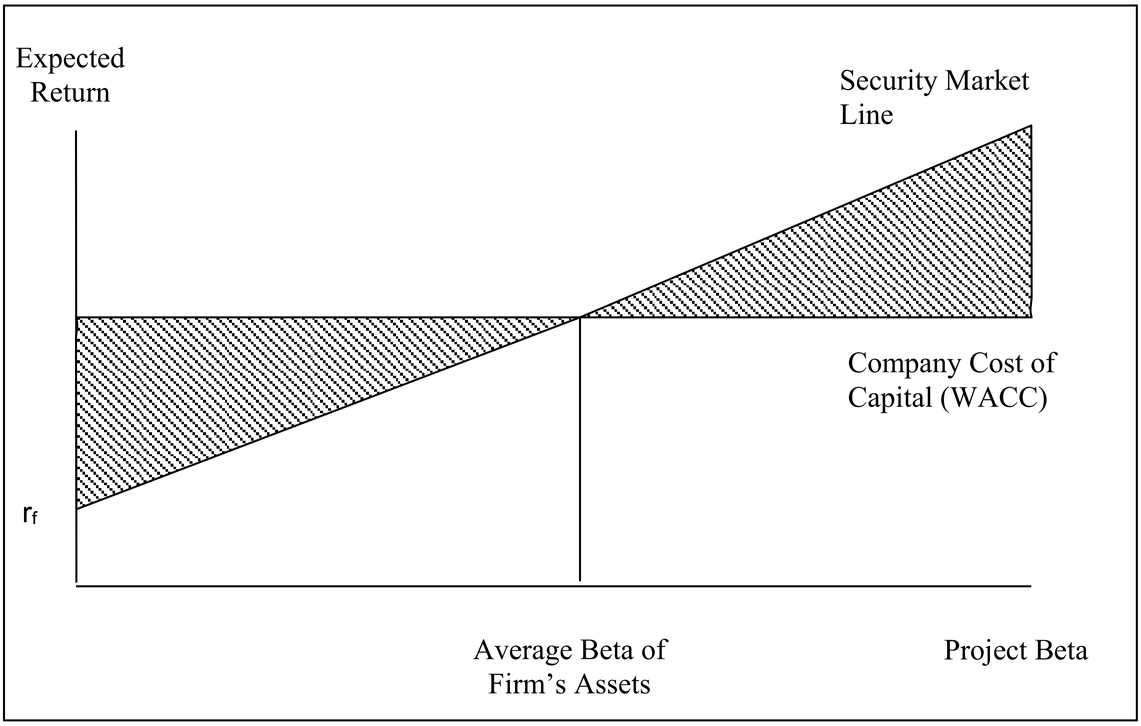

Figure 9.1: The SML, WACC and Project Betas

Even if diversification is not contemplated, the project's beta factor may not conform to the *average* for the firm's assets. For example, the investment proposal may exhibit high *operational gearing (*the proportion of fixed to variable costs) in which case the project's beta will exceed the average for existing operations.

A serious conflict (an *agency* problem) can also arise for those companies producing few products, or worse still a single product, particularly if management approach their capital budgeting decisions based on self-interest and short-termism, rather than shareholder preferences. Shareholders with well-diversified corporate holdings who dominate such companies may prefer to see projects with high risk (high beta coefficients) to balance their own portfolios. Such a strategy may carry the very real threat of corporate bankruptcy but in the event may have very little impact on their overall returns. For the firm's management, other employees, its suppliers and creditors, however, the policy may be economic suicide.

Fortunately, if a beta is required to validate the CAPM for project appraisal, help is at hand. Management can obtain factors for companies operating in similar areas to the proposed project by subscribing to the many commercial services that regularly publish beta coefficients for a large number of companies, world wide. Their listings also include stock exchange classifications for *industry* betas. These are calculated by taking the market average for quoted companies in the same industry. Research reveals that the measurement errors of individual betas cancel out when industry betas are used. Moreover, the larger the number of comparable beta constituents, the more reliable the industry factor.

So, if management wish to estimate a project's beta, it can identify the industry in which the project falls, and use that industry's beta as the project's beta. This approach is particularly suitable for companies that are highly *diversified* and *divisionalised* because their WACC or market beta would be of little relevance as a discount rate for its divisional operations.

As an alternative to stock market data, management can also estimate a project's beta from first principles by calculating its *F-value*.

> The F-value of a project is rather like a beta factor in that it measures the variability of a project's performance, *relative* to the performance of an entity for which a beta value exists.
>
> The entity could be the industry in which the project falls, the firm undertaking the project, or a division within the firm that is responsible for the project.

A project's F-value is defined as follows:

$$6) \quad F = \frac{\text{Percentage change in the project's performance}}{\text{Percentage change in the "entity's" performance}}$$

As a result, we can obtain an estimate of a project's beta through one of three routes:

$$\left.\begin{array}{ll} \text{(i)} & \dfrac{\text{\% change in the company's performance}}{\text{\% change in the industry's performance}} \times \beta_{\text{industry}} \\[2em] \text{(ii)} & \dfrac{\text{\% change in the project's performance}}{\text{\% change in the company's performance}} \times \beta_{\text{company}} \\[2em] \text{(iii)} & \dfrac{\text{\% change in the project's performance}}{\text{\% change in the division's performance}} \times \beta_{\text{division}} \end{array}\right\} \beta_{\text{project}}$$

> **Activity 1**
>
> A company's divisional management is considering a capital project, whose performance may be affected 15 per cent either way, depending on whether the division's overall performance rises or falls by 10 per cent. In other words, the project's profitability is expected to be more volatile than that of the division because of specific economic factors.
>
> Calculate the project's F-value and beta coefficient, given the division's beta factor is 0.80.

Using Equation (6) we can calculate the F-value as follows:

$F = 15\% / 10\% = \underline{1.5}$

If the divisional beta value is 0.80, then the project beta (β_{project}) can be estimated as follows:

(% change in the project's performance / % change in the division's performance) $\times \beta_{\text{division}}$

$\beta_{\text{project}} = 1.5 \times 0.80 = \underline{1.2}$

9.3 Capital Gearing and the Beta Factor

The CAPM defines an individual investment's risk relative to a well-diversified portfolio as *systematic risk*. Measured by the beta coefficient, it is the only risk that a company, or an investor, will pay a premium to avoid. You will recall from Chapter Eight (Figure 8.1) that systematic risk can be sub-divided into:

- *Business risk* that arises from the variability of a firm's earnings caused by market forces,
- *Financial risk* associated with dividend policies and capital gearing, both of which may amplify business risk

Without getting enmeshed in the dividend debate (covered in Part Two) if we accept the 1961 Modigliani and Miller (MM) hypothesis as a benchmark, namely that dividends are *irrelevant* (based on their economic "law of one price") *financial* risk should not matter for an all-equity company. Applied to the CAPM, the *systematic* risk of all investors (who are shareholders) can therefore be defined by the *business* risk of the firm's underlying asset investments.

The *equity* beta of an unlevered (all-equity) firm equals an *asset* beta, which measures the business risk of its total investment relative to the market for ordinary shares (common stock).

Using earlier notation:

$$\beta_{E\,U} = \beta_A$$

The CAPM return on project (r_j) is then defined by:

7) $r_j = r_f + (r_m - r_f)\,\beta_A$

If there is no debt in the firm's capital structure, the company's asset (equity) beta equals the *weighted average* of its individual project betas (β_i) based on the market value of equity:

8) $\beta_A = \Sigma\, w_i\, \beta_i = \beta_{EU}$

where w_i represents the individual weights.

But what about companies who decide to fund future investments by gearing up, or the vast majority who already employ debt finance?

To make rational decisions, it would appear that management now require an asset beta, which measures a firm's business risk that an ungeared equity beta can no longer provide. For example, an all-equity company may be considering a take-over that will be financed entirely by debt. To assess the acquisition's viability, management will now need to calculate their overall CAPM return on investment, using an asset beta that reflects a *leveraged* financial mix of fixed interest on debt and dividends on shares.

Later in this Chapter we shall resolve the dilemma, using the predictions of MM's capital structure hypothesis (*op.cit.*). Based on their "law of one price", whereby similar firms with the same risk characteristics (except capital gearing) cannot sell at different prices, it confirms their dividend hypothesis, namely that financial policy is irrelevant. First, however, let us develop the CAPM, to illustrate the relationship between an asset beta and the equity and debt beta coefficients for a geared company.

You perhaps recall from Part Three that when a firm is financed by a debt-equity mix, its earnings stream and associated risk is divided between the firm's shareholders and providers of corporate debt. The proportion of risk reflects the market values of debt and equity respectively, defined by the *debt-equity ratio*. So, the equity beta will be a *geared* equity beta. It not only incorporates business risk. It also determines shareholders' exposure to financial risk defined by the proportion of contractual, fixed interest securities in the capital structure.

For this reason, as mentioned earlier, the equity beta of an unlevered company is always lower than the beta of a levered company.

$$\beta_{EU} < \beta_{EG}$$

Given a geared equity beta (β_{EG}) and debt beta (β_D), the asset beta (β_A) for a company's investment in risky capital projects can be expressed as a weighted average of the two:

9) $\beta_A = \beta_{EG} [V_E / (V_E + V_D)] + \beta_D [V_D / (V_E + V_D)]$

Where:

V_E and V_D define the *market* values of equity and debt, respectively,

V_E plus V_D define the firm's total market value (V).

Activity 2

A firm with respective market values of €60m and €30m for equity and debt has an equity beta of 1.5. The debt beta is zero.

Use Equation (9) to calculate the asset beta (β_A) and explain its mathematical structure.

- The asset beta (β_A) calculation

9) $\beta_A = \beta_{EG} [V_E / (V_E + V_D)] + \beta_D [V_D /(V_E + V_D)]$

$= 1.5 [60/(60 +30)] + 0 [30/(60 +30)] = \underline{1.0}$

- The mathematical structure of β_A

When a company is financed by debt and equity, management need to derive an asset beta using the *weighted average* of its geared equity and debt components. The market values of debt and equity provide the weightings for the calculation. Note, however, that because the market risk of debt (β_D) was set to zero, the right hand side of Equation (9) disappears.

This is not unusual. As explained in Part Three, debt has priority over equity's share of profits and the sale of assets in the event of liquidation. Thus, debt is more secure and if it is risk-free, there is no variance. So, if β_D equals zero, our previous equation for an asset beta reduces to:

10) $\beta_A = \beta_{EG} [V_E / (V_E + V_D)]$

For example, if a company has an equity beta of 1.20, a debt-equity ratio of 40 per cent and we assume that debt is risk-free, the asset beta is given by:

$$\beta_A = 1.20\ [100 / (100 + 40)]$$
$$= \underline{0.86}$$

Perhaps you also recall that debt is a *tax deductible* expense in many economies. Incorporating this fiscal adjustment into the previous equations (where t is the tax rate) we can redefine the mathematical relationship between the asset beta and its geared equity and debt counterparts as follows.

11) $\beta_A = \beta_{E\,G}\ \{V_E / [V_E + V_D(1\text{-}t)]\} + \beta_D\ \{[V_D(1\text{-}t) / (V_E + V_D(1\text{-}t))]\}$
12) $\beta_A = \beta_{E\,G}\ \{V_E / [V_E + V_D(1\text{-}t)]\}$ if debt is risk-free

Despite the tax effect, our methodology for deriving a company's asset beta still reveals a *universal* feature of the CAPM that financial management can usefully adopt to assess individual projects.

- Whenever risky investments are combined, the asset beta of the resultant portfolio is a *weighted average* of its component parts.

Activity 3

Consider a company with a current asset beta of 0.90. It accepts a project with an asset beta of 0.5 that is equivalent to 10 per cent of its corporate value after acceptance.

Confirm that:

1. The *new* (*ex-post*) asset beta coefficient of the company equals 0.86.
2. The *new* project reduces the original risk of the firm's *existing* portfolio.

- The *ex-post* asset beta coefficient

  After the project's acceptance the beta factor equals a weighted average of the "old and new" defined as:

$$\beta_A = (0.90 \times 0.9) + (0.5 \times 0.1)$$
$$= \underline{0.86}$$

- The revised portfolio

  The significance of the project's acceptance is that with an asset beta of 0.86 compared to 0.90, the firm's overall portfolio of investments is now less risky than it was.

9.4 Capital Gearing and the CAPM

For a given level of systematic risk, we know that the CAPM determines the mean rate of return for any investment, *via* its beta value.

- The required return is equal to the risk-free rate of interest, plus the product of the market premium and the investment's beta coefficient.

From a capital market perspective, for example, the required return on equity that provides adequate compensation for holding shares is the value obtained by substituting the appropriate equity beta into the CAPM. From a managerial viewpoint, we therefore have an important policy prescription:

- The *shareholders'* equilibrium rate of return, given by the basic CAPM, must equal the *company's* cost of equity capital.

Turning to individual companies, the CAPM also defines a project's discount rate as a return equal to the risk-free rate of interest, plus the product of the market premium and the project's asset beta (a risk premium) to compensate for systematic (business) risk. However, we now know that the financial risk associated with capital gearing can affect beta factors. So, the discount rate derived from the CAPM for investment appraisal must also be affected; but how?

Let us first consider a company funded entirely by equity that is evaluating a new project with the same level of risk as its existing activities. The firm's unlevered equity beta (β_{EU}) can be used as the project's asset beta (β_A) because the shareholders' unlevered return (K_{eU}) equals the company's return (r_j) on a new project of equivalent risk. So, the project return that provides adequate compensation for holding shares in the company is the equity return (K_{eU}) obtained by substituting the appropriate equity beta (β_{EU}) into the familiar CAPM formula.

13) $K_{eU} = r_j = r_f + (r_m - r_f)\ \beta_{EU}$

The CAPM therefore offers management an important alternative to the derivation of project discount rates that use the traditional dividend or earnings share valuation models explained earlier in our text. For an *unlevered* (all-equity) firm, the *shareholders'* return (K_{eU}) defines the *company's* cost of capital (K_U) as follows:

14) $K_U = K_{eU} = r_j = r_f + (r_m - r_f)\ \beta_{EU}$

The question we must now ask is whether Equation (14) has any parallel if the firm is geared?

Ignoring fiscal policy, the short answer is yes. You will recall from Part Three, that in the long run, overall corporate returns are distributed between shareholders and debt-holders, which represent the cost of satisfying each capital provider. If no tax benefit is conferred on the company through the acquisition of debt, we can therefore redefine this overall cost by using the CAPM. The firm's levered WACC (K_G) which obviously varies as the value of debt and equity (V_D and V_E) moves with the market, will equal the return on the company's assets in equilibrium.

Rather than use traditional dividend, earnings and interest valuation models to derive a managerial WACC explained in Part Three, we can substitute an appropriately *geared* asset beta for an *all-equity* beta into the CAPM to estimate the overall return on debt and equity capital for project appraisal as follows:

15) $K_G = r_j = r_f + (r_m - r_f)\ \beta_A$

Of course, this relationship between WACC and the CAPM only applies in equilibrium when equity and debt are both fairly priced and the tax system is *neutral*. So, what happens when the system is biased in favour of the tax deductibility of debt, which figured so prominently throughout Part Three?

In the presence of taxation, there is no connection between the CAPM's required return on assets and a company's WACC. The former is independent of the financing of assets, whereas the latter is distorted by the fiscal relief that interest brings. If the effect of debt financing is to be eliminated, we must discover a company identical in every respect to our own, but without any gearing in its capital structure. Such a company should have the same asset beta factor, since its business risk (the variability of asset returns) is identical. However, because there is no debt, the firm's asset beta and the equity beta will exhibit the same values. Thus, we can conclude that:

- The asset beta for any company, irrespective of its financial policy (with or without tax) equals the equity beta of an ungeared company in the same class of business risk.

9.5 Modigliani-Miller and the CAPM

This discussion of companies within the same risk class reminds us yet again of MM's "law of one price" (*op.cit.*). But isn't this logical?

- There is no theoretical objection to combining MM's dividends and capital structure hypotheses within the CAPM. Both their models are entirely consistent with one another, whilst the assumptions that underpin MM and Modern Portfolio Theory (MPT) also stem from a common source, namely the Separation Theorem of Fisher (1930) where:

  - Investors are rational and risk averse.
  - Investors face the same opportunity set of investments, have the same expectations about the future and make one period decisions.
  - Investors measure risk by the standard deviation of expected returns.
  - Information concerning the mean-variance characteristics of investments is freely available.
  - There are no transaction costs.
  - The tax system is neutral.

So, let us conclude our analysis with *equilibrium* formulae for the relationship between the equity betas of companies in the same risk class (whose asset betas are obviously identical) by comparing the MM cost of capital hypothesis (WACC) with the CAPM.

Without debt in it capital structure, a company's asset beta equals its equity beta for projects of equivalent risk. However, according to MM's capital structure theory and their *arbitrage* process (explained in Chapter Seven) companies that are identical in every respect apart from their gearing should also have the same asset betas. Because their business risk is the same, the factors are not influenced by methods of financing.

To summarise MM's CAPM position, which is entirely consistent with their cost of capital hypothesis and the derivation of WACC:

- An ungeared company's asset beta equals its equity beta.
- A geared company's asset beta is lower than its equity beta.
- Irrespective of gearing, the asset beta for any company equals the equity beta of an ungeared company with the same business risk.
- The asset beta (equity beta) of an unlevered company can be used to evaluate projects in the same risk class without considering their finance.

$$\beta_j = \beta_A = \beta_{EU} < \beta_{E\,G}$$

You will recall from previous Chapters that MM's capital theory (like their dividend irrelevancy hypothesis) depends on perfect market assumptions. However, because these assumptions also underpin much else in finance (including the CAPM) we shall accept them to illustrate the MM relationship between the beta factors of all-equity and geared companies with the same systemic business risk.

Let us begin with the following CAPM equation, based on Equation (9), in a taxless world.

16) $\beta_A = \beta_{E\,U} = \beta_{E\,G}\,[V_E / (V_E + V_D)] + \beta_D\,[V_D / (V_E + V_D)]$

If we now rearrange terms, divide through by V_E and solve for β_{EG}, the mathematical relationship between the geared and ungeared equity betas can be expressed as follows:

17) $\beta_{EG} = \beta_{EU} + (\beta_{EU} - \beta_D). V_D / V_E$

This equation reveals that the equity beta in a geared company equals the equity beta for an all-share company in the same class of business risk, *plus* a premium for systemic financial risk. The premium represents the difference between the all-equity beta and debt beta multiplied by the debt-equity ratio. However, the important point is that the *increase* in the equity beta measured by the risk premium is exactly *offset* by a lower debt factor as the firm gears up, leaving the asset beta unaffected. In other words, irrespective of leverage, the asset betas of the two firms are still identical and equal the equity beta of the ungeared firm.

$\beta_A = \beta_{EU} < \beta_{EG}$

For those of you familiar with MM's capital structure hypothesis outlined in Chapter Seven, the parallels are striking.

According to MM, the expected return on equity for a geared firm (K_{eG}) relative to the return (K_{eU}) for an all-share firm in a taxless world equals:

18) $K_{eG} = K_{eU} + (K_{eU} - K_d). V_D / V_E.$

This states that the return for a geared firm equals an all-equity return for the same class of business risk, *plus* a financial risk premium defined by the difference between the all-equity return and the cost of debt multiplied by the debt-equity ratio. The premium compensates shareholders for increasing exposure to financial risk as a firm gears up. And as we also observed in Part Three, because the cheaper cost of debt exactly offsets rising equity yields, the overall cost of capital (WACC) is unaffected. So, irrespective of leverage, all firms with the same business risk can use the cost of equity for an all-share firm as a project discount rate before considering methods of financing.

Turning to a world of taxation, where debt is a *tax-deductible* expense with a tax rate (t), we can redefine the expected return on equity for a geared firm (K_{eG}) relative to the return (K_{eU}) for an all-share firm using a WACC formulation as follows:

19) $K_{eG} = K_{eU} + (K_{eU} - K_d) (1\text{-}t). V_D / V_E.$

Likewise, using Equation (17), the equity beta of a geared company is defined by:

20) $\beta_{EG} = \beta_{EU} + [(\beta_{EU} - \beta_D) (1\text{-}t). V_D / V_E]$

And if debt is risk-free with *zero* variance, so that β_D equals zero, the formula simplifies to:

$$21)\ \beta_{EG} = \beta_{EU} + [(\beta_{EU}(1\text{-}t).\ V_D / V_E]$$
$$= \beta_{EU}\{1 + [(1\text{-}t).\ (V_D/V_E)]\}$$

Review Activity

To illustrate the union between MM and the CAPM, consider Clapton plc, a leveraged company in an economy where interest is tax deductible at a 20 per cent corporate rate.

200 million ordinary shares (common stock) are authorised and issued at a current market value of £2.00 each (*ex-div*). The equity beta is 1.5.

Debt capital comprises £100 million, irredeemable 10 per cent loan stock, currently trading at par value and the risk-free rate.

Required:

1. Calculate the company's asset beta and briefly explain the result.
2. If you are mathematically minded, review the previous relationships between the asset beta, the CAPM and WACC outlined in this Chapter as a basis for project appraisal.

An Indicative Outline Solution

1. The Beta factor

Since the equity beta for an *ungeared* company equals the asset beta for any company in the same risk class, we can use Equation (20) or better still (21) to solve for β_{EU} and hence β_A as follows.

First, define the market values of Clapton's equity and debt

V_E = £2.00 × 200 million = £400 million
V_D = £100 million

Next, define the geared equity beta of 1.5 assuming that debt sold at par is risk-free (β_D = 0).

$$\beta_{EG} = 1.5 = \beta_{EU} + [\beta_{EU}(1\text{-}0.2)\ (100/400)]$$
$$= \beta_{EU}\{1 + [(1\text{-}0.2)\ (100/400)]\}$$

Finally, rearrange terms to solve for β_{EU} and β_A:

$$\beta_A = \beta_{EU} = 1.5/1.2 = \underline{1.25}$$

The result is to be expected. The asset beta should be smaller than the geared equity beta (*i.e.* 1.25 < 1.5) since the systemic risk associated with the asset investment is only one component of the total risk associated with the shares. The asset beta measures business risk, whereas the geared beta measures business and financial risk

2. The Asset Beta, CAPM and WACC

If management use the CAPM, rather than WACC, to obtain a risk-adjusted discount rate for project appraisal, first they need to resolve the sequential questions summarised in Table 9.2.

Question:	**Is the business risk of a project equivalent to that for the company?**	
Answer:	YES	NO
Solution:	Use the company's current equity beta	Use an equity beta for similar companies with similar projects
Question:	**Is the chosen equity beta affected by capital gearing?**	
Answer:	YES	NO
Solution:	De-leverage "ungear" the equity beta to derive an asset beta	Use an equity beta equivalent to an asset beta if it is not affected by gearing

Table 9.2: Derivation of an Asset Beta

Having obtained an appropriate asset beta, the project discount rate may then be calculated using our previous equations, beginning with the basic CAPM formula:

7) $r_j = r_f + (r_m - r_f)\ \beta_A$

According to MM, the asset betas of companies, or projects, in the same class of business risk are identical irrespective of leverage. Higher equity betas are offset by lower debt betas, just as higher equity yields offset cheaper debt financing when a firm gears up.

Even in a world where debt interest is tax deductible, it is possible to establish a connection between MM and the CAPM.

The MM cost of equity for a geared firm (WACC) is given by:

19) $K_{eG} = K_{eU} + [(K_{eU} - K_d)\ (1\text{-}t).\ V_D / V_E]$

According to the CAPM, the *shareholders'* return (K_{eU}) for an *unlevered* (all-equity) firm, defines the *company's* cost of capital (K_U) as follows:

13) $K_{eU} = r_j = r_f + (r_m - r_f)\ \beta_{EU}$

And for a *geared* firm, the corresponding equity return (K_{eG}) is given by:

22) $K_{eG} = r_j = r_f + (r_m - r_f)\ \beta_{EG}$

Where: $\beta_A = \beta_{EU} < \beta_{EG}$

If we assume that the company's pre-tax cost of debt (K_d) in Equation (19) equals the risk-free rate (r_f) in Equations (13) and (22) remember we can rewrite r_f for K_d in Equation (19).

If we then substitute Equations (13) and (22) into Equation (19) rearrange terms and simplify the result, we can confirm our earlier equations for a *geared* equity beta:

20) $\beta_{EG} = \beta_{E\,U} + [(\beta_{EU} - \beta_D)\ (1\text{-}t).\ V_D \,/\, V_E\,]$

And if debt is risk-free with *zero* variance, so that β_D equals zero, the formula simplifies to:

21) $$\beta_{EG} = \beta_{EU} + [(\beta_{EU}(1\text{-}t).\ V_D / V_E]$$
$$= \beta_{EU}\{1 + [(1\text{-}t).\ (V_D/V_E)]\}$$

Practical applications of these equations and the derivation of an equilibrium cost of equity and WACC using the CAPM are referenced in the companion Exercise Text (the next in my *bookboon* series). It also contains detailed examples of MM's comprehensive contribution to modern finance, which support all the previous Chapters, as a guide to your future studies.

9.6 Summary and Conclusions

This entire study is based upon a mean-variance analysis of investment decisions within a framework of uncertainty, using the normative objective of shareholder wealth maximisation and the assumptions of a perfect capital market, which we initially accepted without criticism.

Because ownership is divorced from control (the *agency* principal) we then argued that if management wish to maximise shareholders' wealth (using equity value as a proxy), companies ought to consider the consequences of their actions. According to conventional financial theory, every capital *investment* decision is inextricably tied to a firm's operational and strategic *financial* decisions, which include:

- The expected NPV maximisation of all a firm's projects (Part One).
- The relevance of an optimal dividend policy, rather retained earnings (Part Two).
- The determination of an optimal capital structure through the issue of debt, rather than equity (Part Three).

So, for any given investment policy, the pivotal issue is whether a firm can maximise its total value by manipulating its financial policies.

Modigliani and Miller comprehensively rubbished this view nearly sixty years ago. Using *arbitrage* in perfect capital markets, they demonstrated that financial policy does not matter.

The total market value of any company is independent of its dividend policy and capital structure, and is found by capitalising expected returns at a discount rate appropriate to its class of business risk.

In Part Four, the analysis of investment returns and the pricing of risk within a portfolio framework confirms their hypotheses. A detailed consideration of MPT, based on Markowitz efficiency, the beta coefficient in its various guises and the CAPM revealed that:

- The value of a levered firm in general equilibrium is equal to its unlevered counterpart.
- The sum of the values of debt and equity are based on the returns to each.
- The sum of the returns to debt-holders and shareholders must therefore equal the net operating cash flows of an all-equity firm.

Even within Markowitz's original frame of reference:

- The sum of the covariances of returns to both providers of capital must equal the covariance of the firm's net operating cash flows.

Given the normative assumptions of traditional capital market theory, upon which our study is based, Fisher's Separation Theorem, the MM hypotheses and Modern Portfolio Theory (MPT) are theoretically united. The critical question is whether the relaxation of their common assumptions invalidates their real world applicability. If so, we must continue the search for more realistic explanations of investor behaviour.

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