Electrical Power

W. J. R. H. Pooler

bookboon
The eBook company

W.J.R.H. POOLER

ELECTRICAL POWER

Electrical Power
4th edition

ISBN 978-87-403-2134-0

CONTENTS

About the author	vi
Summary	6
Electromagnetism	14
Induced EMF	25
Electrostatics	33
DC Circuits	40
Alternating Current (AC)	45
Resistance, Inductance and Capacitance on AC	53
AC Circuits	58
Magnetic Properties of Materials	68

DC Motors and Generators	**75**
AC Synchronous Machines	**101**
AC Induction Motors	**137**
Insulation	**144**
Transformers	**145**
Rectifiers	**151**
Power Lines	**157**
Neutral Earthing	**160**
Switchgear	**163**
Instruments	**171**
Protection	**179**
Power Systems	**184**
Generator Response to System Faults	**189**
Calculation of Fault Currents	**219**
Symmetrical Components	**224**
Commissioning and Fault Finding	**232**
Program Manual	**236**
Index	**242**

ABOUT THE AUTHOR

W. J. R. H. Pooler

ONC, MA (Cantab) class 1, CENG, MIEE, MIMechE

I studied for and obtained Ordinary National Certificate while working as an apprentice at the English Electric Co, Stafford. This included time in their high voltage laboratory. I then went to Cambridge University and passed the Mechanical Sciences Tripos after two years with First Class Honours. For the third year, I carried out further studies on heavy electrical power machines. After graduating, I joined the Iraq Petroleum Company in Kirkuk, Iraq and was later appointed Protection Engineer and System Control Engineer responsible for the operation of the high voltage network and for the hands on commissioning of all new electrical plant including 66kv and 11kv cables and lines and transformers up to 5MVA and motors up to 2000 hp.

I was then appointed Head of Electrical Engineering at Basrah Petroleum Co responsible for four power stations, 33kv and 11kv transformers, cables and lines and motors up to 1500 hp. The operation of all Instrumentation in the Production Plants and all Telecommunications in the Company was later added to my responsibilities.

This book is based on experiences gained during this period.

You can contact me on john.pooler@tiscali.co.uk

SUMMARY

Notation
A full stop in Bold (**.**) is a decimal point.
A mid line dot (·) means multiplied by.
A large mid line dot (•) means vector dot product. **A • B** = A · B Cos θ.

Units
cm/gm/sec (cgs) units are;
dyne = force to accelerate 1 gm at 1 cm/sec^2
erg = work done by 1 dyne cm
unit pole = magnetic pole that exerts 1 dyne on an identical pole 1 cm distant in a vacuum
***G* = gauss** = magnetic field that exerts 1 dyne on a unit pole
maxwell (previously **lines**) = magnetic flux = magnetic flux of field of 1 gauss crossing 1 cm^2
emu of current = current flowing through an arc of a circle 1 cm radius, length 1 cm which causes a magnetic field of 1 gauss at the centre of the arc
A coil carrying an electrical current generates a magneto motive force (**mmf**), which is measured in **Gilberts**. The mmf of a coil is (4 π/10) · (Ampere Turns) Gilberts.
Oe = **oersted** = magnetizing force in Gilberts per cm length of the magnetic circuit. The symbol for magnetizing force per unit length is ***H***.
Permeability is a property of a magnetic material. The symbol for permeability is μ.
In a vacuum, $\mu = 1$. In air, μ = approx 1. For iron, μ can be over 1000 but is not a constant.
A magnetizing force of 1 Oe produces a magnetic field of μ gauss.

Engineering units are;
N = newton = force to accelerate 1 kg at 1 m/sec^2 = 10^5 dynes
J = **joule** = work done by 1 newton metre = 10^7 ergs
W = **watt** = 1 joule/ second = 10^7 ergs/sec
kW = **kilowatt** = 1000 watts
HP =**horse power** = 550 ft lbs/sec ≈ 746 watts
amp = 1/10 of emu of current. The symbol for current is ***I***
T = **tesla** = magnetic field strength = 10^4 gauss. The symbol for magnetic field is ***B***
Wb = **weber** = magnetic flux = magnetic flux of magnetic field of 1 tesla crossing 1 m^2.
Wb = 10^8 maxwells. The symbol for magnetic flux is Φ.

Corkscrew Rule As current flows along a wire, the magnetic field rotates in the direction of a corkscrew.
MMF in a solenoid, N turns and current I mmf = $(4\pi/10) \cdot N I$ Gilberts.
Magnetizing Force at the centre of a long solenoid
$H = (4\pi/10)\, N I / L = 1.26\, N I / L$ Oersteds
where L is the length in cm and $(N I)$ is the ampere turns
Magnetic field $B = \mu H$ where B is in tesla and $H = 1.26\, N I\, 10^{-6}$ / metre
Magnetic field at the centre of a long solenoid
In Engineering units $B = (4\pi/10)\, \mu N I / L \cdot 10^{-6} = 1.26\, \mu N I / L \cdot 10^{-6}$ tesla
B = 1.26 μ (Ampere Turns/metre) · 10^{-6} tesla.
In magnetic materials, μ is not a constant. The maximum useful value of B is about 1**.**5 Tesla
Magnetic flux $\Phi = B A$ where Φ is in weber, B is in tesla and A is in square metres.
Magnetic flux in a uniform closed magnetic circuit, length L metres and cross section A square metres is $\Phi = 1.26\, N I \mu A \cdot 10^{-6} / L$ weber.

Closed magnetic circuit eg a ring with an air gap or the field circuit of an electrical machine, mmf = sum of mmfs to drive same Φ in each part, therefore $\Phi = 1.26\,N\,I \cdot 10^{-6} / \Sigma(L_l / \mu_l A_l)$
Where Φ is in weber, I in amps, A in m^2 and L in metres.
Force on a conductor in a magnetic field $F = B\,I\,L$ Newtons where B in tesla, I in amps and L in metres
Force on parallel conductors $F = [\,2\,I^2 / d] \cdot 10^{-7}$ Newtons/metre where I is in amps and d is in metres**.** With currents in opposite directions, the force is pushing the conductors apart
Pull of Electromagnet Pull **=** $B^2 \cdot 10^7 / (8\,\pi)$ newtons per m^2 of magnet face where B is in tesla

Definition of Volt. The **potential difference** between two points is 1 **volt** if 1 watt of power is dissipated when 1 amp flows from one point to the other. $W = V\,I$
Ohms Law (for a direct current circuit with resistance R ohms) $V = I\,R$
Power loss in a resistor $W = I^2\,R = V^2 / R$
Resistance $R = \rho\,L\,(1 + \alpha T) / A$ ohms where ρ is resistivity in ohms per cm cube, L cm is the length, A cm^2 is the cross sectional area, α is temp co-eff and T is the temperature in degrees Celsius.
Several sources give Copper $\rho = 1.7 \cdot 10^{-6}$ ohms per cm cube and $\alpha = 0.004$.
At very low temperatures, the resistance of some materials falls to zero
Resistance R_1 in series with R_2. Equivalent resistance = $R_1 + R_2$
Resistance R_1 in parallel with R_2. Equivalent resistance = $1/(1/R_1 + 1/R_2)$

Kirchoff's first law
The total current leaving a point on an electrical circuit = total current entering
Kirchoff's second law.
The sum of the voltages round any circuit = net "$I\,R$" drop in the circuit

Induced emf
$E = -N\,d\Phi/dt$ where E is in volts, N is number of turns and $d\Phi/dt$ is in Wb/sec
This equation is the foundation on which Electrical Engineering is based.
Self Inductance
$E = -L\,dI/dt$ where E is in volts, L is inductance in henries and dI/dt is in amps/sec
Self inductance of a coil wound on a ring of permeability μ is $L = 1.26\,N^2\,\mu\,A / S \cdot 10^{-6}$ Henries where N is number of turns, A is cross sectional area in m^2 and S metres is the length of the magnetic circuit. Experimental results for a coil length S metres, diameter d metres and radial thickness t metres with air core indicate $L = 3\,d^2\,N^2 / (1.2\,d + 3.5\,S + 4\,t)$ micro Henries. ($t = 0$ for a single layer coil).
Energy stored in an inductance $= \frac{1}{2}\,L\,I^2$ Joules where L is in henries and I is in amps
Capacitance $q = C\,V$ where q is in Coulombs (ie amps times seconds), C is Farads and V is volts. Capacitance of a parallel plate condenser area A cm^2 and separated d cm and dielectric constant k $C = 1.11 \cdot 10^{-6}\,A\,k / (4\,\pi\,d)$ microfarads
Capacitance of co-axial cylinders radii a and b
$C = 1.11 \cdot 10^{-6}\;k / [\,2\ln(b/a)\,]$ microfarads per cm
Energy stored in a capacitance $= \frac{1}{2}\,C\,V^2$ Joules where C is in farads and V in volts

DC Motors and Generators

Motors obey the **left hand rule** and generators the **right hand rule**, (the gener - righter rule).

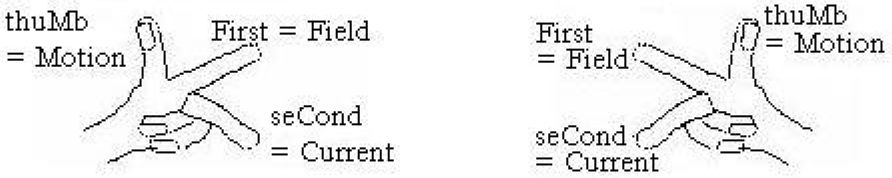


Figure 1; Left hand and Right hand rules

Back emf in DC machine $E = 2p\, Z_S\, \Phi\, rps$ where E is volts, $2p$ is number of poles, Z_S is number of conductors in series, Φ is in Wb and rps is speed in rev/sec

Power $W = 2p\, Z_S\, \Phi\, I_a\, rps$ where W is watts, I_a is the armature current in amps

Torque Torque $= 2p\, Z_S\, \Phi\, I_a\, /(2\,\pi\,)$ Newton metres $= E\, I_a\, /(2\,\pi\, rps\,)$ Newton metres

In Imperial units Torque$= 0.117 \cdot 2p\, Z_S\, \Phi\, I_a$ lb ft $= 0.117\, E\, I_a\, /\, (\, rps)$ lb ft)

Shunt motor $n = n_0 - m\,T$ where n is speed, n_0 is no load speed, m is approximately constant and T is Torque. $n_0 = V/(2p\,\Phi\,Z_S)$ and $m = 2\,\pi\,R_a/(2p\,\Phi\,Z_S)^2$

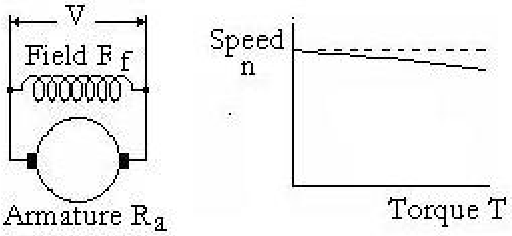


Figure 2; DC Shunt motor

Series motor $T = T_0/(1 + \alpha\,n)^2$ where T_0 and α are approximately constant

$T_0 = 2p\,K\,Z_S\,V^2/(2\,\pi\,R^2)$ and $\alpha = 2p\,K\,Z_S/R^2$ and $K = \Phi/I = 4\,\pi\,N\cdot 10^{-7}/\Sigma(L/\mu\,A)$

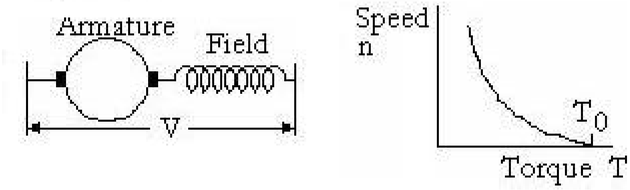


Figure 3: DC Series motor

Compound motor has shunt and series windings. This can increase the starting torque for a shunt motor. If wound in opposition, the motor speed can be made nearly constant.
Armature reaction causes a magnetizing force centred between the poles distorting the field and slightly reducing it.
Compensating windings between the main poles cancel the armature reaction.
Interpoles are small poles carrying armature current between the main poles to improve commutation.
Armature windings can be **lap** or **wave** wound.

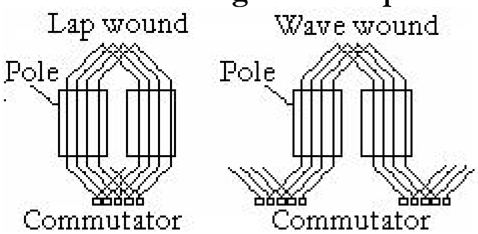


Figure 4; DC machine armature winding

DC shunt generators will fail to excite if there is no residual magnetism or the field resistance is above the critical value for the speed.
DC series or compound generators require special treatment especially when two or more are in parallel.

Alternating Current (AC) emf $E = E_p\,\mathrm{Sin}\,(\omega\,t) = E_p\,\mathrm{Sin}\,(2\,\pi\,f\,t)$
where E_p is peak value, f is frequency and t is seconds.
Mean value of E for a half cycle $= 2\,E_p/\pi = 0.636\,E_p$.
Root mean square (rms) value $= E_p/\sqrt{2} = 0.707\,E_p$
peak factor = (peak value)/(rms value).
form factor = (rms value)/(average value for ½ cycle)
Square wave peak factor = 1, form factor = 1
Sine wave peak factor = 1.41, form factor = 1.11
Triangular wave peak factor = 1.73, form factor = 1.15

Vector representation of AC voltage and current.

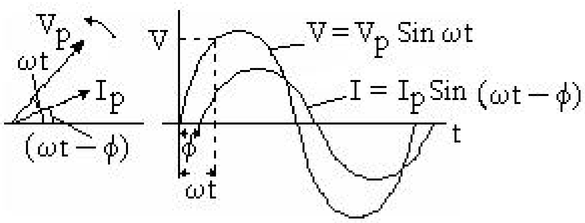


Figure 5; Vector representation of AC

The projection on a vertical surface of a vector rotating at constant speed anti clockwise is equal to the value of an AC voltage or current. The phase angle between V and I is the same as the angle between their vectors. The diagram shows the Vector representation of current and voltage where the current lags the voltage This diagram shows the vectors as the peak values. However the rms values are 0.707 times the peak value. Thus the vector diagram shows the rms values to a different scale. Vector diagrams are rms values unless stated otherwise.

Power Factor is Cos φ where φ is the angle between the vectors for V and I
Power in a single phase AC circuit $W = V I \text{ Cos } \varphi$ watts
Three phase ac. Three voltages with phase angles of 120 degrees between each.
Power in a three phase AC circuit $W = \sqrt{3}\, V I \text{ Cos } \varphi$ watts
where V is the voltage between lines

Resistance is higher on AC due to eddy current loss.
$R_f = R_0 [1 + 100\, \pi^4 f^2 a^4 / (3 \rho^2)]$
where R_f and R_0 are the AC and DC resistances, f is the frequency, a is the radius of the conductor in metres and ρ is the resistance in microhms/cm cube.
$V = I R$ and the voltage V is in phase with the current I.

Inductance $V = I X_L$ where $X_L = 2 \pi f L$ where L is in Henries. I lags V by $\pi/2$.
At 50 cps, $X_L = 314 L$

Capacitance $V = I X_C$ where $X_C = 1/(2 \pi f C)$ where C is in farads. I leads V by $\pi/2$.
At 50 cps $X_C = 3183/C$ where C is in micro farads.

Inductive Impedance $Z = R + jX$. $V = I \sqrt{(R^2 + X^2)}$. I lags V by arc tan (X/R)
Capacitive Impedance $Z = R + jX$. $V = I \sqrt{(R^2 + X^2)}$ I leads V by arc tan (X/R)
Impedance R_1 + jX_1 in series with R_2 + jX_2 Equivalent impedance = $(R_1 + R_2) + j(X_1 + X_2)$
Impedance R_1 + jX_1 in parallel with R_2 + jX_2 Put X +ive for inductance, –ive for capacitance
Put $Z_1 = \sqrt{(R_1^2 + X_1^2)}$ and $Z_2 = \sqrt{(R_2^2 + X_2^2)}$
Put $A = R_1 / Z_1^2 + R_2 / Z_2^2$ and $B = X_1 / Z_1^2 + X_2 / Z_2^2$
Equivalent impedance is $R = A / (A^2 + B^2)$ and $X = B / (A^2 + B^2)$

Sum of two AC currents (or voltages).

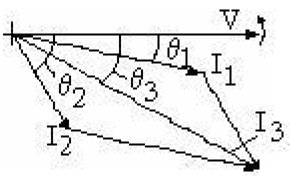


Figure 6; Sum of two AC currents or voltages

Add I_1 at phase angle θ_1 to current I_2 at phase angle θ_2 and the result is I_3 at phase angle θ_3
I_3 and θ_3 are obtained by the vector addition of I_1 and I_2.

Hysteresis loss Loss = f (area of hysteresis loop) watts/cubic metre where the hysteresis loop is in tesla and ampere turns/metre
Energy in magnetic field Energy = $B^2 \cdot 10^7 / (8\,\pi)$ joules per cubic metre where B is in tesla

Eddy current loss in laminated core Loss = $\pi^2 f^2 B_M^2 b^2 / (6\,\rho)$ watts per cm^3
Where $B = B_M$ Sin $(2\pi f t)$ is parallel to the lamination, f is the frequency in Hz, b is the thickness in cm of the lamination and ρ is the resistivity in ohms/ cm cube.

Star/Delta transformation
Three impedances $R + jX$ in star = three impedances $3R + 3jX$ in Delta

AC generators and motors
Fundamental EMF of generator $E_{RMS} = 4.44\, kp\, kd\, N\, f\, \Phi_{TOTAL}$
where N is (number of turns)/(pairs of poles) and kp is the pitch factor. If each coil spans an angle of 2λ instead of the full angle π between the poles, then kp = Sin (λ).
kd is the distribution factor due to the phase difference of the emf in each conductor.
kd = (vector sum of emfs)/(scalar sum of emfs)

For N^{th} harmonic, knp = Sin ($n\lambda$), and knd = Sin ($n\theta/2$) /[c Sin ($n\theta/2c$)]
where $\theta = \pi$ /(no of phases) and c = slots / phase/pole.
Harmonic content can be kept small by suitable values for λ, θ and c.
The **third harmonic** is blocked by delta/star transformation.
Armature reaction of a current in phase with V gives an mmf between the poles distorting the field. Armature reaction of lagging currents give an mmf opposing the main field. Armature reaction of leading currents give an mmf boosting the main field.

Vector Diagram of the emfs, current and mmfs of a salient pole synchronous generator.

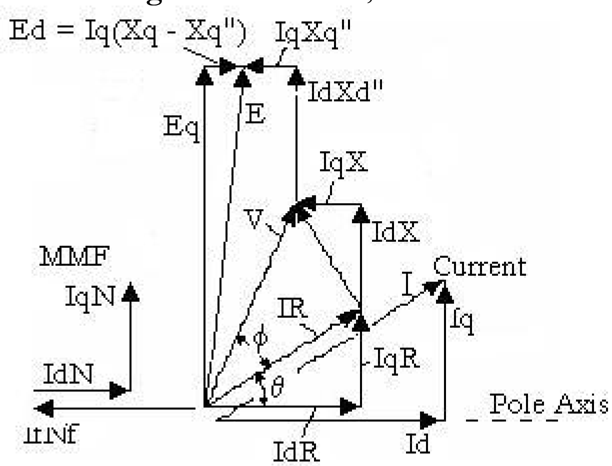


Figure 7; Vector diag AC generator

Automatic Voltage Regulator adjusts the excitation so that at the system design power factor, the voltage is correct whatever the current. If however it adjusts the excitation to give the correct voltage at other power factors, then two machines will not run in parallel. One can supply a huge leading current and the other a huge lagging current. A "droop" is needed to give a lower voltage if the power factor lags by more than the system design. This is achieved by the **compounding**. Faulty Compounding causes unstable sharing of $kVAr$ which can be quite violent.

System Faults. When a fault occurs, initially dc currents are induced in the damping winding and main field circuit opposing the demagnetizing effect of the low power factor fault current. These currents die away exponentially causing the fault current to fall. In extreme cases it can fall below the full load value.

Induction motor Power = $3\,V^2\,(1-\Sigma)\,R_r\,\Sigma\,/(R_r^2 + X^2\,\Sigma^2)$ watts
where the slip $\Sigma = (n_0 - n)\,/\,n_0$
Power = $2\,\pi\,T\,(1-\Sigma)\,n_0$ watts
where T is the torque in Newton metres and n_0 is the synch speed
Torque = $3\,V^2\,R_r\,\Sigma\,/[\,2\,\pi\,n_0\,(R_r^2 + X^2\,\Sigma^2)\,]$ Torque is a maximum when $\Sigma = R_r\,/X$
Put $\Sigma = 1$, Starting Torque = $3\,V^2\,R_r\,/[\,2\,\pi\,n_0\,(R_r^2 + X^2)]$
If $R_r = X$, the maximum torque occurs when the speed is zero but the motor would be very inefficient. However large motors sometimes have slip rings allowing an external resistance to be added for starting.

The Induction motor speed torque curve. Sometimes there is a kink in the curve at a speed below the speed for maximum torque due to harmonics in the supply. In such cases, the motor may get stuck at this speed , called "crawling".

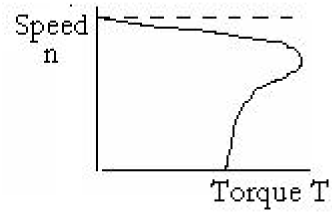


Figure 8; Induction motor

Transformers

Power transformers are usually delta primary and star secondary. The primary is supplied through three conductors.

Flux $\Phi_{max} = [4\,\pi\,\mu\,A\,N\,I_{max}\,/L] \cdot 10^{-7}$ weber

EMF $E_{rms} = 4.44\,N\,\Phi_{max}\,f$ volts

Delta/Star transformer

Three phase load

$V_1 \cdot I_1 = V_2 \cdot I_2$

But a single phase load on the secondary results in a current on two lines in the primary governed by the turns ratio, not the voltage ratio.

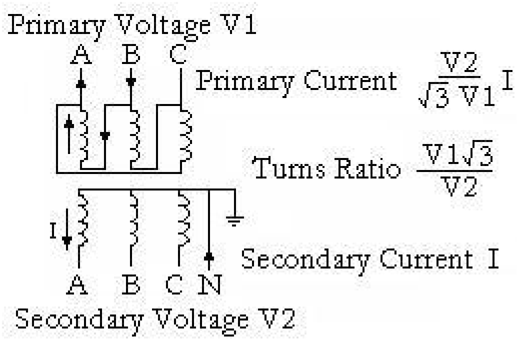


Figure 9; Delta/Star Transformer

Third harmonic voltages are the same at each end of a winding connected between phases. Therefore no third harmonic current flows in the primary of a delta/star transformer and no third harmonic voltage appears in the secondary.

ELECTROMAGNETISM

Notation

The symbol " · " or adjacent variables will be used to signify "multiplied by".
The symbol " • " signifies vector dot product. **A•B** = A · B Cos θ
Variables are in italics for example *V*, *I* and *X*.

Magnets, Magnetic Fields and Direct Currents
Units of Force, Work and Power

cm/gm/sec (cgs) units are **dyne** = force to accelerate 1 gm at 1 cm/sec^2
erg = work done by 1 dyne cm

Engineering units are **newton** = force to accelerate 1 kg at 1 m/sec^2
= 10^5 dynes
joule = work done by 1 newton metre = 10^7 ergs
watt = 1 joule/ second = 10^7 ergs/sec
kilowatt = 1000 watts
horse power = 550 ft lbs/sec ≈ 746 watts

Magnet
A magnet has two poles, a North pole and a South pole. When suspended, the North pole aligns towards the North and the South pole towards the South.

Unit Pole
The pole of a magnet, whose strength is one **unit pole**, placed in a vacuum one cm from an identical pole repels it with a force of one dyne.

Magnetic Field
The magnetic field at any point is the force in dynes on a unit North pole placed at that point, provided it does not disturb the magnetic field. The symbol used for magnetic field is $\boldsymbol{B}$. It is a vector quantity. The cgs unit is dynes/unit pole or **gauss** (G).
The Engineering unit is the **tesla** (T) = 10^4 gauss.

(value in tesla) = $10^{-4} \cdot$ (value in gauss).

Iron filings sprinkled on a card placed over the magnet align into lines from the North to the South pole and show the direction of the magnetic field.

Magnetic Flux
The magnetic flux crossing an area normal to a magnetic field is the product of the magnetic field and the area. The symbol used for magnetic flux is $\boldsymbol{\Phi}$
The cgs unit is the **maxwell** (previously **lines**) = gauss cm^2.
The Engineering unit is the **weber** (Wb) = tesla m^2.

1 weber = 10^8 maxwells.

(value in weber) = $10^{-8} \cdot$ (value in maxwells).

Flux $\Phi = \int B \, dA$ where Φ is in weber, B is in Tesla and A is m^2

Magnetic Flux from a unit pole
The magnetic field 1 cm from a unit pole is 1 Gauss by definitions of a Unit Pole and Magnetic Field.
But the surface area of the sphere radius 1 cm is 4π cm^2
Thus the total magnetic flux leaving a unit pole is 4π maxwells.
In Engineering units, the total flux leaving a unit pole is $4\pi\ 10^{-8}$ weber.

Magnetic Field due to a unit pole
At a distance r cms from a unit pole, the magnetic flux of 4π maxwells is spread over the surface of a sphere area $4\pi r^2$ square cms. Therefore the field is $4\pi/4\pi r^2 = (1/r^2)$ gauss. A unit pole exerts a force of $(1/r^2)$ dynes on a unit pole at a distance of r cms.
In Engineering units the Magnetic Field r metres from the unit pole is $(1/r^2)\ 10^{-8}$ tesla

Magnetic Field due to an Electrical Current

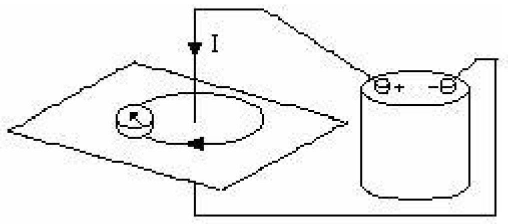


Figure 10; Magnetic field due to an electric current

Faraday was carrying out experiments in his laboratory with an electrical cell. There happened to be a magnetic compass on his table. He noticed that the compass needle was deflected whenever he switched on the electrical current. He investigated further and found that the current caused a magnetic field in a circular path round the wire. The direction of the magnetic field is in the clockwise direction when viewed in the direction of the current which flows from the positive to the negative terminal of the cell.
This is the Corkscrew Rule. As the corkscrew is wound forward in the direction of the current, it rotates in the direction of the magnetic field.

The electric current is actually a flow of negatively charged electrons through the metal. The electric current is shown by convention from the positive to the negative. This direction is the reverse of the electron flow. For engineering purposes, the current flows from the positive to the negative.

Magnetizing Force
The current produces a magnetizing force which is proportional to the current and inversely proportional to the square of its distance from the wire. The symbol for magnetizing force is $\boldsymbol{H}$ and the emu unit is the **oersted** (*Oe*).
In a vacuum, a magnetizing force of one *Oe* produces a magnetic field of one $\boldsymbol{G}$.
In a medium with permeability $\boldsymbol{\mu}$, one *Oe* produces a magnetic field of $\boldsymbol{B} = \boldsymbol{\mu G}$.
The value of μ in a vacuum is 1 and in air is very close to 1.
For small values of H, μ for iron is approximately constant with a value in the order of 1000 or more.

$B = \mu \cdot H$ where μ is approximately constant provided H is small.

However the relation between B and H in iron is not linear. For magnetic fields above about 1.5 tesla, the gradient falls dramatically. Thus magnetic circuits are often designed for a maximum magnetic field B of about 1.5 tesla.

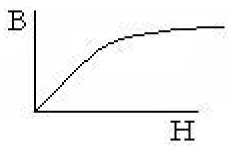


Figure 11; *B-H* Curve
Furthermore, iron retains some magnetism when the magnetizing force is switched off. The magnetic field B therefore depends on both H and on what has gone before.

Electromagnetic unit of current, Ampère's or La Places' Rule
Electrical current, strength one electromagnetic unit (1 emu), flowing through an arc of wire one cm long and one cm radius produces a magnetizing force of one oersted at the centre. In air, the magnetising force due to 1 emu of current produces a magnetic field of 1 *G*.

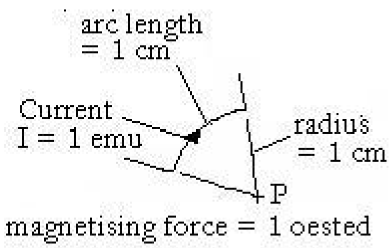


Figure 12; Magnetising Force

The symbol for current is ***I***
The Engineering unit is the ampère or **amp**.

1 amp = 1/10 em unit of current.

(value in amps) = 10 · (value in emu)

Quantity of Electricity

The emu for quantity of electricity is the quantity of electricity passing through a cross section of the wire carrying 1 emu of current for 1 second.
The symbol for quantity of electricity is ***q*** and the Engineering unit is the **coulomb.**

1 coulomb = 1 amp second = 1/10 emu of quantity.

$I = dq/dt$ where I is in amps, q is in coulombs and t is in seconds

Magnetizing force due to an element

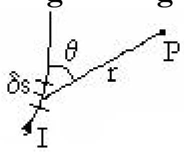


Figure 13; Magnetising Force due to an element

The magnetizing force at P due to I emu of current through the element δs is;

$\delta H = I \cdot \delta s \cdot \mathrm{Sin}\ \theta\ /\ r^2$ where H is in oersted, I is in emu, δs and r are in cm.

Mechanical Force on a conductor in a magnetic field
Place two unit poles P and Q a distance r cm apart in air ($\mu = 1$).
Pole P exerts a force of $1/r^2$ dynes on pole Q in the direction shown.
By the definition of magnetic field, the field at Q due to P is $1/r^2$ gauss.

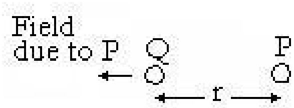


Figure 14; Force on a Unit Pole

Replace pole Q by a length of wire δs long at an angle θ to the direction of P. The magnetic field at Q due to pole P remains at $1/r^2$ gauss.
Pass a current of I emu through the wire.
It will cause a magnetic field of $I \cdot \delta s \cdot \text{Sin}\,\theta \,/\, r^2$ gauss at P
By the definition of magnetic field, the element of wire exerts a force of $I \cdot \delta s \cdot \text{Sin}\,\theta \,/ r^2$ dynes on pole P. By the corkscrew law the direction of the force is into the paper.

Action and reaction are equal and opposite, therefore the magnetic field of $1/r^2$ gauss due to pole P exerts a force of $I \cdot \delta s \cdot \text{Sin}\,\theta \,/ r^2$ dynes on the element of wire.

Therefore a field of B gauss would exert a force of $B \cdot I \cdot \delta s \cdot \text{Sin}\,\theta$ dynes on the element of wire, the force being in a direction out of the paper.

In a uniform field of B gauss, a length of wire L cm long at right angles to the field and carrying a current of I emu will experience a force $F = B \cdot I \cdot L$ dynes.

1 emu of current = 10 amps and 1 Gauss = 10^{-4} tesla. and 1 cm = 10^{-2} metres
Therefore
1 amp flowing through 1 metre wire in a field of 1 tesla will experience a Force of 10^5 dynes.
But 10^5 dynes = 1 Newton
Converting to Engineering units,
$F = B \cdot I \cdot L$ newtons
where B is in tesla, I is in amps and L is in metres.

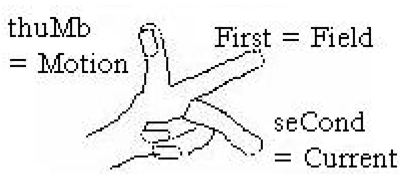


Figure 15; Left Hand rule

Using the corkscrew law as above, the direction that the wire tries to move complies with the left hand rule, (motors in UK keep to the left).

In diagrams, the direction of a vector into or out of the paper can be represented as a circle containing the tail or the point of an arrow.

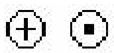


Figure 16; Notation for vectors

The magnetizing force at the centre of a circle of wire radius *r* carrying a current *I*

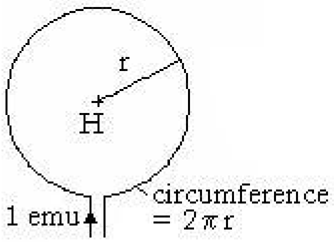


Figure 17; Magnetising force at centre of a circular wire

$H = 2 \cdot \pi \cdot r \cdot I / r^2 \quad = 2 \cdot \pi \cdot I / r$
where H is in oersted, I is in em units and r is in cms.
Magnetic field at the centre in air
$B = (2 \cdot \pi \cdot \mu \cdot I / r)\ 10^{-7}$
where B is in tesla, I is in amps and r is in metres.

The magnetizing force δ*H* at a point P distant *d* cm from a long straight wire due due to current *I* emu in element δ*s*

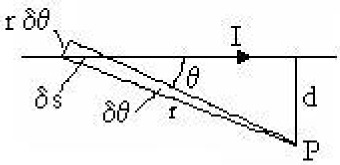


Figure 18; Magnetising Force due to a straight wire

$\delta H = I \cdot \delta s \cdot \text{Sin}\ \theta / r^2$
But $r \cdot \delta\theta = \delta s \cdot \text{Sin}\ \theta$ and $d = r \cdot \text{Sin}\ \theta$
$\delta H = I \cdot r \cdot \delta\theta / r^2 = I \cdot \delta\theta / r$
$\delta H = I \cdot \text{Sin}\ \theta \cdot \delta\theta / d$
Integrate from 0 to π
$H = \int (I \cdot \text{Sin}\ \theta / d)\ \delta\theta = (I/d)\ (-\text{Cos}\ \pi + \text{Cos}\ 0)$
$H = 2 \cdot I / d$
where H is in oersted, I is in em units and d is in cm.

Magnetising Force within a conductor
Consider the magnetizing force in a circular element radius x cms and thickness δx within the conductor.
The current enclosed by this element is $I\, \pi\, x^2 / (\pi\, a^2)$ emu units where a is the conductor radius
Current enclosed by the element = $I\, x^2 / a^2$ emu units
Magnetising Force due to the current enclosed by the circilar element $H = 2 \cdot (I\, x^2 / a^2) / d$
Put $d = x$ to get the M agnetising Force at radius x within the conductor.
$H = 2 \cdot I\, x / a^2$ oersted
.

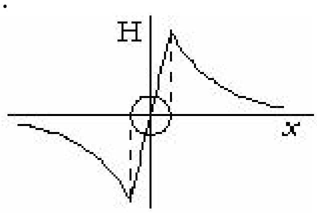


Figure 19; Magnetising Force near and within a wire

Therefore the magnetic field a distance x metres from the centre of a conductor carrying I amps is given by;

Within the conductor Magnetic field $B = (2 \cdot I \cdot \mu\, x / d^2) \cdot 10^{-7}$ tesla
Outside the conductor Magnetic field $B = (2 \cdot I \cdot \mu / x) \cdot 10^{-7}$ tesla
Where $\mu = 1$ for air or non magnetic materials.

The force between two adjacent conductors in air

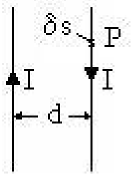


Figure 20; Force between conductors

Consider two parallel conductors d metres apart in air ($\mu = 1$) each carrying a current I amps in opposite directions.
Magnetic field at P due to left hand conductor
$B = (2 \cdot I / d) \cdot 10^{-7}$ tesla into the paper (corkscrew rule)
Mechanical force on δs at P = $B \cdot I \cdot \delta s$ newtons where δs is in metres
Mechanical force = $[2 \cdot I^2 / d] \cdot 10^{-7}$ newtons/metre
where I is in amps and d is in metres. The force is pushing the conductors apart (left hand rule).

Example
A conductor carries a current of 100 amps and a length of 20 cms is in a uniform magnetic field of 0.8 tesla perpendicular to the conductor. Find the mechanical force on the conductor.
Force $F = B \cdot I \cdot L$ newtons where B is in tesla, I is in amps and L is in metres.
$F = 0.8 \cdot 100 \cdot 0.2 = 16$ newtons = 1.63 kg wt.

Example
Two conductors are 2 cm apart and each carries a current of 400 amps in opposite directions. Find the force each exerts on the other.
The Force is $[2 \cdot 400^2 / (2 \cdot 10^{-2})] \cdot 10^{-7} = 400^2 \cdot 10^{-5} = 1.6$ newtons / metre

The magnetizing force on the axis of a circle of wire carrying a current *I* em units

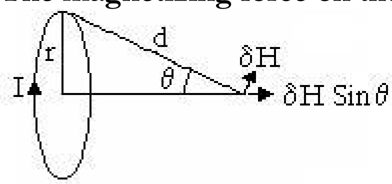


Figure 21; Magnetising Force
$\delta H = I \cdot \delta s / d^2$
Component along axis
$\delta H \cdot \text{Sin}\,\theta = I \cdot \delta s \cdot \text{Sin}\,\theta / d^2$
$H = I \cdot 2\pi r \cdot \text{Sin}\,\theta / d^2$
$= 2\pi \cdot I \cdot \text{Sin}^3\,\theta / r$
By symmetry, the sum of the components of H perpendicular to the axis is zero

Magnetic field due to I,
$B = [2\pi\,\mu\,I\,\text{Sin}^3\,\theta / r]\,10^{-7}$
where B is in tesla, I is in amps and r is in metres.

The magnetizing force on the axis of a solenoid

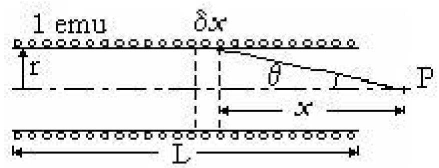


Figure 22; Magnetising Force due to a solenoid
N turns uniformly wound
$N\,\delta x / L$ turns in element δx

Magnetizing force at P due to element δx
$\delta H = 2\,\pi\,I\,\text{Sin}^3\theta\;N\,\delta x / (L\,r)$
But $x = r\,\text{Cot}\,\theta$
$\delta x = -\,\text{r}\,\text{Cosec}^2\theta\,\delta\theta$
$\delta H = -\,2\,\pi\,I\,\text{Sin}^3\theta\;N\,r\,\text{Cosec}^2\theta\,\delta\theta / (L\,r)$
$\delta H = -\,(2\,\pi\,I\,N / L)\,\text{Sin}\,\theta\;\delta\theta$
$H = \int [-\,(2\,\pi\,I\,N / L)\,\text{Sin}\,\theta]\,\delta\theta$ from θ_1 to θ_2
$H = (2\,\pi\,I\,N / L)\,(\text{Cos}\,\theta_2 - \text{Cos}\,\theta_1)$

If P is at the centre of the solenoid,

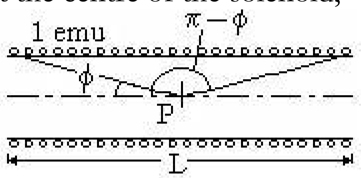


Figure 23; Magnetising Force due to a solenoid

$\theta_2 = \phi$ and $\theta_1 = \pi - \phi$

$H = (4 \pi I N \cos \phi / L$ where I is in emu

If the solenoid is very long, then $\cos \phi = 1$

$H = (4\pi/10) IN / L$

$H = 1.26\, I N / L$

where H is in oersted, I is in amps and L is in cm

Magneto Motive Force

The current in a coil is said to produce a Magneto Motive Force (mmf) of $(4 \pi / 10) \cdot$ (Ampere Turns) **Gilberts**. Thus one Gilbert / cm produces a Magnetizing Force of one oersted. The symbol for mmf is ***F***.

$F = (4 \pi / 10) \cdot$ (Ampere Turns) where F is in Gilberts

Magnetic field (or Flux density) at the centre of a solenoid

$B = 1.26\, \mu\, I\, (N/L)$ gauss

where B is in gauss, I is in amps and L is in cms

In Engineering units

$B = 1.26\, \mu\, I\, (N/L) \cdot 10^{-6}$ tesla where I is in amps and L is in metres

$B = 1.26\, \mu$ (Ampere Turns/metre) $\cdot\, 10^{-6}$ tesla

Magnetic Circuit

If the solenoid is wound on a ring, the magnetic circuit is complete within the ring.

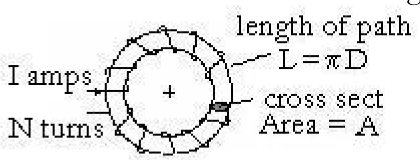


Figure 24; Magnetic Circuit in a ring

The mmf $F = (4\, \pi/10)\ N\ I$ Gilberts

This causes flux $\Phi = (\mu\, F\, A \,/\, L)\, 10^{-6}$ weber

$\Phi = (4\, \pi/10)\ N\ I\ \mu\, A \,/\, L\, 10^{-6}$ weber

where I is in amps, A in m^2 and L in metres

If the magnetic circuit consists of different materials, eg a ring with an air gap or the field circuit of a motor, then the total mmf to produce the flux is the sum of the mmfs to produce the flux in each part.

$F = F_1 + F_2 + F_3 +$ etc

where $F_1 = \Phi L_1 / (\mu_1 A_1)\, 10^6$ etc

$\Phi = (4\, \pi/10)\, NI \cdot 10^{-6} / \Sigma(L_1/\mu_1 A_1)$

where Φ is in weber, I in amps, A in m^2 and L in metres

Example

An iron ring, mean diameter 20 cms, with an air gap of 5 mm is wound with 680 turns. It takes 5 amps to give a flux density of 0.8 tesla. Find μ for the iron.

Length of iron = $\pi \cdot 20 / 100 = 0.628$ metres

NI = ampere turns = $5 \cdot 680 = 3400$ ampere turns

The cross section area of iron A_1 = cross section of air gap $A_2 = A$

$B = \Phi / A = (4\pi/10)\, N I \cdot 10^{-6} / [L_1/\mu_1 + L_2]$ tesla

$0.8 = (4\pi/10) \cdot 3400 \cdot 10^{-6} / [0.628/\mu + 5/1000]$

$0.628/\mu + 0.005 = 1.26 \cdot 3400 \cdot 10^{-6} / 0.8 = 5.355 \cdot 10^{-3}$

$1/\mu = 0.355 \cdot 10^{-3} / 0.628$

$\mu = 1770$

What current is needed to give the same flux in a ring with the same number of turns and same air gap but twice the diameter.

$B = (4\pi/10)\ \ N I \cdot 10^{-6} / [L_1/\mu_1 + L_2]$

$0.8 = (4\pi/10)\, 680 \cdot I \cdot 10^{-6} / [2 \cdot 0.628/1770 + 0.005]$

$I = 5.33$ amps

Example
Part of the B – H curve for a ring of iron is;

AmpTurns/cm	5.4	1.3	– 0.4	– 1.0	– 1.4	– 1.6
Tesla	1.1	1.0	0.8	0.6	0	– 0.3

The mean diameter of the ring is 15 cms and it is in two parts separated by 0.2mm
The iron is magnetized by a uniformly distributed coil to a maximum flux density of 1.1 tesla.
What are the ampere turns?

The ampere turns for the iron $= 5.4 \cdot \pi \cdot 15 = 254$
The ampere turns for the air gaps are given by
1.26 (ampere turns) $\cdot 10^{-6} /(2 \cdot 0.2/1000)$ tesla $= B = 1.1$
ampere turns = 349
Total ampere turns = 254 + 349 = 603

Without altering the current, the ring is separated by a further 0.4 mm. Find B
Ampere turns for the air gaps
$B = 1.26 \cdot$ (ampere turns) $\cdot 10^{-6} /(2 \cdot 0.6/1000)$ tesla
ampere turns $= B \cdot (2 \cdot 0.6/1000) \cdot 10^{6} /1.26 = 955\ B$

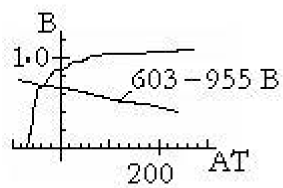


Figure 25; Flux Density

Plot the *B-H* curve in tesla against ampere turns
And plot (total *AT* – *AT* for air gaps)
ie $AT = 603 - 955\ B$
The plots cross when *AT* for iron + *AT* for air gaps = 603

This occurs at a negative *AT* of 44 and a flux density of 0.621 tesla
The mmf for the air gap is provided partly by the residual magnetism and partly by the ampere turns.

INDUCED EMF

Potential Difference (pd)
The potential difference between two points is one volt if one watt of power is produced when one amp flows from one point to the other. The symbol for pd is ***V*** and the Engineering unit is the **volt**. One joule of work is done when one coulomb of electricity flows through a pd of one volt.

$W = V \cdot I$ where W is in watts, V is in volts and I is in amps
joule is 10^7 ergs and amp is 10 em unit units of current
therefore the volt is joules / sec / amp = 10^8 emu of voltage

Electro Motive Force (emf)
An emf is generated when the magnetic flux linking with a coil is changed. It is measured in volts. The generated emf is one volt when one amp generates one watt of power.

Faraday's Law
Consider two parallel conductors L metres apart with a third lying across them which carries a current of I amps. Apply a uniform magnetic field B tesla perpendicular to the plane of the conductors.
Move the top conductor at a constant velocity v metres/sec against the force on the conductor.

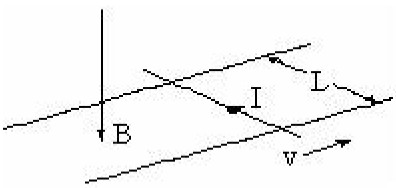


Figure 26; Induced EMF

The Force on the conductor is $B \cdot I \cdot L$ newtons.
Mechanical power supplied to move the conductor at velocity v is $B \cdot I \cdot L \cdot v$ watts
This power is used to generate an emf E in the conductor generating power at the rate $E \cdot I$ watts.

Hence $E = B \cdot L \cdot v$
Where E is in volts, B is in tesla , L in metres and v in metres/sec

But $B \cdot L \cdot v$ = rate at which the top conductor cuts the magnetic flux = $d\Phi / dt$.

Lenz's law states that the generated emf E opposes the change.
Therefore the polarity of E is usually chosen so that it is negative when the flux linked with the circuit is increasing.

If there are N turns of a coil linking the flux, then
$E = -N \cdot d\Phi / dt$ where E is in volts and $d\Phi/dt$ is in Wb/sec
This is the fundamental equation connecting emf and magnetic flux.

Experiment shows that if a conductor is Moved in the direction of the right hand thuMb in a Field in the direction of the right hand First finger then an emf will be generated causing a Current to flow in the direction of the seCond finger.

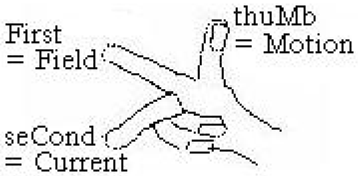


Figure 27; Right Hand Rule

This is the right hand rule, the "gener-right-or" rule.

Resistance (Ohm's law)
At constant temperature, the current in a wire is proportional to the potential difference between the ends.
The ratio Volts / Amps is called the resistance in Ohms. The symbol for Ohms is **Ω.**
$R = V/I$ where R is in ohms, V is in volts and I is in amps
Legal Ohm is the resistance of a column of mercury 106.3 cms long and 1 sq cm cross section at 0^0 C

Resistance is proportional to the length of the wire and inversely proportional to the cross section.

$R = (\rho L / A)$ where R is in micro Ω, ρ is the resistivity in micro Ω per metre cube, L is the length in metres and A is the cross sectional area of the wire in metres^2.

Power loss in a resistor

Power loss $W = V \cdot I = I^2 \cdot R = V^2/R$

Resistance Temperature Coefficient of conductors

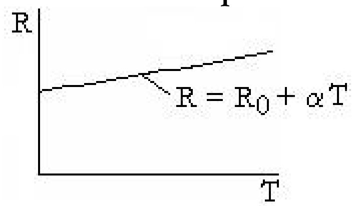


Figure 28; Resistance - Temperature

Resistance increases with temperature, the increase being approximately linear.

$R = R_0 (1 + \alpha T)$ where R_0 is the resistance at 0^0 Celsius, T is the temperature in Celsius and α is the temperature coefficient

Typical values of ρ and α at 15^0C

Copper	ρ = 1.7 micro Ω per cm cube	α = 0.004
Aluminium	ρ = 2.9 micro Ω per cm cube	α = 0.004
Silver	ρ = 1.6 micro Ω per cm cube	α = 0.004
Iron	ρ = 10 to 160 micro Ω per cm cube	α = 0.002 to 0.006

Absolute zero temperature is – 273 ^{0}C. At a temperature near absolute zero, R for some materials becomes zero. The material is said to be supercooled and can carry a huge current with no energy loss. This property is used in some large electro-magnets.

Temperature coefficient of insulating materials.

Increase in temperature reduces the insulation resistance and the effect is logarithmic. An increase in temperature of 65^0 C reduces the insulation resistance by a factor of 10. The insulation resistance is also dependent on how dry it is. Records of insulation resistance should give the temperature at which the measurement was taken.

Self Inductance

The current in a coil causes a magnetic field that links with the coil. Therefore any change in the current will induce an emf in the coil. The coil is said to have self inductance. The symbol for self inductance is **L** and the Engineering unit is the **henry**. A coil is said to have an inductance of one henry if a rate of change in current of one amp per second induces an emf of one volt.

$E = -L \cdot dI/dt$ where E is in volts, L is in henries, I is in amps and t is in seconds. The minus sign signifies that the emf opposes the change.

Self Inductance of a coil on a ring

A coil is wound with N turns on a ring D metres mean diameter and cross sectional area A square metres and permeability μ.

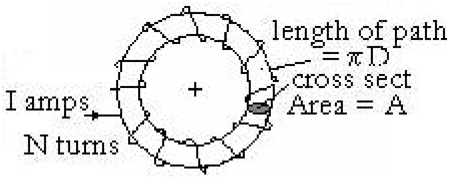


Figure 29; Self Inductance of a coil on a ring

Let the current be I amps
Magnetising force, $H = 4\,\pi\ \ N I\ \ /(\pi\ D) \cdot 10^{-7}$
$\Phi = \mu\ H\ A$ Weber
$\Phi = 4\ \pi\ \mu\ N I A\ /(\pi\ D) \cdot 10^{-7}$ Wb.
emf due to change in I is given by;
$E = -N\, \mathrm{d}\Phi/\mathrm{d}t = -N\, 4\, \pi\, \mu\, N\, A\ /(\pi\ D) \cdot 10^{-7}\ \mathrm{d}I/\mathrm{d}t$ volts
$= -(4\,N^2 \mu\, A\ /D) \cdot 10^{-7}\ \mathrm{d}I/\mathrm{d}t$ volts
But $E = -L\ \mathrm{d}I/\mathrm{d}t$
Therefore $L = (4\,N^2 \mu\, A\ /\ D) \cdot 10^{-7}$ henries
For a magnetic circuit length S metres
$L = 1.26\ N^2 \mu\, A\ /\ S \cdot 10^{-6}$ henries

Example
A coil of 500 turns is wound on a wooden ring 20 cms diameter
The cross section of the ring is 4 cms diameter. Estimate the self inductance

$L = (4\pi/10) \cdot 500^2 \cdot [\pi \cdot (2/100)^2\,] \cdot 10^{-6}\ /\ (\pi \cdot 0.2) = 2\pi \cdot 10^{-4}$ henries

Self Inductance per metre length of two parallel conductors spaced d metres apart
Conductors A and B form a coil of one turn in air
Each carries a current I amps

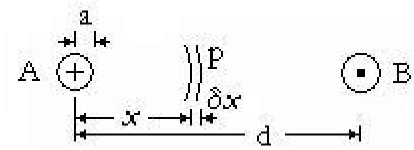


Figure 30; Self Inductance of parallel lines

Field t P due to conductor A = $(\,2\,I\ /\ x\,)\ 10^{-7}$ tesla
Flux at P due to conductor A per metre length through element δx =$(\,2\,I\ /\ x\,)\ 10^{-7}\ \delta x$ Weber
Total flux linkage per metre length = $(2\,I\,)\ 10^{-7}\ \int (1/x)\ \mathrm{d}x$ from a to d
= $[2\,I\ \ \ln(d/\mathrm{a})\,]\ 10^{-7}$

But there is also flux linkage within conductor A where conductor diameter is a
Flux linkage in element within conductor A per metre length =$(\,2\,I\,x\ /\ a^2\,)\ 10^{-7}\ \ \delta x$ Weber
But this links with only $(\ x^2\ \ /\ \ a^2\ )$ of the conductor
Total flux linkage within conductor A per metre length =$(\,2\,I\,x^3\ /\ a^4\,)\ 10^{-7}\ \ \delta x$ Weber
Total flux linkage within the conductor per metre length = $(2\,I\ /\ a^4\,)\ 10^{-7}\ \int x^3\ \mathrm{d}x$ from 0 to a
$= (2\,I\ /\ a^4\,)\ 10^{-7}\ \ (a^4\ /\ 4)\ = (\,I\ /\ 2\,)\ 10^{-7}$ Weber
Hence the total linkage per metre length due to I amps in A = $[2\,I \ln(d/a) + I\ /\ 2\,]\ 10^{-7}$
Total linkage per metre length due to I amps in conductors A and B = $I\,[1 + 4 \ln(d/a)\,]\ 10^{-7}$

Self Inductance per metre length = $[1 + 4 \ln(d/a)\,]\ 10^{-7}$ Henries

Energy stored in an inductance

An inductance L henries carries a current I amps.

Let the inductance be disconnected from the supply but allowed to discharge through a resistor.

The power supplied by the inductance;

$$w = e\,i \text{ watts}$$

where w, e and i are the values of power, emf and current at any instant during the discharge.

$$e = -L\,\mathrm{d}i/\mathrm{d}t$$

$$w = -L\,i\,\mathrm{d}i/\mathrm{d}t$$

Energy released during the discharge $= \int w\,\mathrm{d}t = \int -L\,i\,\mathrm{d}i/\mathrm{d}t\;\;\mathrm{d}t$

$$= \int -L\,i\,\mathrm{d}i \text{ from } i = I \text{ to } i = 0$$

$$= -(1/2)\,L\,i^2 \text{ from } i = I \text{ to } i = 0$$

$$= (1/2)\,L\,I^2 \text{ joules}$$

Discharge Resistor

When the current in an inductor is suddenly switched off, $\mathrm{d}i/\mathrm{d}t$ has a very high negative value. In practice this means that attempting to switch the current off results in severe arcing at the switch contacts.

Thus the switch for a large inductor is usually double pole which connects the inductor to a discharge resistor before the connection to the supply is broken.

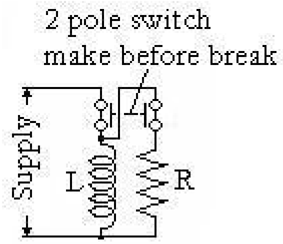


Figure 31; Discharge Resistor

Inductance discharged through a resistance

When an inductance L henries is discharged through a resistance R ohms, the current decays exponentially with a time constant T

Let I_0 be the current at $t = 0$

$E = -L\,\mathrm{d}i/\mathrm{d}t$ and $E = iR$

$(L/i)\mathrm{d}i = R\,\mathrm{d}t$

$\int(1/i)\,\mathrm{d}i$ from I_0 to $i = -(R/L)\int \mathrm{d}t$ from 0 to t

$\ln(i/I_0) = -(R/L)t$

$i = I_0\,e^{-(R/L)\,t}$

But $i = I_0\,e^{-(t/T)}$ for exponential decay with time constant T

Hence the current decays exponentially with a time constant $T = L/R$

Inductance charged through a resistance from constant volt supply

When $t = 0$, the current is zero

At time t, $V = L\,\mathrm{d}i/\mathrm{d}t + iR$

Multiply by integrating factor $e^{R/L\,t}$

$V e^{R/L\,t} = L e^{R/L\,t}\,\mathrm{d}i/\mathrm{d}t + R\,i\,e^{R/L\,t}$

$= L\,\mathrm{d}/\mathrm{d}t\,[\,i\,e^{R/L\,t}\,]$

Integrating

$(L/R)\,V e^{R/L\,t} = L[\,i\,e^{R/L\,t}\,] + \text{Const}$

When $t = 0$, $i = 0$ therefore Constant $= (L/R)V$

$i = V/R - V/R\,e^{-R/L\,t} = (V/R)\,(1 - e^{-R/L\,t})$

The current rises exponentially with a time constant $T = L/R$ towards $I = V/R$

Power to inductance charged through a resistance

Power to inductance $W = v\,i = i\,L\mathrm{d}i/\mathrm{d}t$

$W = [(V/R)\,(1 - e^{-R/L\,t})]\,[\,L\,(V/R)\,(R/L)\,e^{-R/L\,t}]$

$W = (V^2/R)\,(e^{-R/L\,t})\,(1 - e^{-R/L\,t})$

Put $(e^{-R/L\,t}) = x$

$W = (V^2/R)\,(x - x^2)$

$\mathrm{d}W/\mathrm{d}x = (V^2/R)\,(1 - 2x)$

$\mathrm{d}W/\mathrm{d}x = 0$ when $x = ½$

$\mathrm{d}_2W/\mathrm{d}x^2 = (V^2/R)\,(-2)$ which is negative

Therefore W is a maximum when $x = ½$

$W_{MAX} = (V^2/R)\,[\,½ - (\,½)^2]$

$= V^2/(4R)$ and is independent of L

Work done in taking a unit pole round a closed path through a coil of *N* turns carrying a current of *I* amps

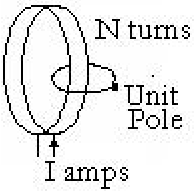


Figure 32; Unit pole moved round a coil

The total flux leaving the pole links with the coil.
Thus the total flux linkage is $4\pi N \cdot 10^{-8}$ weber
Thus the emf generated by moving the pole is;

$$E = 4\pi N \cdot 10^{-8} / t \text{ where } t \text{ is the time taken.}$$

Power generated = $E\,I$ watts = $4\pi N\,I\,10^{-8} / t$ for t secs
Work done = $4\pi N\,I\,10^{-8}$ joules

Change in Flux and Quantity

A coil with N turns is connected to a circuit with a total resistance R ohms.

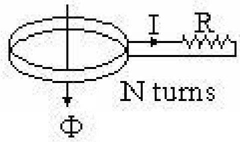


Figure 33; Flux and Coulomb

The Flux through the coil is Φ

$$-N\,d\Phi/dt = I\,R = dq/dt\,R$$

Integrate wrt t from 1 to 2

$N\,(\Phi_1 - \Phi_2) = R\,(q_2 - q_1)$ where Φ is in weber, q is in coulombs and R is in ohms

ELECTROSTATICS

Electrostatic units (esu)

If an insulating particle becomes charged, it sticks to another insulated body. Particles of polystyrene stick to the window. A nylon shirt pulled over your hair charges up the nylon and it can discharge with visible sparks. This introduces a new set of units, electrostatic units or esu. A unit electrostatic charge placed one cm away from a similar unit charge in air experiences a force of one dyne. The work done in taking an esu unit of charge through an esu unit of voltage is one erg. Many textbooks state that this set of esu units is related to emu units by a factor equal to the velocity of light = $2.998 \cdot 10^{10}$ cm / sec

Force on electrostatic charge

A charge q_1 esu distant r cms from another charge q_2 esu in dielectric constant k experiences a force

$$F = q_1\, q_2 \,/\, (k\, r^2) \text{ dynes}$$

The emu of electric charge is $2.998 \cdot 10^{10}$ times the esu of charge
Amp = 1/10 emu, Coulomb = amp second = 1/10 emu, metre =100 cms
newton = 10^5 dynes
$F = Q_1\, Q_2 \,/\, k\, m^2 \cdot\ [2.998 \cdot 10^{10} \cdot 10^{-1}]^2 \cdot\ [10^{-2}]^2 \cdot\ 10^{-5}$ newtons

$F = Q_1\, Q_2 \,/\, k\, m^2 \cdot 8.988 \cdot 10^9$ newton
where Q_1 and Q_2 are in Coulombs, k is the dielectric constant (air = 1) and m is in metres

Electrostatic field

The electrostatic field at any point is the force in dynes exerted on a unit charge.

$f = q \,/\, (k\, r^2)$ where f is the electrostatic field in esu

Electrostatic flux

Electrostatic field at the perimeter of a sphere radius r cm in air (k = 1) with a charge q at the centre
Electrostatic field $f = q \,/\, (1 \cdot r^2) = q \,/\, r^2$
Electrostatic flux = electrostatic field times area
Surface area of sphere radius r cm = $4\pi\, r^2$
Therefore the total electrostatic flux = $q\, 4\pi$

Gauss' theorem.

The total electrostatic flux leaving a charge q is $4\pi q$

Electrostatic field inside a conducting sphere

Let the density of charge on the surface of a hollow conducting sphere be σ per unit area

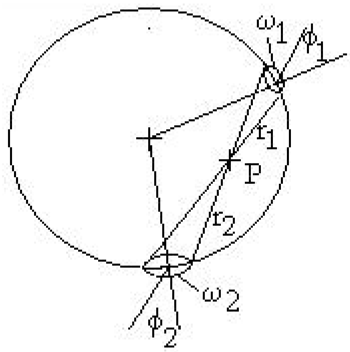


Figure 34; Electrostatic Field inside a hollow body

.

Point P experiences a force due to area ω_1 of

$F = \sigma\,\omega_1 \text{ Cos } (\phi_1\,) \,/(\,k\, r_1^{\,2}\,) \;-\; \sigma\,\omega_2 \text{ Cos } (\phi_2\,) \,/(k\, r_2^{\,2}\,)$

ϕ_1 and ϕ_2 are equal solid angles

Therefore $\omega_1 \text{ Cos } (\phi_1\,) \,/\, r_1^{\,2} = \omega_2 \text{ Cos } (\phi_2\,) \,/\, r_2^{\,2}$

The charge σ on the surface of the sphere exerts no Force on a charge at P

Hence there is no electrostatic field inside the sphere.

Electrostatic field external to the sphere
Force on a unit charge external to the sphere = $q / k r^2$
where r is the distance of the unit charge from the centre of the sphere

In any set of units, the voltage difference between two points is the work done in taking a unit charge from one point to the other. The electrostatic unit of voltage on the sphere is the work done in ergs when an electrostatic unit of charge is brought from infinity to the surface of the sphere.

$V = (q / k) \int [r^{-2}]$ dr from infinity to the radius of the sphere a cm.

$= q / k a$

Therefore $q = k a V$ where all quantities are in esu

Capacitor
A capacitor stores a quantity of electricity q.
The storage is proportional to the voltage and capacitance.
hence $q = CV$ where C is the symbol for capacitance.
The SI unit is the farad = coulombs/volt but this is too large for practical use. The Engineering unit is the microfarad (μF) = 10^{-6} farad.

$q = C \cdot V$ where C, q and V are in the same units

$q = C V \cdot 10^{-6}$ where C is in μF, q is in coulombs and V is in volts

The quantity of electricity stored by an isolated sphere $q = k a V$
Therefore the capacitance of the sphere $C = k a$ in electrostatic units

Conversion of esu unit of capacitance to the Engineering unit microfarad (μF)
1 esu of quantity = $1 / (3 \cdot 10^{10})$ emu of quantity = $1/(3 \cdot 10^9)$ coulomb

where $3 \cdot 10^{10}$ cm/sec is the speed of light (actually $2.998 \cdot 10^{10}$)

The unit for voltage is the energy required to move a unit charge through a potential difference of one. Therefore one esu of pd = $3 \cdot 10^{10}$ emu of pd = $3 \cdot 10^{10} \cdot 10^{-8}$ volts = 300 volts

thus 1 esu of capacitance = (1 esu of quantity) / (1 esu of pd)

= $[1/(3 \cdot 10^9)$ coulomb$] / [3 \cdot 10^2$ volts$]$

= $[1/(9 \cdot 10^{11})]$ coulombs / volt

= $[1.11 \cdot 10^{-12}]$ coulombs / volt or farads

= $[1.11 \cdot 10^{-12} \cdot 10^6]$ μF

= $[1.11 \cdot 10^{-6}]$ μF

hence (value of C in μF) = $1.11 \cdot 10^{-6}$ · (value of C in es units)

Isolated sphere

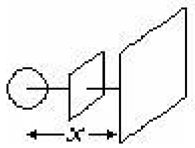


Figure 35; Capacitance of an isolated sphere

$C = k a$ in es units
$C = 1.11 \cdot 10^{-6} \cdot k a$ μF where a is in cm and k is the dielectric constant (air = 1)
Voltage gradient = $V a / x^2$ Volts / cm
Where V is the potential of the sphere in Volts
a cm is the radius of the sphere and x cm is the distance from the sphere centre

Concentric spheres
Smaller sphere radius a cm, larger sphere radius b cm
Apply a charge q to sphere radius a and a charge $-q$ to sphere radius b.
Voltage on inner sphere $V = q / (k\, a)$ due to its own charge
Voltage inside the outer sphere $V = -q / (k\, b)$
Resultant potential of inner sphere $V = q / (k\, a) - q / (k\, b) = (q / k)\,[\,(b - a) / (ab)\,]$
$C = q / V = k\, a\, b / (b - a)$
$C = 1.11 \cdot 10^{-6} \cdot k\, a\, b / (b - a)$ microfarads

Parallel plate capacitor

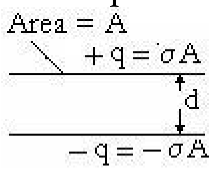


Figure 36; Capacitance of parallel plates

Let area of plate = A sq cm and charge on the plate $q = \sigma A$
Gauss' theorem, Electrostatic flux = $4\pi q = 4\pi \sigma A$
Electrostatic field $f = 4\pi \sigma / k$
V equals the work done in taking the charge from one plate to the other
$V = \int f\, dx = 4\pi \sigma\, d / k$ where d is the distance between the plates in cms
The capacitance $C = q / V = A\sigma / V = A\, k / (4\pi\, d)$ in es units
$C = 1.11 \cdot 10^{-6} \cdot A\, k / (4\pi\, d)$ microfarads
Voltage gradient is constant = V/d

Parallel plate capacitor with two insulations

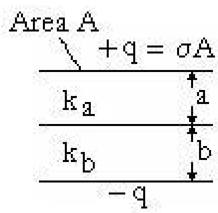


Figure 37; Capacitance of parallel plates with different insulations

Electrostatic charge $q = \sigma A$
Electrostatic flux = $4\pi q = 4\pi \sigma A$
Electrostatic field $f = 4\pi \sigma A / (kA) = 4\pi \sigma / k$

$f_a = 4\pi \sigma / k_a$
$f_b = 4\pi \sigma / k_b$

$V = 4\pi \sigma\, a / k_a + 4\pi \sigma\, b / k_b$
Capacitance per unit area = $\sigma / V = 1/[4\pi\,(a / k_a + b / k_b)\,]$ es units
$C = 1.11 \cdot 10^{-6} / [\,4\pi\,(a / k_a + b / k_b)\,]$ µF per square cm

Co-axial cylinders radius *a* and *b*, length *L*

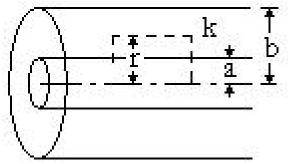


Figure 38; Capacitance of co-axial cylinders

Let charge per unit length $= \sigma$

Gauss' theorem, electrostatic flux $= 4\pi\, \sigma\, L$

At radius r the electrostatic field $= 4\pi\, \sigma\, L\, /\, 2\pi\, r\, L\, k = 2\, \sigma\, /\, (\, r\, k)$

$f = 2\, \sigma\, /\, (r\, k)$

$V = \int f\, \mathrm{d}r$ from a to b $= (2\, \sigma\, /\, k)\, \ln\, (b\, /\, a)$

Capacitance $C = \sigma\, /\, V = k\, /\, [2\, \ln\, (b\, /\, a)\,]$ in es units

$C = 1.11 \cdot 10^{-6} \cdot k\, /\, [2\, \ln\, (b\, /\, a)\,]$ µF per cm

Voltage gradient $= 2\, \sigma\, /\, (r\, k) = V\, /\, [\, r \ln\, (b/a)\,]$ Volts / cm at r cm from the axis

Parallel conductors

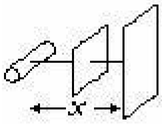


Figure 39; Capacitance of parallel conductors

The field at x from a sphere is $f = q / (k\,x^2)$ since the area covered by each line of flux is proportional to x^2

The field at x from a long cylinder is $f = q / (k\,L\,x)$ since the area covered by each line of flux is proportional to x not x^2

This is true if the cylinder is isolated and half the electrostatic flux leaves on the opposite side of the conductor.

The presence of a second conductor with opposite charge means that all the flux leaves on the same side in effect doubling the flux and therefore $f = 2\,q / (k\,L\,x)$

Conductors A and B are each radius r with distance d between centres

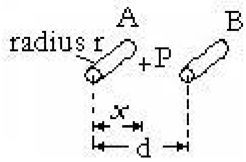


Figure 40; Capacitance of parallel conductors

Let charge per unit length on A = σ. Therefore $q = L\,\sigma$

And charge per unit length on B = $-\sigma$

Electrostatic field at P in due to A $= 2\,\sigma\,L / x\,L\,k$

Electrostatic field at P in due to B $= 2\,\sigma\,L / [\,L\,(d - x)]$ in the same direction

Therefore $V = \int 2\,\sigma\,L\,[\,1/x + 1/(d - x)\,]\,dx$ from r to $(d - r)$

$= 2\,\sigma\,L\,[\,\ln(x) - \ln(d - x)\,]$ from r to $(d - r)$

$= 4\,\sigma\,L\,[\,\ln\{(d - r) / r\}]$

Capacitance $C = \sigma / V = k / [4\ \ln\{(d - r)/r\}\,]$ in es units per unit length

$C = 1.11 \cdot 10^{-6} \cdot k / [4 \ln\{(d - r) / r\,]$ µF per cm

Voltage gradient at distance x cm from one of the conductors

$= V\,d / [x\,(d - x)\,\ln\{(d - r) / r\}\,]$ volts/cm

Where V is difference in potential of the conductors in Volts, d cm is the distance between the conductor centres and r cm is the radius of each conductor

Example 1

Calculate the capacitance per kilometer of a concentric lead covered cable where a = 2.5 mm and insulation is 2 mm thick and k = 4. Therefore b = 4.5 mm.

Capacitance / cm = $1.11 \cdot 4 \cdot 10^{-6} / [2 \ln (4.5 / 2.5)\,]$ µF

Capacitance / kilometer = $1.11 \cdot 4 \cdot 10^{-6} / [2 \ln (4.5 / 2.5)\,] \cdot 100 \cdot 1000 = 0.38$ µF

Example 2

Calculate the capacitance if the outer 1 mm of the insulation has k = 2

$\varphi = 2\,\sigma / r$ throughout the cable.

For inner 1 mm, $F_1 = \varphi / 4 = 2\,\sigma / (4\,r)$

For outer 1 mm, $F_2 = \varphi / 2 = 2\,\sigma / (2\,r)$

$V = \int F_1 \, dr$ from .25 to .35 + $\int F_2 \, dr$ from .35 to .45
$= 2\,\sigma\,[(1/4)\ln(.35/.25) + (1/2)\ln(.45/.35)]$
Hence $\sigma / V = 2.37$ esu / cm $= 2.37 \cdot 10^5$ esu / km
Capacitance $= 1.1 \cdot 10^{-6} \cdot 2.37 \cdot 10^5$ µF per km $= 0.26$ µF per km

Energy stored in a capacitor
Work done in increasing the charge by $\delta q = V\,\delta q$
But $q = C\,V$
$\delta q = C\,\delta V$
Work done $= CV\,\delta V$
Energy stored $\int CV\,dV$ from $V = 0$ to $V = \mathrm{V}$
Energy stored $= (1/2)\,C\,\mathrm{V}^2$ where energy is in joules, C is in farads and V is in volts

Capacitor discharged through a resistance
At time t, $q = Cv$ and $v = i\mathrm{R}$ and $i = dq/dt$
$q = CR\,dq/dt$
Integrate from q_0 to q, $CR\ln(q/q_0) = -t$
$q = CR\,i$, therefore $CR\ln(i/i_0) = -t$ hence $i = i_0\,e^{-(1/CR)\,t}$
The current decays exponentially with a time constant $T = CR$

Capacitor charged through a resistance from a supply at constant voltage *V*
When $t = 0$, $q = 0$
At time t, $q = Cv$ and $V = i\mathrm{R} + q/C$ and $i = dq/dt$
$q/CR + dq/dt = V/\mathrm{R}$
Multiply by the integrating factor $e^{(1/CR)\,t}$
$d/dt[\,q\,e^{(1/CR)\,t}\,] = (V/\mathrm{R})\,e^{(1/CR)\,t}$
Integrating
$q\,e^{(1/CR)\,t} = (V/\mathrm{R})\,(C\mathrm{R})\,e^{(1/CR)\,t}$ + constant $= (CV)\,e^{(1/CR)\,t}$ + constant
When $t = 0$, $q = 0$ therefore constant $= -CV$
$q = CV\,[\,1 - e^{-(1/CR)\,t}\,]$
The charge rises exponentially with a time constant $T = CR$ towards $Q = CV$

DC CIRCUITS

Internal resistance of a cell

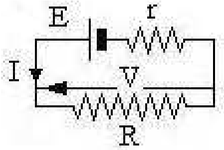


Figure 41; Internal resistance of a cell

The current I in a circuit of resistance R connected to a cell of voltage E and internal resistance r

$$I = E/(R + r)$$

If V is the voltage across R
then $V = IR$
hence $r = (E - V)/I$
E is the voltage on open circuit and can be measured with a potentiometer
hence r can be found.

Alternatively;
If I_1 is the current with Resistance R_1 and I_2 is the current with Resistance R_2
Then $I_1 (R_1 + r) = E = I_2 (R_2 + r)$
Thus $r = (I_2 R_2 – I_1 R_1) / (I_1 – I_2)$

Resistances in series

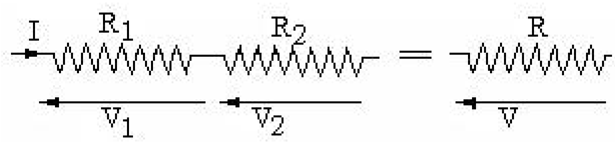


Figure 42; Resistances in series

$V_1 = I R_1$ $V_2 = I R_2$
$V = I R$ $V = V_1 + V_2$
$I R = I R_1 + I R_2$
Hence $R = R_1 + R_2$

Resistances in parallel

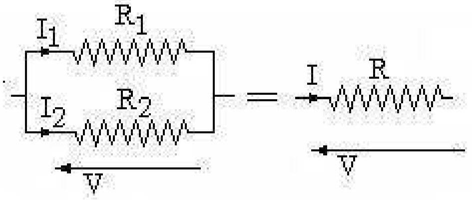


Figure 43; Resistances in parallel

$V = I_1 R_1$ $V = I_2 R_2$ $V = I R$
$I_1 = V / R_1$ $I_2 = V / R_2$ $I_1 + I_2 = I = V / R$
$V / R = V / R_1 + V / R_2$
$R = 1 / [(1 / R_1) + (1 / R_2)]$

Kirchoff's first law

The total current leaving any portion of a network is equal to the total current entering that portion.

Kirchoff's second law

The algebraic sum of all the "IR" drops around any circuit is equal to the total emf applied in that circuit.

Example 1
In the diagram find I_2

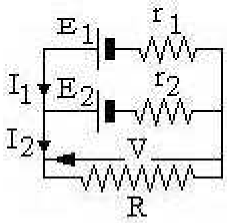


Figure 44; Kirchoff's laws

$E_1 - I_1 r_1 = V = I_2$ R
$E_2 - (I_2 - I_1)\ r_2 = V = I_2$ R

Hence $I_1 = (E_1 - I_2\ \mathrm{R})\ /\ r_1$
$E_2 + I_1\ r_2 = I_2\ (\mathrm{R} + r_2)$
$E_2 + (E_1 - I_2\ \mathrm{R})\ r_2\ /r_1 = I_2\ (\mathrm{R} + r_2)$
$E_1\ r_2 + E_2\ r_1 = I_2\ (\mathrm{R}\ r_1 + \mathrm{R}\ r_2 + r_1 r_2)$
$I_2 = (E_1\ r_2 + E_2\ r_1)/(\mathrm{R}\ r_1 + \mathrm{R}\ r_2 + r_1 r_2)$

Example 2
Each edge of a tetrahedron is resistance R
Find the resistance between adjacent corners.

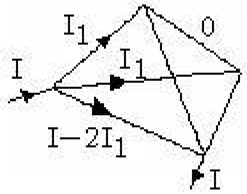


Figure 45; Resistance of wires in the shape of a tetrahedron

Let the voltage between adjacent corners be V
By symmetry, four bars carry I_1 , one bar carries zero and
the sixth carries $I - 2I_1$
$V = I_1 R + I_1 R$
therefore $I_1 = V\ /\ (2R)$
$V = (I - 2\ I_1)R = IR - V$
$2V = IR$
Resistance between adjacent corners = $V/I = \mathrm{R}/2$

Thevenin's Theorem
To find the current in a resistance r, a branch of a network, remove the branch and find the voltage E across the ends of the branch. Short circuit all sources of emf and find the resistance R of the network between the ends of the branch with the branch removed. The current in the branch is $E\ /\ (\mathrm{R} + r)$.

Example
Find the current in branch QS
All values are in ohms.

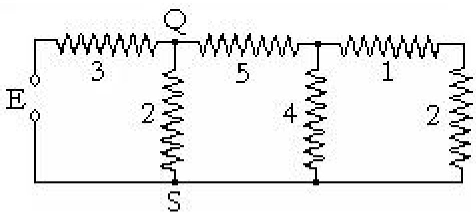


Figure 46; Example of a circuit

4 ohm resistor in parallel with (1 + 2) ohms = 1/(1/4 + 1/3) = 12/7 ohms
Remove QS and
voltage between Q and S = E (5 + 12/7) / (8 + 12/7) = 0.691 E
Resistance between Q and S = 1/[1/3 +1/(5 + 12/7)] = 2.074
Current in QS = 0.691 E / (2.074 + 2) = 0.170 E

Wheatstone Bridge

A Wheatstone Bridge consists of four resistances connected as shown.
D is a galvanometer to detect any current.

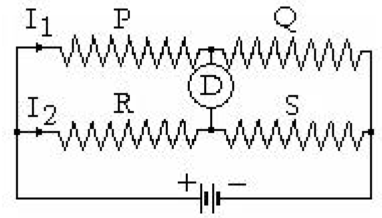


Figure 47; Wheatstone Bridge

If the galvanometer cannot detect any current, then

$I_1\, P = I_2\, R$

$I_1 Q = I_2\, S$

$P/Q = R/S$.

For example, P, Q and R are resistances that can be switched to any value within a range. An unknown resistance S is connected and P, Q and R adjusted till the galvanometer shows the bridge to be balanced. Hence the value of S can be found.

Potentiometer

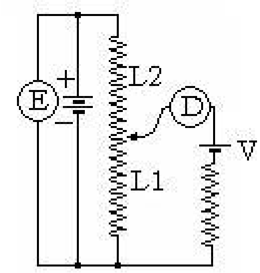


Figure 48; Potentiometer

A voltage V connected internally through a high resistance cannot be measured by a meter. As soon as the meter takes current, the voltage drops.

A potentiometer allows the voltage to be measured without taking any current. A dc supply and voltmeter E are connected across a uniformly wound rheostat
The slider is connected to the unknown voltage through a galvanometer D.
The slider is adjusted till the galvanometer shows no current.

$V = E\, L_1 / (L_1 + L_2)$
As no current is flowing from V, there is no volt drop in its resistance.

The output from a thermocouple which measures temperature is usually connected to a potentiometer. The voltage is small (in the order of millivolts) and any resistance drop would introduce a significant error.

ALTERNATING CURRENT (AC)

Generating an AC voltage

The coil, area A, is rotated in a uniform magnetic field B at constant angular velocity ω

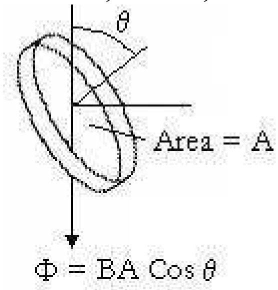


Figure 49; AC voltage

The flux linking the coil is $\Phi = BA \text{ Cos } \theta$
Let the maximum value of Φ be Φmax
$\Phi max = BA$ and $\Phi = \Phi max \text{ Cos } \theta$
Let the coil be rotated at constant speed ω radians/sec. Therefore $\theta = \omega t$
emf $E = -N\, d\Phi/dt = -N\, d(\Phi max \text{ Cos } \omega t)/dt$
$= N\omega\, \Phi max \text{ Sin } \omega t$
$= Emax \text{ Sin } \omega t$
where $Emax = N\omega\, \Phi max$ and is the maximum value of E

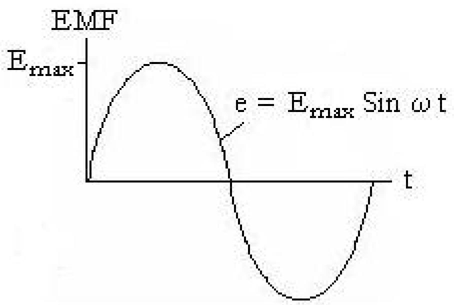


Figure 50; Alternating voltage

When this emf is applied to a circuit, the current flows in one direction for half a cycle then flows in the reverse direction for the next half cycle. This is called Alternating Current. Throughout the world, mains electricity is usually AC.

Average value

The average value for a full cycle is zero
Let the average value for half a cycle be Eav
Integrate over a half cycle
$Eav\ \pi = \int Emax \text{ Sin } \theta \ d\theta$ from 0 to π = $Emax\,[-\text{Cos } \theta]$ from 0 to π = $Emax\,[1 + 1]$
$Eav = 2\, Emax/\pi = 0.636\, Emax$

Root Mean Square value (rms value)

Integrate the value of E^2 over a full cycle to obtain the average value of E^2
2π (average value of E^2) = $\int Emax^2 \text{ Sin}^2 \theta \ d\theta$ from 0 to 2π
$= \frac{1}{2} Emax^2 \int (1 - \text{Cos } 2\theta)\, d\theta$ from 0 to 2π

$$= \tfrac{1}{2}\, Emax^2\, [\theta - \text{Sin } 2\theta] \text{ from } 0 \text{ to } 2\pi$$
$$= \tfrac{1}{2}\, Emax^2\, (2\pi - 0 - 0 + 0) \;=\; \pi\, Emax^2$$

Therefore average value of $E^2 = \tfrac{1}{2}\, Emax^2$

Take the square root to obtain the root mean square or rms value

$$Erms = Emax / \sqrt{2} = 0.707\, Emax$$

The power of an electrical circuit is proportional to E^2 therefore use the *Erms* value for power.

Unless stated otherwise, the value given for an AC voltage or current is the rms value.

The power in a DC circuit is equivalent to the power in an AC circuit with the same rms values.

Peak factor and Form factor

Peak factor = (peak value) / (rms value)

Form factor = (rms value) / (av value for ½ cycle)

Sine wave Form factor $= (0.707) / (0.636\, Emax) = 1.11$

Triangular wave form $E = Emax\, \theta / (\pi/2)$ between 0 and $\pi/2$

$$Eav = 0.5\, Emax$$
$$Erms = Emax \sqrt{[\textstyle\int \{\theta^2/(\pi/2)^2\} d\theta / (\pi/2)]} \text{ with the integral from } 0 \text{ to } \pi/2$$
$$Erms = [Emax / (\pi/2)^{3/2}] \sqrt{[\textstyle\int \theta^2\, d\theta\,]}$$
$$Erms = [Emax / (\pi/2)^{3/2}] \sqrt{[\, \theta^3/3\,]} \text{ from } 0 \text{ to } \pi/2$$

$$Erms = [Emax/(\pi/2)^{3/2}]\ \sqrt{[(\pi/2)^3/3]} = Emax/\sqrt{3} = 0.577\ Emax$$

Triangular wave Form factor = 0.577 $Emax$ /0.5 $Emax$ = 1.15

Square wave $Eav = Emax$ and $Erms = Emax$
Square wave form factor = $Emax$ / $Emax$ = 1

Summarising

Square wave form factor = 1	peak factor = 1
Sine wave form factor = 1.11	peak factor = $\sqrt{2}$ = 1.41
Triangular wave form factor = 1.15	peak factor = $\sqrt{3}$ = 1.73

Thus the form factor and peak factor are a measure of how peaky the wave is.

Example
Find the average value for half cycle, the rms value and the form factor of the wave shown where $3T = \pi$

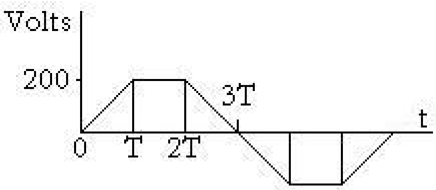


Figure 51; Wave form example

Average value for ½ cycle $Eav = 2 \cdot 200/3 = 133$ volts
$Erms = \sqrt{[\{2 \int 200^2 (t/T)^2 dt + 200^2\ T\}/\pi]}$ where the integral is from 0 to T
$= 200 \sqrt{[\{2t^3/(3T^2) + T\}/\pi]}$ where the integral is from 0 to T
$= 200 \sqrt{[\{2T^3/(3T^2) + T\}/\pi]} = 200 \sqrt{[(5T/3)/\pi]}$ where $T = \pi/3$
$= [200 \sqrt{(5\pi/9)/\pi}] = 200 \sqrt{[5/9]}$
$= 149$ volts
Form factor = $Erms/Eav$ = 149 / 133 = 1.12

Frequency
The number of complete cycles per second is the frequency in Hertz (Hz)
Mains electricity is at a frequency of 50 Hz in Europe and 60 Hz in America.
$E = Emax \operatorname{Sin} \omega t$ so the time taken for N cycles is given by $\omega T = 2\pi N$
Therefore $T = 2\pi N/\omega$ But the time taken for N cycles = N/f
Therefore $N f = 2\pi N/\omega$
Therefore $\omega = 2\pi f$
AC Voltage $E = Emax \operatorname{Sin} (2\pi f t)$

Vector representation of AC

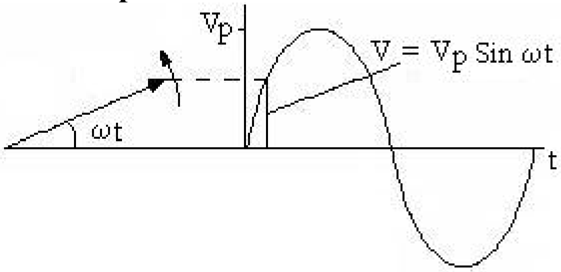


Figure 52; Vector representation of AC

Let the vector *Vp* rotate anti-clockwise at a speed of ω radians/sec.
The projection of *Vp* on a vertical is sinusoidal with respect to time.

Let another vector *Ip* at an angle of ϕ with *Vp* also rotate anti-clockwise at the same speed of ω radians/sec.
The projection of *Ip* on the vertical is sinusoidal displaced ϕ behind *Vp*.

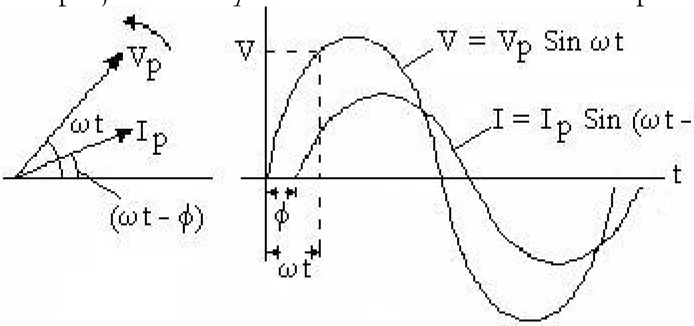


Figure 53; Vector representation of voltage and current
The vectors *Vp* and *Ip* represent the two sinusoidal quantities.
Thus alternating currents and voltages can be represented by vectors.

Addition of two AC voltages or two AC currents
Let *V1* and *V2* be two AC voltages at the same frequency but of different magnitude and phase angle.

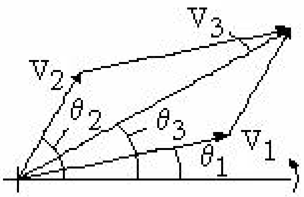


Figure 54; Addition of two AC currents or voltages
$V_1 = Vp_1 \text{ Sin } (\omega t + \theta_1)$ and $V_2 = Vp_2 \text{ Sin } (\omega t + \theta_2)$
$V_1+V_2 = Vp_1[\text{Sin } \omega t \text{ Cos}\theta_1 + \text{Cos } \omega t \text{ Sin}\theta_1] + Vp_2[\text{Sin } \omega t \text{ Cos}\theta_2 + \text{Cos } \omega t \text{ Sin}\theta_2]$
$V_1+V_2 = \text{Sin } \omega t\, [Vp_1 \text{ Cos}\theta_1 + Vp_2 \text{ Cos}\theta_2] + \text{Cos } \omega t\, [Vp_1 \text{ Sin}\theta_1 + Vp_2 \text{ Sin}\theta_2]$
Put $V_1+V_2 = Vp_3[\text{Sin } (\omega t + \theta_3)]$
$= \text{Sin } \omega t\, (Vp_3 \text{ Cos}\theta_3) + \text{Cos } \omega t\, (Vp_3 \text{ Sin}\theta_3)$
Thus $[Vp_1 \text{ Cos}\theta_1 + Vp_2 \text{ Cos}\theta_2] = Vp_3 \text{ Cos}\theta_3$
And $[Vp_1 \text{ Sin}\theta_1 + Vp_2 \text{ Sin}\theta_2] = Vp_3 \text{ Sin}\theta_3$

Inspection of the vector diagram of the two voltages shows that this result could have been obtained directly from the vector diagram.

$\tan \theta_3 = Vp_3 \text{ Sin}\theta_3 \;/\; (Vp_3 \text{ Cos}\theta_3)$
$= [Vp_1 \text{ Sin}\theta_1 + Vp_2 \text{ Sin}\theta_2] \;/\; [Vp_1 \text{ Cos}\theta_1 + Vp_2\text{Cos}\theta_2]$

and $Vp_3 = \sqrt{[(Vp_3 \text{ Sin}\theta_3)^2 + (Vp_3 \text{ Cos}\theta_3)^2]}$
$= \sqrt{[(Vp_1 \text{ Sin}\theta_1 + Vp_2 \text{ Sin}\theta_2)^2 + (Vp_1 \text{ Cos}\theta_1 + Vp_2 \text{ Cos}\theta_2)^2]}$

If two AC voltages V_1 and V_2 (at the same frequency but different phase) are added together, the result is another AC voltage whose magnitude is the vector addition $V_1 + V_2$.

Similarly if two AC currents I_1 and I_2 (at the same frequency but different phase) are added together, the result is another AC current whose magnitude is the vector addition $I_1 + I_2$.

The vectors have been shown as the peak value of the vector. However the rms value of a sine wave is always $1/\sqrt{2}$ times the peak value. Thus the vector diagram of the rms values is exactly the same to a different scale as the vector diagram for the peak values.

The vector diagrams of voltage and current are the rms values unless stated otherwise.

Power in a single phase AC circuit

The power in an AC circuit is the product of Volts and Amps.

Let the phase angle between voltage and current be ϕ.

Let $V = Vp \text{ Sin } \omega t$ and $I = Ip \text{ Sin } (\omega t + \phi)$

$W = Vp\, Ip \text{ Sin } \omega t \text{ Sin } (\omega t + \phi)$

$\quad = Vp\, Ip \text{ Sin } \omega t \text{ (Sin } \omega t \text{ Cos } \phi + \text{Cos } \omega t \text{ Sin } \phi)$

$\quad = Vp\, Ip\, [\text{Sin}^2\, \omega t \text{ Cos } \phi + (1/2) \text{ Sin } 2\, \omega t \text{ Sin } \phi]$

The mean value of Sin 2 ωt over a complete cycle is zero,

$W = Vp\, Ip \text{ Cos } \phi\ \text{ Sin}^2\, \omega t$

Put $\omega t = x$

The mean value of $\mathrm{Sin}^2 x = (1/2\pi)\int \mathrm{Sin}^2 x\, dx$ from 0 to 2π

$= (1/2\pi)\int[(1 - \mathrm{Cos}\, 2x)/2]\, dx$ from 0 to 2π

$= (1/4\pi)\,[x - \mathrm{Sin}\, 2x]$ from 0 to 2π

$= (1/4\pi)\,[2\pi - 0 - 0 + 0] = ½$

But $Vp\ \mathrm{Ip} = 2Vrms\ Irms_r$

Therefore $W = Vrms\ Irms\ \mathrm{Cos}\ \phi$

In a single phase circuit

$W = V\,I\,\mathrm{Cos}\ \phi$ where V and I are the rms values

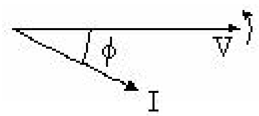


Figure 55; Power in a single phase AC crcuit

This can be written by the vector equation $W = \boldsymbol{V} \bullet \boldsymbol{I}$

Cos ϕ is called the power factor (or pf).

Three phase system

If the generator has three coils at 120^0 spacing, three voltages will be produced with a phase angle of 120^0 between them.

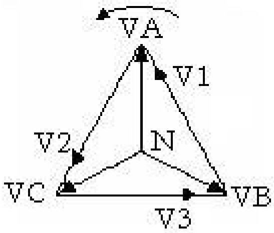


Figure 56; Three phase system

If each voltage is connected to a circuit with the same impedance and the three currents return along the same conductor, then the vector sum of the three return currents is zero. Thus instead of three full sized return cables, only one of smaller size is needed. If none of the load is single phase, then the neutral is not needed at all. High voltage supplies are nearly always three phase without a neutral conductor. There is a great economy in distribution costs if the electricity can be supplied in three phases.

The vector diagram shows the common return point, called the Neutral point, at N and a three phase supply with voltages *VA*, *VB* and *VC*.

These are called the phase voltages or *Vph*.

The voltages *V1*, *V2* and *V3* are called the line voltages or *Vline*.

It can be seen by the 30^0 and 60^0 triangles that the magnitude of the line voltage is √3 times the magnitude of the phase voltage.

The voltage of a three phase supply is defined by the line voltage.

Thus a 415 volt three phase supply can provide three separate 240 volt single phase domestic supplies .

If all three phases have currents of the same magnitude and power factor, then

Total power $W = 3\ Vph\ I\ \cos\phi = \sqrt{3}\ Vline\ I\ \cos\phi$

$W = \sqrt{3}\ Vline\ I\ \cos\phi$

Power to a balanced three phase system is constant

$Ea\ Ia = \sqrt{2}E \sin(\omega t) \cdot \sqrt{2}I \sin(\omega t - \phi)$

$Eb\ Ib = \sqrt{2}E \sin(\omega t + 120^0) \cdot \sqrt{2}I \sin(\omega t + 120^0 - \phi)$

$Ec\ Ic = \sqrt{2}E \sin(\omega t - 120^0) \cdot \sqrt{2}I \sin(\omega t - 120^0 - \phi)$

where E and I are rms values

$\sin A \sin B = ½[\cos(A - B) - \cos(A + B)]$

$Ea\ Ia = E\ I[\cos(\phi) - \cos(2\omega t - \phi)]$

$Eb\ Ib = E\ I[\cos(\phi) - \cos(2\omega t + 240^0 - \phi)]$

$Ec\ Ic = E\ I[\cos(\phi) - \cos(2\omega t - 240^0 - \phi)]$

$W = Ea\ Ia + Eb\ Ib + Ec\ Ic$

$W = E\ I\ [3\cos(\phi) - \cos(2\omega t - \phi) - \{\cos(2\omega t + 240^0 - \phi) + \cos(2\omega t - 240^0 - \phi)\}]$

$\cos A + \cos B = 2\cos\{(A + B)/2\}\ \cos\{(A - B)/2\}$

$W = E\ I\ [3\cos(\phi) - \cos(2\omega t - \phi) - 2\{\cos(4\omega t - 2\phi)/2\ \ \cos(480^0)/2\}]$

$W = E\ I\ [3\cos(\phi) - \cos(2\omega t - \phi) - 2\{\cos(\omega t - \phi)\ \ \cos(240^0)\}]$

$\cos 240 = - ½$

$W = E\ I\ [3\cos(\phi) - \cos(2\omega t - \phi) + \cos(\omega t - \phi)]$

$W = E\ I\ [3\cos\phi]$

$W = 3E\ I\cos\phi$ where E is the phase voltage

$W = \sqrt{3}\ VI\cos\phi$ where V is the line voltage

This does not include t, therefore it is constant for all values of t.

Measurement of power

The power in a three phase, three wire, system can be measured by two single phase wattmeters.

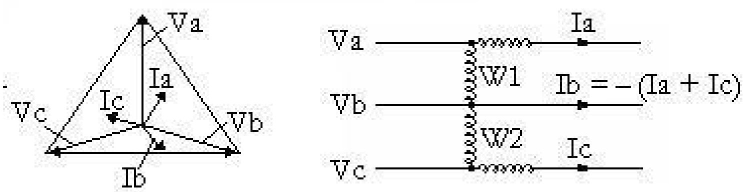


Figure 57; Power measured by two wattmeters

$W1$ reads the vector dot product $(Va - Vb)\bullet Ia$

$W1 = (Va - Vb)\bullet Ia$

$W2 = (Vc - Vb)\bullet Ic$

$W1 + W2 = Va\bullet Ia - Vb\bullet(Ia + Ic) + Vc\bullet Ic$

For a three wire system, $Ia + Ib + Ic = 0$

$W1 + W2 = Va\bullet Ia + Vb\bullet Ib + Vc\bullet Ic$ = total power in the three phases

The sum of two wattmeter readings gives the power in a three phase three wire system. The phases do not need to carry the same current or have the same power factor.

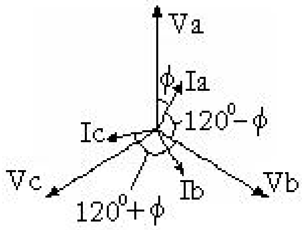


Figure 58; Power measured by two wattmeters

Power Factor measurement by two wattmeters.

If the load is an electric motor, the voltages are usually balanced three phase system with the currents and power factors the same on each phase. In this case, the power factor can be obtained from the two wattmeter readings.

$W1 = Va\,Ia\,\mathrm{Cos}\,\phi - Vb\,Ia\,\mathrm{Cos}(120^0 - \phi)$

$\quad = V\,I\,\mathrm{Cos}\,\phi - VI\,\{\mathrm{Cos}(120^0)\mathrm{Cos}(\phi) + \mathrm{Sin}(120^0)\mathrm{Sin}(\phi)\}$

$W2 = Vc\,Ic\,\mathrm{Cos}\,\phi - Vb\,Ic\,\mathrm{Cos}(120^0 + \phi)$

$\quad = V\,I\,\mathrm{Cos}\,\phi - VI\,\{\mathrm{Cos}(120^0)\mathrm{Cos}(\phi) - \mathrm{Sin}(120^0)\mathrm{Sin}(\phi)\}$

$W2 - W1 = 2\,V\,I\,\mathrm{Sin}(120^0)\mathrm{Sin}(\phi) = 2\,V\,I\,(\surd 3/2)\mathrm{Sin}(\phi) = V\,I\,(\surd 3)\,\mathrm{Sin}(\phi)$

But $W1 + W2 = 3\,V\,I\,\mathrm{Cos}(\phi)$

$(W2 - W1)/(W1 + W2) = (1/\surd 3)\,\mathrm{Tan}(\phi)$

$\mathrm{Tan}(\phi) = \surd 3\,(W2 - W1)/(W1 + W2)$

Power factor $= \mathrm{Cos}(\phi) = 1\,/\,\surd\{1 + \mathrm{Tan}^2\,(\phi)\}$

RESISTANCE, INDUCTANCE AND CAPACITANCE ON AC

AC Current through a Resistance
When an AC voltage is applied to a pure resistance,
at any instant $I = V/R$

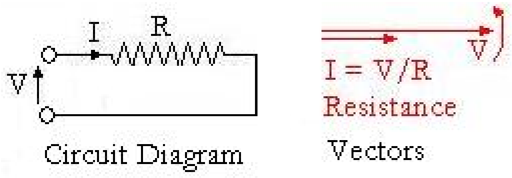


Figure 59; Current in a resistor

Irms = Vrms / R
where *Vrms* is in volts, *Irms* is in amps and R is in ohms
*I*rms is in phase with *Vrms*

However the Resistance is higher on AC than on DC. This is due to eddy currents causing a power loss.

I^2R = copper loss + eddy current loss.

AC Current through an Inductor
Let the inductance be L henries

$L\,di/dt$ = back emf
= applied voltage v
$= V_p \sin 2\pi f t$

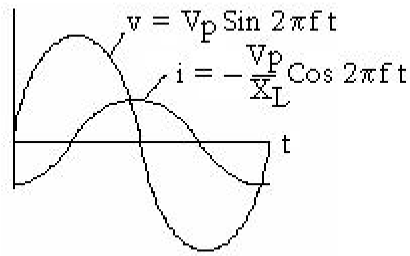


Figure 60; AC current in an inductor

Integrating,
$L\,i = -V_p\,(1/2\,\pi\,f)\,\cos 2\pi f t$ + Const
The constant is a DC current (usually zero)

$i = -[V_p/(2\pi f L)] \cos 2\,\pi\,f\,t$
$I_{rms} = V_{rms}/(2\pi f L)$

But $V_{rms} = I_{rms}\,X_L$
where X_L is the reactance of the inductor
Thus $X_L = 2\pi f L$
At 50 Hz $X_L = 314\,L$
where X_L is in ohms and L is in henries

The current lags the voltage by 1/4 cycle

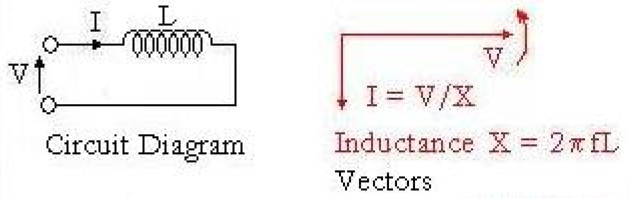


Figure 61; Current in an inductor

AC Current through a Capacitance (or Condenser)

Let the capacitance be C farads
Thus the charge $q = C\,v$
where $dq/dt = i$ amps
$q = C\,V_p \text{ Sin } 2\pi f\,t$
Differentiating wrt t

$$i = C\,V_p\,2\pi f \text{ Cos } 2\pi f\,t$$

Hence $I_p = 2\pi f\,C\,V_p$
$I_{rms} = I_p/\sqrt{2}$ and $V_{rms} = V_p/\sqrt{2}$
Hence $I_{rms} = 2\pi f\,C\,V_{rms}$
But $V_{rms} = I_{rms}\,X_C$
where X_C is the reactance of the capacitor in ohms (ie volts/amps)
Thus $X_C = 1/(2\pi f\,C)$
With C in μF then $X_C = 10^6/(2\pi f\,C)$
At 50 Hz $X_C = 3183/C$
where X_C is the reactance of the capacitor in ohms (ie volts/amps) and C is in μF

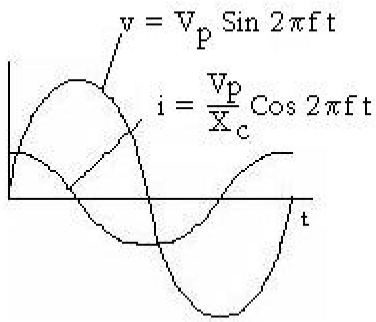


Figure 62; Current leads the voltage

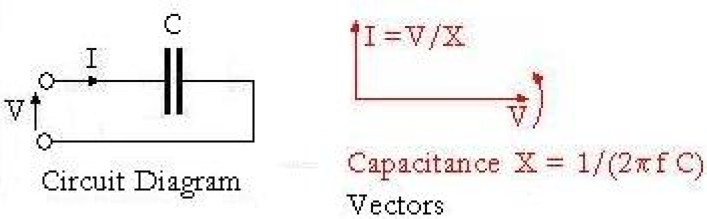


Figure 63; Current in a Capacitance

The current is established over several cycles, but it then lags by 3/4 cycle.
Thus, in effect, Capacitive Current leads the voltage by 1/4 cycle

Eddy Currents in a conductor
AC Resistance is higher than DC Resistance due to eddy current loss.
A conductor, radius *a* metres carries a current I Sin(pt) amps

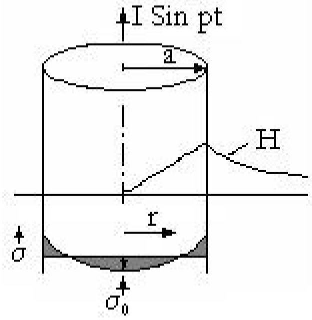


Figure 64; Current in a conductor

H outside the conductor is $(2I/r)$ Sin (pt) 10^{-7}
where r is the distance in metres from the centre of the conductor

At the surface of the conductor
$$H = (2I/a) \text{ Sin } (pt)\ 10^{-7}$$
At the centre of the conductor, $H = 0$
Thus inside the conductor
$H = (2I\ r/a^2)$ Sin (pt) 10^{-7}

Let σ be the eddy current in amps/m^2 at radius r metres
and σ_0 be the current in amps/m^2 at the centre of the conductor

$$(\sigma + \sigma_0)\,\rho = -\,\partial\Phi / \partial t$$
$$= -\int [\,\partial H / \partial t]\, \mathrm{d}r \quad \text{from } r = 0 \text{ to } r = r$$
$$= -\int [\,(2\,\mathrm{p}\,\mathrm{I} / a^2)\,\mathrm{Cos}\,(pt)\,]\,10^{-7}\, r\,\mathrm{d}r \quad \text{from } r = 0 \text{ to } r = r$$
$$= (p\,I\,r^2 / a^2)\,\mathrm{Cos}\,(pt)\,10^{-7}$$

therefore $\quad \sigma = \{p\,I\,r^2 / (\rho\, a^2)\}\,\mathrm{Cos}\,(pt)\,10^{-7} - \sigma_0$

But $\quad 2\pi \int \sigma\, r\,\mathrm{d}r = 0$ from $r = 0$ to $r = a$

$$2\pi \int [\,\{p\,I\,r^2 / (\rho\, a^2)\}\,\mathrm{Cos}\,(pt)\,10^{-7} - \sigma_0\,]\, r\,\mathrm{d}r = 0 \quad \text{from } r = 0 \text{ to } r = a$$
$$2\pi\,[\,\{p\,I\,r^4 / (4\,\rho\, a^2)\}\,\mathrm{Cos}\,(pt)\,10^{-7} - \sigma_0\, r^2/2] = 0 \quad \text{from } r = 0 \text{ to } r = a$$

$$p\,I\,a^2 / (4\,\rho)\,\mathrm{Cos}\,(pt)\,10^{-7} - \sigma_0\, a^2/2] = 0$$
$$\sigma_0 = p\,I / (2\,\rho)\,\mathrm{Cos}\,(pt)\,10^{-7}$$

therefore $\quad \sigma = (p\,I / \rho)\,\mathrm{Cos}\,(pt)\,[r^2 / a^2 - \tfrac{1}{2}]10^{-7}$ and $\sigma = 0$ when $r = a/\sqrt{2}$
Total current density $\; i = \sigma + [\,I\,\mathrm{Sin}\,(pt)\,] / (\pi\, a^2)$
Put $\; [\,I\,\mathrm{Sin}\,(pt)\,] / (\pi\, a^2) = C$

$$i^2 = \sigma^2 + 2\sigma C + C^2$$

Energy loss $= 2\pi \int i^2\, \rho\, r\,\mathrm{d}r$ from $r = 0$ to $r = \mathrm{a}$
$\quad = 2\pi \int [\sigma^2 + 2\sigma C + C^2\,]\,\rho\, r\,\mathrm{d}r$ from $r = 0$ to $r = a$
But as before, $\quad 2\pi \int \sigma\, r\,\mathrm{d}r = 0$ from $r = 0$ to $r = a$
Therefore $\; 2\pi \int 2\,\sigma C\,\rho\, r\,\mathrm{d}r = 0$ from $r = 0$ to $r = a$

Energy loss $= 2\pi \int [\sigma^2 + C^2\,]\,\rho\, r\,\mathrm{d}r$ from $r = 0$ to $r = a$
Additional loss due to eddy currents = integral from 0 to a

$$2\pi \int \sigma^2\,\rho\, r\,\mathrm{d}r = 2\pi\,(p^2\, I^2 / \rho)\,\mathrm{Cos}^2\,(pt)\,10^{-14} \int [r^5/a^4 - r^3/a^2 + r/4]\,\mathrm{d}r$$
$$= 2\pi\,(p^2\, I^2 / \rho)\,\mathrm{Cos}^2\,(pt)\,[\,a^2/6 - a^2/4 + a^2/8]\,10^{-14}$$
$$= \pi\, p^2\, I^2\, a^2 / (12\,\rho)\,\mathrm{Cos}^2\,(pt)\,10^{-14}$$

Mean value of $I_{max}{}^2\,\mathrm{Cos}^2\,(pt) = I_{rms}{}^2$
Mean value of energy loss $= \pi\, p^2\, I_{rms}{}^2\, a^2 / (12\,\rho)\,10^{-14}$
Total loss $= I_{rms}{}^2\,[\rho / (\pi\, a^2) + \pi\, p^2\, a^2 / (12\,\rho)\,10^{-14}]$
$\quad = I_{rms}{}^2\, \mathrm{R}_0\,[\,1 + \pi^2 p^2 a^4 / (12\,\rho^2)\,10^{-14}\,]$
where a is in metres and ρ is in ohms per metre cube
hence $\; \mathrm{R}_f / \mathrm{R}_0 = 1 + \pi^2 p^2 a^4 / (12\,\rho^2)\,10^{-14}$
but $p = 2\pi f$

$$\mathrm{R}_f / \mathrm{R}_0 = 1 + 100\pi^4 f^2 a^4 / (3\,\rho^2)$$

where f is in Hz, a is in metres and ρ is in $\mu\Omega$ per cm cube

Example
Conductor 1.29 cm radius, 50 Hz, $\rho = 1.65\ \mu\Omega$ per cm cube

$$\mathrm{R}_f / \mathrm{R}_0 = 1 + 100 \cdot 3.14^4 \cdot 50^2 \cdot (1.29/100)^4 / (3 \cdot 1.65^2)$$
$$= 1.08$$

Iron cored inductor
The iron core of an inductor can saturate as the sine wave approaches peak value. If an AC voltage is applied, the current will increase as the peak value is approached. The current wave has a peaky form factor.

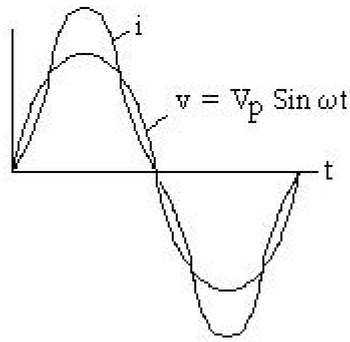


Figure 65; Current in an inductor

There is another effect. If the rms current through an iron cored reactor is increased, the reactance falls sharply as the iron saturates. If the current is increased in stages from zero and the voltage drop across the reactor is measured, the graph will have a kink as shown. This means that applying a fixed voltage Vc to the reactor, the current can take any one of three values. One of these values, I2 in the diagram, is unstable but the other two are stable. So while applying a voltage within this range, any transient can make the current flip to the other value.

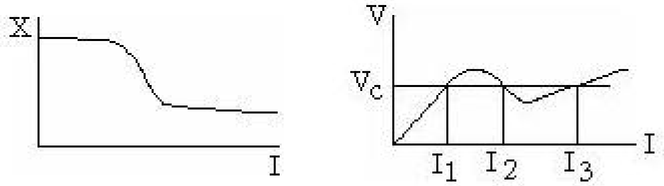


Figure 66; Current in an inductance

Examples Find the current taken from the 240 volt 50 Hz mains by;
(i) 0.12 henry inductance (ii)40 μF capacitance.

(i) $X = 314 \cdot 0.12 = 37.7$ ohms $I = V/X = 6.37$ amps
(ii) $X = 3183/40 = 79.6$ ohms $I = V/X = 3.02$ amps

AC CIRCUITS

Series and Parallel Circuits

Resistance, Inductance and Capacitance in series

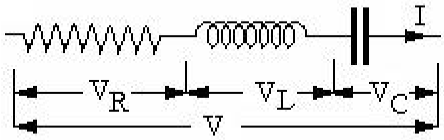


Figure 67; RLC Impedance

Let a circuit consist of a Resistor, Inductor and a Capacitor is series all carrying an AC current I
V_R is in phase with I, V_C lags I by 90^0 and V_L leads I by 90^0
$V_R = IR$ and $V_L = \text{j}\, IX_L$ and $V_C = -\text{j}\, IX_C$
where X_L and X_C are the reactances of L and C

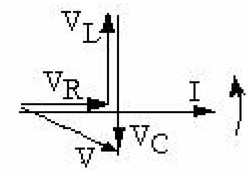


Figure 68; Voltages

These voltages can be represented by vectors on the same diagram as I
It can be seen from the vector diagram that
$V^2 = V_R^2 + (V_C - V_L)^2$
And the current leads the voltage by arc tan $[(V_C - V_L)/ V_R]$ = arc tan $[(X_C - X_L)/R]$

Where a circuit contains reactance and resistance, the combination is called impedance. The symbol for impedance is Z and the units are ohms.

$V = I Z$

$IZ = IR + \text{j}\, IX_L - \text{j}\, IX_C$

Thus $I Z = I [\text{R} + \text{j}\, X_L - \text{j}\, X_C]$
ie Z can be considered an operator $Z = [\text{R} + \text{j}\, X_L - \text{j}\, X_C]$

Magnitude of $Z = \sqrt{} [\text{R}^2 + (X_L - X_C)^2]$ by pythagoras, see the vector diagram.

Resonance of a Series *LC* circuit with a variable frequency AC supply
Let the coil have resistance R ohms and Inductance L henries and the Capacitance be C farads.

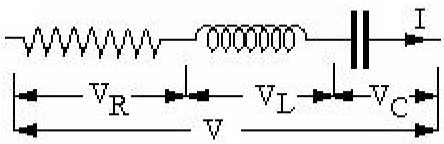


Figure 69; Resonance

The impedance of the circuit
$Z = \text{R} + \text{j}\, 2\pi f L - \text{j} /(2\pi f C)$
Z has a minimum when $\text{j}2\pi f L = \text{j}/(2\pi f C)$ and this minimum value of Z is R
Z has the minimum value when $(2\pi f L)(2\pi f C) = 1$

This occurs at the Resonant Frequency

$f_0 = 1 / [2\pi \sqrt{(LC)}]$

The reactance of the coil at resonance = $2\pi f_0 L = \sqrt{(L/C)}$

If a variable frequency supply at a constant voltage is applied to the circuit, a plot of current against f will be of the form shown.

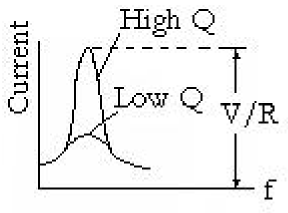


Figure 70; Q curves

The exact shape depends on the relative values of R and L, ie on the Q factor of the coil where Q factor = (reactance at resonance)/R = $[\sqrt{(L/C)}]/R$

At a frequency $(f_0 + \delta f)$ near resonance,

$Z = R + j\,2\pi(f_0 + \delta f)\,L - j\,/[2\pi(f_0 + \delta f)\,C]$

$= R + j\,2\pi f_0\,L + j\,2\pi\delta f\,L - j\,/[2\pi f_0\,C(1 + \delta f/f_0)]$

$= R + j\,2\pi f_0\,L + j\,2\pi\delta f\,L - [j\,/(2\pi f_0\,C)](1 - \delta f/f_0)$

$= R + j\,2\pi\delta f\,L + [j\,/(2\pi f_0\,C)](\delta f/f_0)$

$= R + j\,2\pi\delta f\,L + [j\,(2\pi f_0\,L)](\delta f/f_0)$

$= R + j\,4\pi\delta f\,L$

$= R + j\,2\,X\,(\delta f/f_0)$

Hence near resonance, $I = V/\sqrt{[R^2 + 4X^2\,(\delta f/f_0)^2]}$

where $X = X_L = X_C$ at resonance

The phase angle changes rapidly from positive to negative as resonance is passed.

Resistances, Inductances and Capacitances in series on an AC supply

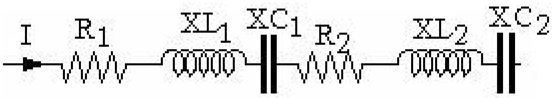


Figure 71; Impedances in series

Let the circuit be equivalent to a single resistance R and a single reactance X

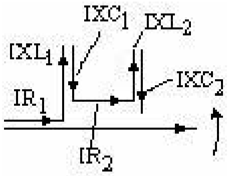


Figure 72; Voltages in series

By inspection of the vector diagram of voltages, it can be seen that

$R = R_1 + R_2$

and $X = XL_1 - XC_1 + XL_2 - XC_2$

If X is positive then X is an inductance

If X is negative then X is a capacitance.

Thus in a circuit containing resistances and reactances in series,

The equivalent circuit is a resistance = sum of all the resistances

and reactance = sum of all the reactances, where inductive reactances are positive and capacitive reactances negative.

Inductive impedances in parallel

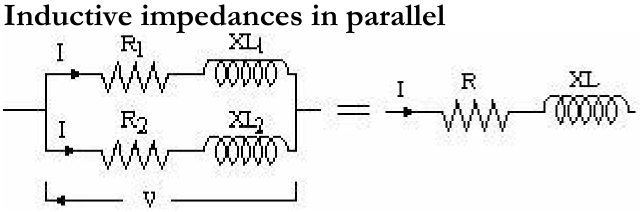


Figure 73; Impedances in parallel

Impedances

$Z_1 = \sqrt{(R_1^2 + X_1^2)}$ and $Z_2 = \sqrt{(R_2^2 + X_2^2)}$ and $Z = \sqrt{(R^2 + X^2)}$

I = vector sum of I_1 and I_2

Hence $I \text{ Cos } \theta = I_1 \text{ Cos } \theta_1 + I_2 \text{ Cos } \theta_2$ and $I \text{ Sin } \theta = I_1 \text{ Sin } \theta_1 + I_2 \text{ Sin } \theta_2$
$I_1 = V / Z_1$ and $I_2 = V / Z_2$ and $I = V / Z$

$\text{Cos } \theta = R / Z$
$\text{Cos } \theta_1 = R_1 / Z_1$
$\text{Cos } \theta_2 = R_2 / Z_2$

$\text{Sin } \theta = X / Z$
$\text{Sin } \theta_1 = X_1 / Z_1$
$\text{Sin } \theta_2 = X_2 / Z_2$

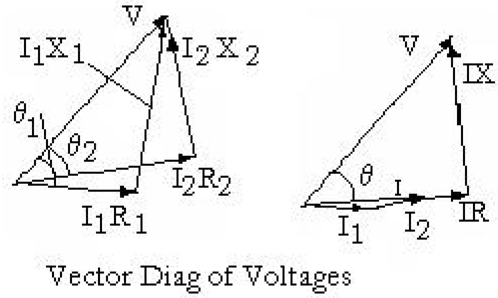


Figure 74; Vector Diag of Voltages

$(V/Z)(R/Z) = (V/Z_1)(R_1/Z_1) + (V/Z_2)(R_2/Z_2)$
$R/Z^2 = R_1/Z_1^2 + R_2/Z_2^2$
Similarly
$X/Z^2 = X_1/Z_1^2 + X_2/Z_2^2$
Put $A = R_1 / Z_1^2 + R_2 / Z_2^2$ and $B = X_1 / Z_1^2 + X_2 / Z_2^2$
$R/Z^2 = A$ and $X/Z^2 = B$
$1 / Z^2 = R^2/Z^4 + X^2/Z^4 = A^2 + B^2$

$R = A / (A^2 + B^2)$
$X = B / (A^2 + B^2)$
Where $A = R_1 / Z_1^2 + R_2 / Z_2^2$ and $B = X_1 / Z_1^2 + X_2 / Z_2^2$

Inductance and Capacitor or two Capacitors in parallel
If X_1 or X_2 (or both) is a capacitance, then the vector diagrams are still valid except that X has the negative value. The evaluation of R and X are still valid.

Resonance of a parallel LC circuit

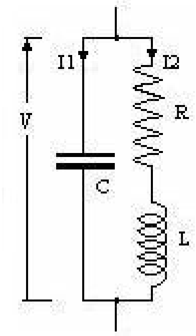


Figure 75; Resonance of a parallel circuit

$I1 = V / [-j / (\omega C)]$

$I2 = V \,/\, [\, \mathrm{R} + \mathrm{j}\,\omega L \,]$
$I1 + I2 = V\,[\,1/\{-\mathrm{j}\,/(\omega C)\} + 1 \,/\, (\mathrm{R} + \mathrm{j}\,\omega L)\,]$
$\quad = V[\mathrm{R} + \mathrm{j}\,\omega L - \mathrm{j}\,/(\omega C)] \,/\, [(\mathrm{R} + \mathrm{j}\,\omega L)\{-\mathrm{j}/(\omega C)\}]$
But $I1 + I2 = V \,/\, Z$
$Z = [(\mathrm{R} + \mathrm{j}\,\omega L)\{-\mathrm{j}/(\omega C)\}] \,/\, [\mathrm{R} + \mathrm{j}\,\omega L - \mathrm{j}\,/(\omega C)]$

At resonance, $\omega = \omega_0$ and $\mathrm{j}\,\omega_0 L = \mathrm{j}\,/(\omega_0 C)$
$Z = [\,(\mathrm{R} + \mathrm{j}\,\omega_0 L)\,(-\mathrm{j}\,\omega_0 L)] \,/\, \mathrm{R}$

$Q = \omega_0 L \,/\, \mathrm{R}$
If Q is large, $\mathrm{R} + \mathrm{j}\,\omega_0 L \approx \mathrm{j}\,\omega_0 L$
$Z \approx \mathrm{j}\,\omega_0 L\,(-\mathrm{j}\,\omega_0 L) \,/\, \mathrm{R} = (\omega_0 L)^2 \,/\, \mathrm{R}$
$Z \approx Q^2\,\mathrm{R}$
Z is a maximum at resonance

Delta / Star transformation of balanced load

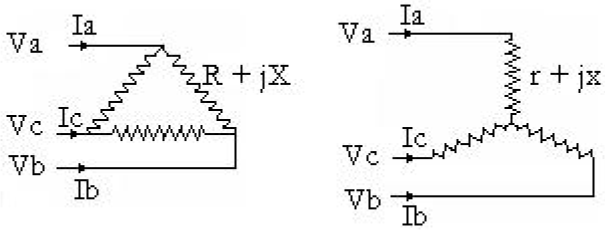


Figure 76; Delta/Star transformation

Delta connection
$Ia = (Va - Vb) / (R + jX)$
$+ (Va - Vb)/ (R + jX)$
$= [2Va - (Vb + Vc)]/(R + jX)$
But $Va + Vb + Vc = 0$
$Ia = 3Va/(R + jX)$

Star connection
$Ia = Va / (r + jx)$
Hence $3 / (R + jX) = 1 / (r + jx)$
$R + jX = 3r + 3jx$
Equate real and imaginary terms $R = 3r$ and $X = 3x$

Example on mains supply power loss
A single phase power line has resistance R ohms per metres in each of the phase and neutral lines.
It supplies a current I_1 to a consumer at the end of the line and a total of I_2 amps to consumers uniformly distributed along its total length of L metres.
Calculate the total power loss in the line.

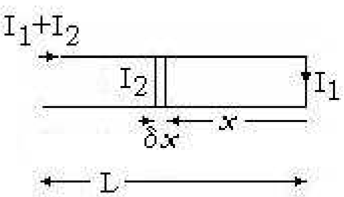


Figure 77; Distributed Load

Current in element δx of the line $= I_1 + I_2\, x / L$
Power loss in each element of line and neutral
$\delta W = (I_1 + I_2\, x/L)^2\ R\, \delta x$

Total power loss in both line and neutral
$W = 2 \int (I_1 + I_2\, x/\mathrm{L})^2\ R\, dx$ from 0 to L
$= 2\, R \int [I_1^{\,2} + 2\, I_1 I_2\, x/L + (I_2\, x/L)^2]\, dx$ from 0 to L
$= 2R\, [I_1^{\,2}\, x + 2\, I_1 I_2\, x^2 /2L + I_2^{\,2}\, x^3/ 3L^2]$ from 0 to L
$= 2\, R\, L\, [I_1^{\,2} + I_1 I_2 + I_2^{\,2} / 3]$

Example on mains supply volt drop
A supply cable has resistance R ohms/km and reactance X ohms/km
It supplies a load I_1 at pf_1 distant L_1 km from the source

and a load I_2 at pf_2 a further L_2 km from the source
and a load I_3 at pf_3 a further L_3 km from the source
Find the voltage required at the source of the supply to give the specified voltage V_3 at the last consumer

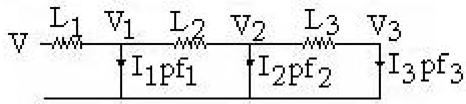


Figure 78; Multiple Loads

Draw the vectors V_3 and I_3 at Arc Cos (pf_3), the angle between them
Draw the vectors $RL_3\, I_3$ and j$XL_3\, I_3$ to obtain V_2
Draw vector V_2 and I_2 at Arc Cos (pf_2) relative to V_2
Complete the parallelogram to get vector ($I_2 + I_3$)
Draw vectors $RL_2\,(I_2 + I_3)$ and j$XL_2\,(I_2 + I_3)$ to obtain V_1
Repeat to obtain V

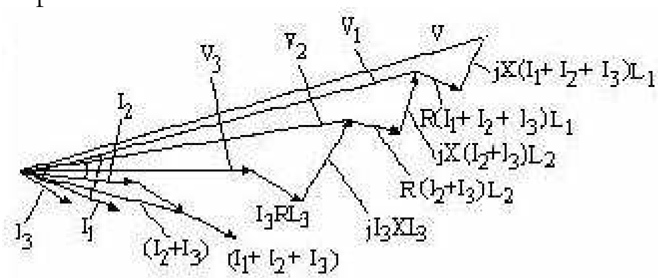


Figure 79; Volt drop with multiple oads

The volt drops are greatly exaggerated to show the construction. In practice the volt drop does not exceed a few per cent.

Example
A motor is taking a load of 10 amps at a power factor of 0.8 at 440 volt 50 Hz.
Find the size of condenser to be connected in parallel to bring the pf to unity.
Phase angle = Arc Cos (0.8) = 36.9^0 Wattless current = 10 Sin 36.9 = 6.0 amps
$X_C = 3183/C$ and $V = IX_C$ therefore $C = 3183/X_C = 3183 \cdot 6.0/440 = 43\mu F$

AC Bridge Circuits

AC Bridges

The total reactance X in each leg is the difference between the inductive reactance (+ ive) and the capacitive reactance (- ive)

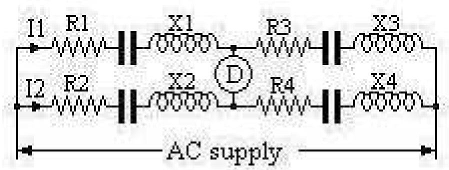


Figure 80; AC Bridge circuit

For balance;

$I1 \cdot R1 + j\, I1 \cdot X1 = I2 \cdot R2 + j\, I2 \cdot X2$

And $I1 \cdot R3 + j\, I1 \cdot X3 = I2 \cdot R4 + j\, I2 \cdot X4$

Therefore $(R1 + j\, X1) / (R3 + jX3) = (R2 + j\, X2) / (R4 + jX4)$

And $(R1 + j\, X1) \cdot (R4 + jX4) = (R2 + j\, X2) \cdot (R3 + j\, X3)$

$R1 \cdot R4 - X1 \cdot X4 + j(R1 \cdot X4 + R4 \cdot X1) = R2 \cdot R3 - X2 \cdot X3 + j(R3 \cdot X2 + R2 \cdot X3)$
Equate real and imaginary terms
$R1 \cdot R4 - X1 \cdot X4 = R2 \cdot R3 - X2 \cdot X3$ and $(R1 \cdot X4 + R4 \cdot X1) = (R3 \cdot X2 + R2 \cdot X3)$
These two conditions must be met for balance.

AC Bridge both reactances capacitive
(Reactances both inductive is similar)

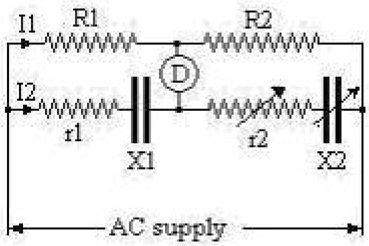


Figure 81; Bridge circuit

When in balance (no current in D)
$I1 \cdot R1 = I2 \cdot (r1 - jX1)$
$I1 \cdot R2 = I2 \cdot (r2 - jX2)$
Therefore
$R1 / R2 = (r1 - jX1) / (r2 - jX2)$

$R1 \cdot r2 - \mathrm{j}\ R1 \cdot X2 = R2 \cdot r1 - \mathrm{j}\ R2 \cdot X1$

Equate real and imaginary terms

$R1 \cdot r2 = R2 \cdot r1$ and $R1 \cdot X2 = R2 \cdot X1$

AC Bridge capacitive reactance balancing an inductive reactance

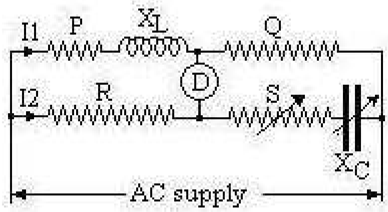


Figure 82; Balanced C and L Bridge

With no current through *D*

$\mathrm{I}1 \cdot (P + \mathrm{j}X_L) = I2 \cdot \mathrm{R}$ and $I1 \cdot Q = I2 \cdot (S - \mathrm{j}X_C)$

Therefore $(P + \mathrm{j}X_L)/Q = \mathrm{R} / (S - \mathrm{j}X_C)$

$P \cdot S - \mathrm{j}\ P \cdot X_C + \mathrm{j}\ S \cdot X_L + X_L \cdot X_C = Q \cdot \mathrm{R}$

Equate real and imaginary terms

$P \cdot S + X_L \cdot X_C = Q \cdot \mathrm{R}$

$P \cdot X_C = S \cdot X_L$

AC Bridge alternative arrangement

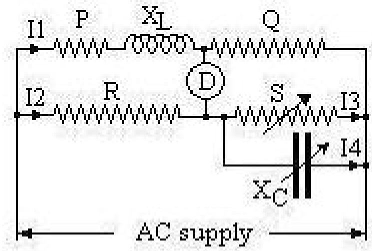


Figure 83; Alternative *C* and *L* bridge

With no current through *D*

$I1 \cdot P + \mathrm{j}\ I1 \cdot X_L = I2 \cdot \mathrm{R}$

$I1 \cdot Q = I3 \cdot S = -\mathrm{j}\ I4 \cdot X_C$

$I2 = I3 + I4$

Therefore

$I2 = [(P + \mathrm{j}\ X_L)/\mathrm{R}] \cdot I1$

$I3 = (Q/S) \cdot I1$

$I4 = [Q/(-\mathrm{j}\ X_C)] \cdot I1$

$[(P + \mathrm{j}\ X_L)/\mathrm{R}] = (Q/S) + [Q/(-\mathrm{j}\ X_C)]$

Equate real and imaginary terms

$P/\mathrm{R} = Q/S$ and $X_L/\mathrm{R} = Q/ X_C$

Bridge circuit to find *R* and *L*

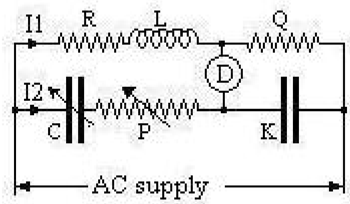


Figure 84; Bridge circuit to measure *R* and *L*

$I1 \cdot (\mathrm{R} + \mathrm{j}\omega L) = I2 \cdot [P - \mathrm{j}/(\omega C)]$

$I1\ Q = I2 \cdot [-j/(\omega K)]$
$(R + j\omega L) \cdot [-j/(\omega K)] = [P - j/(\omega C)] \cdot Q$
$-jR/(\omega K) + L/K = P \cdot Q - jQ/(\omega C)$
Equate real and imaginary terms
$L = P \cdot Q \cdot K$ and $R = Q \cdot K / C$
Note this result does not include ω, thus a buzzer which contains a multitude of sine waves can be used as the AC supply.

MAGNETIC PROPERTIES OF MATERIALS

***B – H* Curves**

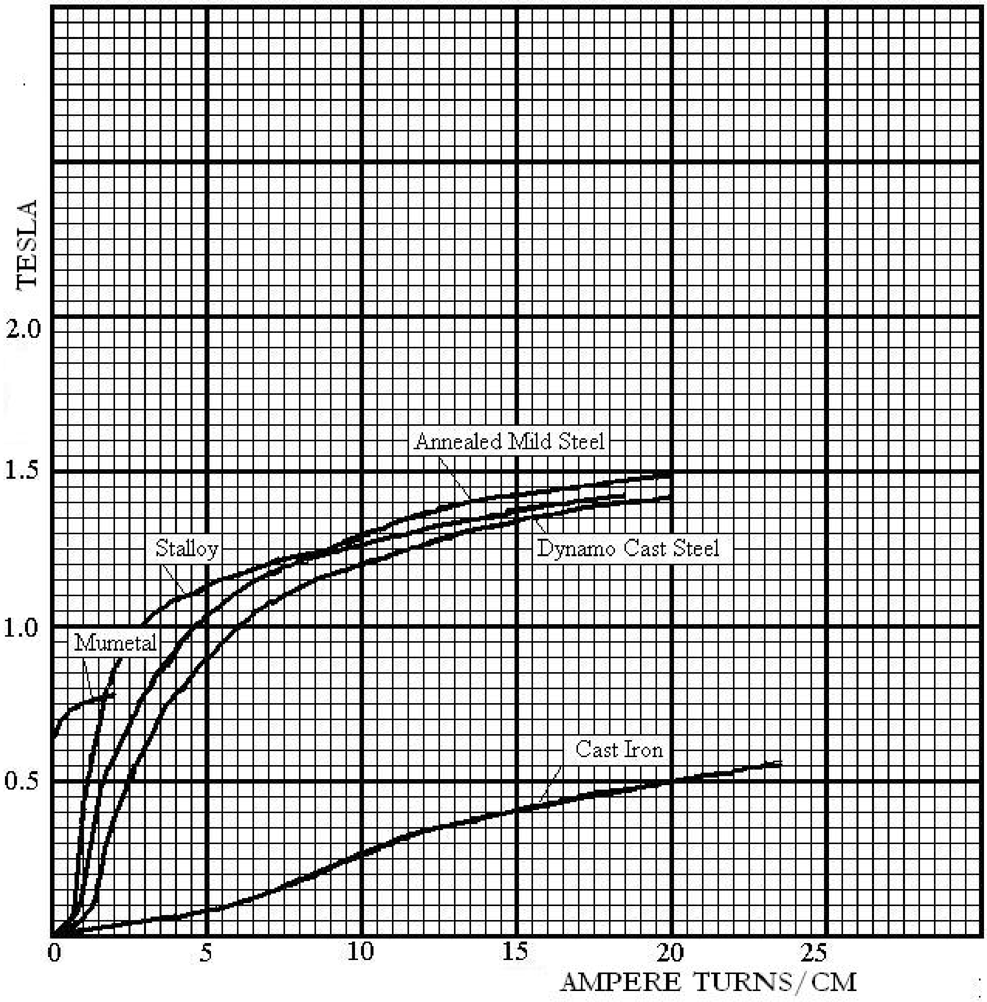


Figure 85; *B-H* Curves

The graph shows typical *B-H* curves for some common magnetic materials. The gradient near the origin is a measure of the permeability μ.

For many materials
$H = \alpha B^{1.6}$ in the range 0.5 to 1.2 tesla

$\mu = B / H$
where B is in tesla
and $H = 1.26 \cdot (NI/m)] \cdot 10^{-6}$

Therefore
$\mu = [B / \{1.26 \cdot (\text{Ampere Turns/cm})\}] \cdot 10^4$
For Dynamo Cast Steel, μ is about $[(1 / 1.26) / 5] \cdot 10^4 = 1600$

Hysteresis

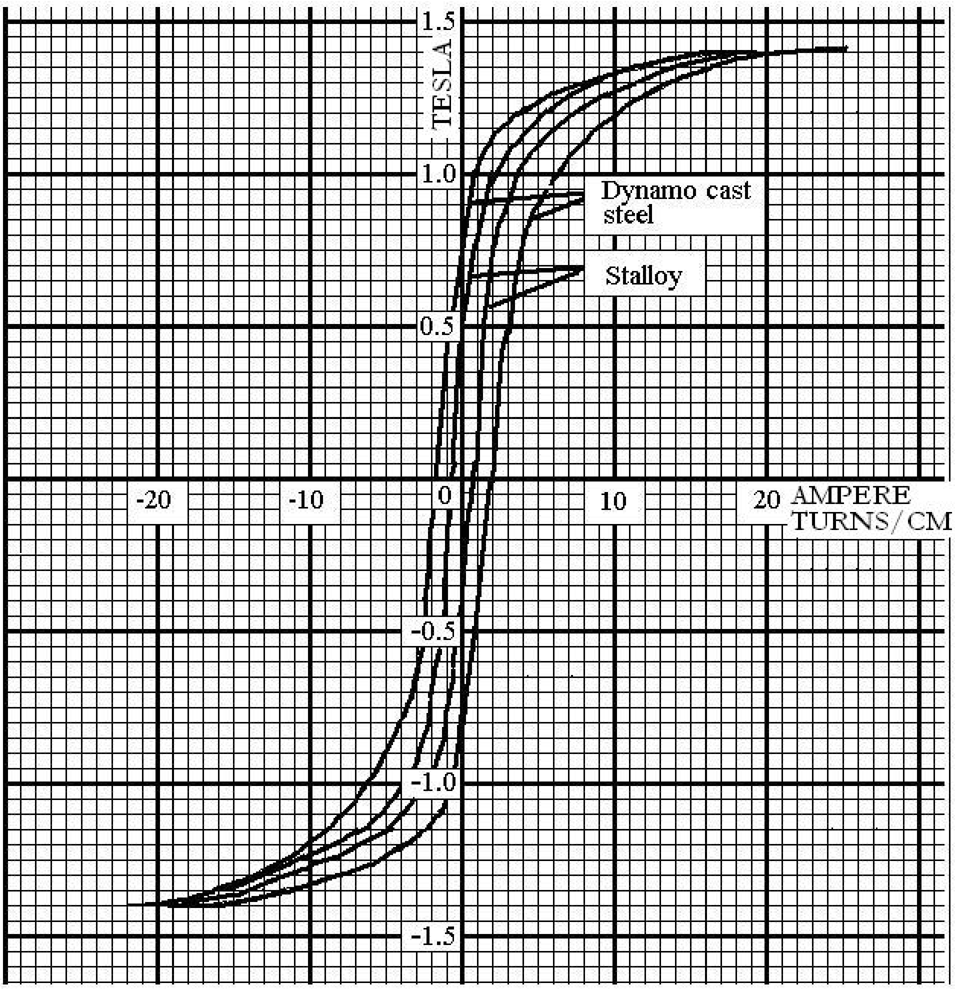


Figure 86; Hysteresis loops

.When a magnetic material is magnetized, it retains some magnetism when the magnetizing force is switched off. If *H* is raised from zero to a positive value then reduced to the same value negative and again to the positive, the magnetic field *B* lags behind the magnetising force. This is called the hysteresis loop. The area of the loop is a measure of work done in magnetizing the iron through this cycle.

The graph shows the hysteresis loops for Dynamo Cast Steel and for Silicon Iron (Stalloy). The Stalloy is the inner loop. As the area is a measure of the energy loss per cycle, the Stalloy has less energy loss per cycle than the Dynamo Cast Steel.

Flux Temperature relation

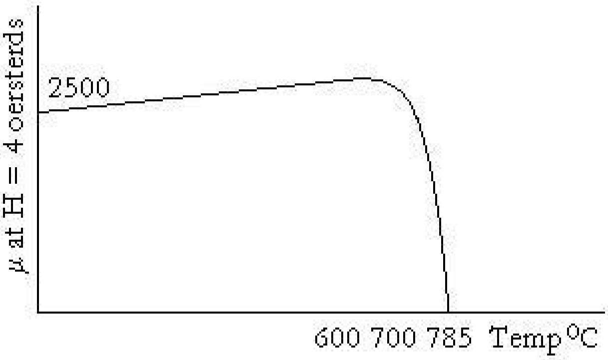


Figure 87; Flux – Temperature curve

With a constant Magnetizing Force, μ rises slightly with Temperature till about 600 ^{0}C and then falls rapidly to unity at the Curie point.

Energy spent in Hysteresis

Consider a laminated ring of the material, cross sectional area A cm^2 and circumference of the ring L cms. The ring has a coil of N turns taking a current i emu at time t.

$$H = 4\pi N i / L$$

(H in oersted, i in emu and L in cm)

Power taken = copper loss + loss in field

Loss in field = ei where e is the back emf

But $e = -N \, d\Phi / dt = -N A \, dB / dt$

Energy put into the field in time δt

$$= e \, i \, \delta t \text{ ergs}$$

$$= [N A \, dB / dt] \, H L / (4\pi N) \; \delta t$$

$$= (1/4\pi) \, H L A \, \delta B$$

Energy put into the field in element of time = $(1/4\pi) \, H L A \, \delta B$ ergs

Energy put into the field in finite time = $(1/4\pi) \, L A \int H \, \delta B$ ergs

$L A$ = volume of the core material in cc

$\int H \, \delta B$ over one cycle = area of the hysteresis loop

Hence Energy loss due to hysteresis = (1/4π) (area of hysteresis loop) ergs / cycle / cc where the hysteresis loop is in oersteds/cm and gauss

Let AT be the ampere turns / metre
AT / 100 is the ampere turns / cm
AT / 1000 is the emu of current turns / cm
H in oersted $= 4\pi\, AT / 1000$
Let F be the flux density in tesla
Flux density in gauss $= 10^4\ F$
Area of the hysteresis loop in oersted and gauss $= 4\pi\, AT\, F \cdot 10$
Hence Energy loss due to hysteresis

$= (1/4\pi)\,(4\pi\, AT\, F \cdot 10)$ ergs / cycle / cc
$= AT\, F \cdot 10 \cdot 10^6$ ergs / cycle / cubic metre
$= AT\, F$ joules / cycle / m^3

At frequency f cycles/sec
Power loss = $f \cdot$ (area of hysteresis loop) watts/ cubic metre
where the hysteresis loop is in tesla and ampere turns / metre

Example
Steel used for an armature has a B-H curve of area 4 sq ins for the relevant cycle.
B_{max} = 1 tesla.
Scales are 1 inch = 2 oersted and 1 inch = 0.5 tesla
Find the approx hysteresis loss if the stampings weigh 200 lb
Machine has 4 poles and runs at 600 rpm.

$H = (4\pi/10)\ NI/\mathrm{cm} = [(4\pi/1000)\ (\mathrm{AmpTurns/metre})]$ oersted
Area of hysteresis loop = $4 \cdot 2 \cdot 0.5$ = 4 oersted · tesla
$= 4 \cdot 1000\ /(4\pi)$ (Amp Turns/metre) · tesla
= 318.3 joules / cu metre

Volume of steel = $200 \cdot 453.6/7.7$ cubic cms = 0.01176 cu metres
Cycles per sec = $2 \cdot 600$ / 60 = 20 cycles / sec

Hysteresis loss = $318.3 \cdot 0.01176 \cdot 20$ joules/sec = 74.9 watts

Energy in an electro-magnetic field in air

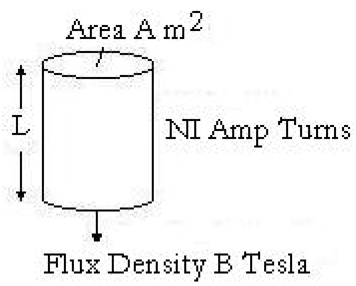


Figure 88; Energy in an electro-magnetic field

Let the current i amps be raised uniformly over T seconds from 0 to the final value I
The back emf during T is constant at $N\, d\Phi/dt$

$E = -N\, d\Phi/dt$

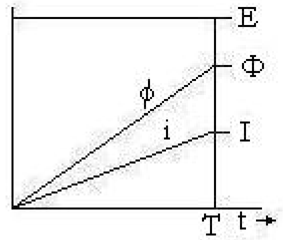


Figure 89; ϕ and i raised in time t

Energy input in joules during $T = \int E\, i\, dt$
$= N(\Phi/T) \int i\, dt$ from 0 to T
$= N(\Phi/T) \int (I\, t/T)dt$ from 0 to T
$= N(\Phi/T)\ (I\, t^2/2T)$ from 0 to T
$= N(\Phi/T)\ (I\, T^2/2T)$
$= NI\, \Phi/2$
But in air, $B = (4\pi/10^7)\ NI\ /\ L$ and $\Phi = B\, A$
where L metres is the length of path
hence $NI = B\, L \cdot 10^7/(4\pi)$
Energy input $= [B\, L \cdot 10^7/(4\pi)]\ \ B\, A/2$ joules
$= B^2 \cdot 10^7/(8\pi)$ joules per cubic metre

Energy stored in a uniform magnetic field in air $= B^2 \cdot 10^7/(8\pi)$ joules per m^3
where B is in tesla

Eddy Currents in laminated iron core all dimensions in metres

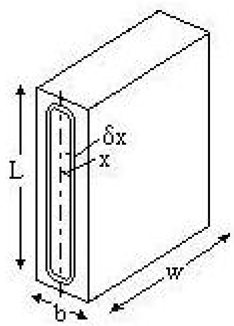


Figure 90; Laminated core

Consider an elemental path, distance x metres from the centreline and width δx in a flux density B tesla in a direction parallel to edge w.
emf induced in the loop $e = d/dt(B\, L2x)$
Resistance $= 2L\,\rho\ /\ (w\,\delta x)$ where ρ is ohms per metre cube
power loss $= e^2/\ R$
$= [d/dt(B\, L2x)]^2\ /\ (2L\,\rho/w\,\delta x)$
$= [(dB/dt)^2\ 2Lx^2\ w/\rho]\ \delta x$
total power loss $= [(dB/dt)^2\ 2L\ w/\rho] \int x^2 \delta x$ from 0 to $b/2$
$= [(dB/dt)^2\ 2L\ w/\rho]\ (1/3)\ (b/2)^3$
$= (dB/dt)^2\ L\ w/\rho]\ (1/12)\ b^3$
Let $B = B_m\ \mathrm{Sin}\ (2\pi f\, t)$
$dB/dt = B_m\ 2\pi f\ \mathrm{Cos}\ (2\pi f\, t)$
Mean value of $(dB/dt)^2 = ½\ (B_m\ 2\pi f)^2 = 2\ \pi^2 f^2\ B_m^{\ 2}$
This is the loss in a volume Lbw

Power loss / cubic metre = $2\,\pi^2 f^2 B_m^{\,2}\, b^2/(12\rho)$ metre = $\pi^2 f^2 B_m^{\,2}\, b^2/(6\,\rho)$

Eddy current loss = $\pi^2 f^2 B_m^{\,2}\, b^2/(6\,\rho)$ watts / cubic metre

where f is the freq, B_m is the maximum magnetic field in tesla, b is in metres and ρ is ohms/metre cube

Eddy current loss = $\pi^2 f^2 B_m^{\,2}\, b^2/(6\,\rho)$ watts / cm^3 where b is in cm and ρ is ohms/cm cube

Empirical formula

Iron loss = eddy current loss + hysteresis loss

$$= K_1 f^2 B^2 + K_2 f B^{1.6}$$

where K_1 and K_2 are constants, f is the number of cycles / sec (or rps) and B is the flux density in the range 0.5 to 1.2 tesla.

K_2 depends on the material and is typically in range 500 – 5000 watts/cu metre

Electromagnet

Let A = total area of pole faces in contact in m^2

assume North and South faces are equal.

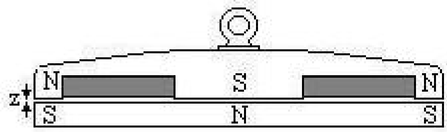


Figure 91; Electro magnet

B = Flux density in the gap in tesla
z = air gap in metres

Let the gap be widened by δz metres and the current in the coil increased by δI to keep the flux density constant in the air gap.

There is no change in the flux turns linked with the winding.
Therefore no emf set up in the winding
Therefore no power change in the winding except for the I^2R loss.

In the non magnetic air gap,

energy stored = $B^2 \cdot 10^7/(8\pi)$ joules per cubic metre.

Increase in energy stored in the air gap = $(A\,\delta x)\, B^2 \cdot 10^7/(8\pi)$

This must have come from mechanical work done = $P\,\delta x$ joules

Hence Pull $P = B^2 \cdot 10^7/(8\pi)$ newtons per m^2
where B is in tesla

Example
The total loss in a cylindrical core of steel stampings running at 400 rpm in a given field is 300 watts. At 600 rpm in the same field, loss is 525 watts. Estimate how much of loss at 400 rpm is due to hysteresis.
Let $W1$ be hysteresis loss and $W2$ be eddy current loss at 400 rpm
Then $W1 + W2 = 300$
At 600 rpm, $W1 \cdot 600/400 + W2 \cdot [600/400]^2 = 525$

$W1 \cdot 400/600 + W2 = 525 \cdot [400/600]^2 = 233$

$W1 \cdot 1/3 = 300 - 233$

$W1 = 200$ and $W2 = 100$ watts

A new core is made of stampings 1.5 times the thickness, other dimensions unchanged.
Estimate the iron loss in a flux density 20% higher and at 500 rpm.

New hysteresis loss = $200 \cdot (120/100)^{1.6} \cdot (500/400) = 335$ watts

New eddy current loss = $100 \cdot (120/100)^2 \cdot (500/400)^2 \cdot (1.5)^2 = 506$

Total loss = 840 watts

Example
At flux density B, the iron loss for Stalloy sheet 0.014" thick
at 50 Hz is 0.89 watts/lb and at 100 Hz is 2.17 watts/lb
Estimate the hysteresis loss at 50 hz and the total loss at 100 hz of 0.02" sheet
Let $W1$ be hysteresis loss and $W2$ be the eddy current loss at 50 hz

$W1 + W2 = 0.89$ watts/lb

$W1 \cdot (100/50) + W2 \cdot (100/50)^2 = 2.17$ watts/lb

$W2 = (2.17 - 2 \cdot 0.89)/2 = 0.195$ and $W1 = 0.89 - 0.195 = 0.695$

New hysteresis loss = $0.695 \cdot (100/50) = 1.39$ watts/lb
New eddy current loss = $0.195 \cdot (100/50)^2 \cdot (0.02/0.014)^2 = 1.59$ watts/lb
Total loss = 2.98 watts/lb

DC MOTORS AND GENERATORS

DC machines

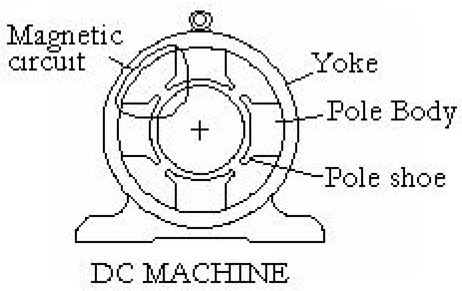


Figure 92; DC Machine

DC Machines are usually built with the field magnetic circuit in the stator, through the Yoke, Pole Body and Pole Shoe. Field windings are round the pole body.
The rotor is made up of stampings, ie laminations of low hysteresis steel.
Dynamo sheet steel is often used.
The armature windings are on the rotor and are put in slots on the laminated core and held in place by strips of insulating material.

Figure 93; Rotor slot

Each slot usually contains two layers, half a coil in each layer. The coil may be anything from a single conductor to a multitude of turns The whole assembly is then impregnated with varnish to remove air and prevent any movement.

The rotor windings are connected to the commutator.

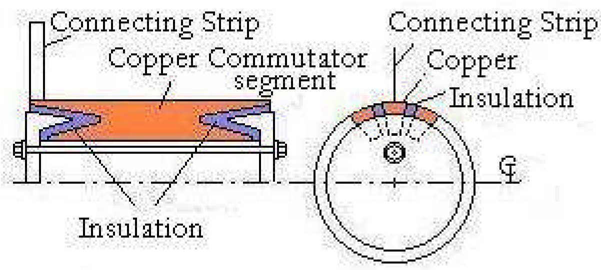


Figure 94; Commutator

The commutator is made with copper strips that have a wedge shaped cross section. These are stacked together with insulation between the strips. The assembly is clamped together onto the shaft between two discs each having a tapered flange. Each segment has a strip of copper or steel soldered in a slot in the segment to connect the segment to the windings.

The connection to the commutator is through the “brushes”.

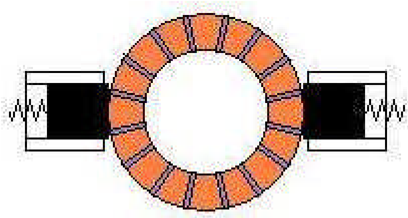


Figure 95; Brushes

The brushes are carbon or graphite blocks spring loaded to rub against the commutator.

Back emf distribution

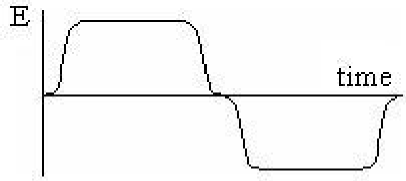


Figure 96; Back *EMF* in one conductor

As each conductor moves past the pole face, an emf E is induced in the conductor.

The armature contains many conductors in series spaced round the armature. The emf in each follows the same pattern. The total due to all the conductors in series is the mean value times the number of conductors in series.

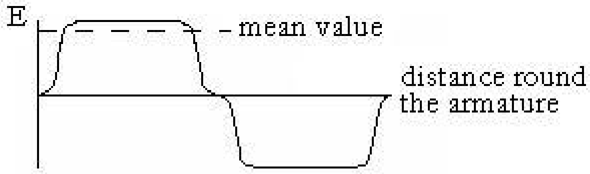


Figure 97; *EMF* in each conductor in series

Total back emf.

In one revolution, each conductor on the rotor cuts the magnetic flux of each pole once. Poles are always in pairs, a North and a South pole.
Let there be p pairs of poles each with a magnetic flux Φ weber.
Therefore each conductor cuts $2p\Phi$ of magnetic flux in one revolution.
Assume that the conductor is connected to the commutator so that the emf generated is the same polarity under a North pole as under a South pole.
In one second, each conductor cuts a flux of $2p\Phi \cdot$ (revolutions per second).
Emf generated in one conductor E = webers cut per second = $2p\Phi\ (rps)$
Let there be Z_S conductors connected in series
$E = 2p\ \Phi\ Z_S\ (rps)$ volts

where p is number of pairs of poles, Φ is flux per pole, Zs is number of conductors in series and (rps) is the speed in revolutions per second.

Output Coefficient

Output coefficient = $kW\ /\ D^2L \cdot (rpm)$
where D is the armature diameter and L is the armature Length and (rpm) is the speed
Output = $E\ I$ watts $\propto B\ \ \pi DL \cdot Z_S \cdot I \cdot (rpm)$
where Z_S is the number of conductors in series.
Output coefficient = $kW\ /\ D^2L \cdot (rpm)$

$$\propto [B \cdot \pi DL \cdot Z_S \cdot I \cdot (rpm)]\ /\ [D^2L \cdot (rpm)]$$
$$\propto B \cdot Z_S \cdot (I/D)$$

Output coefficient $\propto$ (flux density) $\cdot$ (ampere wires/cm)

Power in a DC machine

Power = (back emf) $\cdot$ (Armature Current)
Power = $E\ I_a$ watts
$= 2p\ \Phi\ Z_S\ I_a\ (rps)$ watts

Torque in a DC machine

But Power in watts = (Torque in newton metres) $\cdot$ $(2\pi\ rps)$
Torque = $2p\ \Phi\ Z_S\ I_a\ /(2\pi\ )$ newton metres
$= E\ I_a/[2\pi \cdot (\mathrm{rps}\ )]$ newton metres
Torque = $0.117 \cdot 2p\ \Phi\ Z_S\ I_a$ lb ft
$= 0.117\ E\ I_a\ /\ (rps)$ lb ft

DC Motor Shunt Connected

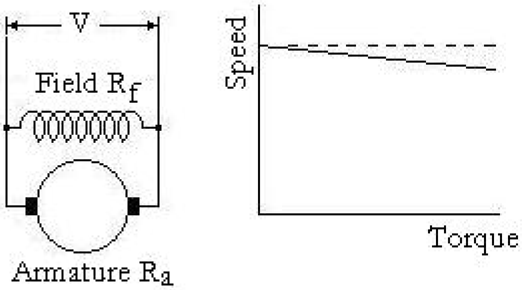


Figure 98; DC Shunt motor

DC Shunt Motors have a field winding of many turns switched directly to the supply.
The flux Φ is the flux due to the field current I_f where $I_f = V/R_f$.
V/R_f is constant, hence the flux Φ is constant.

In the armature circuit $V = E + I_a R_a$

hence the armature current is given by $I_a = (V - E) / R_a$

Substitute for E,

$I_a = (V - 2p\, \Phi\, Z_s\, n) / R_a$ where n is the speed in rps

$T = 2p\, \Phi\, Z_s\, I_a / (2\pi)$ where T is the Torque in newton metres

Thus (Output torque + torque due to losses) is proportional to I_a and Φ

Substitute for I_a,

$T = 2p\, \Phi\, Z_s\, (V - 2p\, \Phi\, Z_s\, n) / (2\pi\, R_a)$

$[(2\pi\, R_a) / (2p\, \Phi\, Z_s)]T = V - 2p\, \Phi\, Z_s\, n$

$n = [V / (2p\, \Phi\, Z_s)] - [(2\pi\, R_a) / (2p\, \Phi\, Z_s)^2]\, T$

$n = n_0 - \mathrm{m}\, T$

where n is the speed in *rps* and T is the input torque in newton metres
ie Torque = output torque + torque loss due to eddy currents, hysteresis, bearing friction, windage and brush friction.

and $n_0 = [V / (2p\, \Phi\, Z_s)]$ is the speed in *rps* when $T = 0$

and $\mathrm{m} = [(2\pi\, R_a) / (2p\, \Phi\, Z_s)^2]$ is the gradient of the n - T curve

R_a is small, hence DC shunt motors run at nearly constant speed whatever the torque. They are used where a constant speed is required, eg to drive rolling mills, pit winding gear, machine tools etc.

DC Motor Series Connected

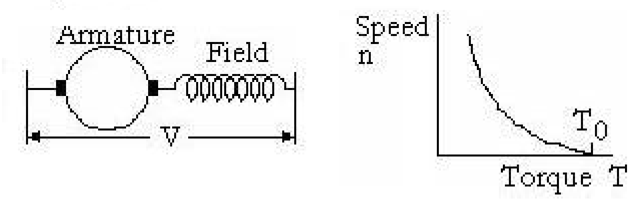


Figure 99; DC Series motor

The DC series motor has a high current field winding in series with the armature.
Let the total resistance of armature and field be R ohms and the current be I amps.

Applied voltage $V = \mathrm{E} + IR$

$E = 2p\, \Phi\, Z_s\, (rps)$

$\Phi = 4\pi\, N\, I / \Sigma[L / \mu A]$

where $\Sigma[L/\mu A]$ is the sum of the several parts of the magnetic circuit
Put $4\pi N / \Sigma[L/\mu A] = K$

$\Phi = K I$ and therefore $E = 2p K I Z_s\ (rps)$

But $V - E = I R$ and $T 2\pi\ (rps) = E I$

$V - I R = E = 2p K I Z_s\ (rps)$

$V = I [R + 2p K Z_s\ (rps)]$

$I = V / [R + 2p K Z_s\ (rps)]$

$E = 2p K Z_s\ (rps)\ V / [R + 2p K Z_s\ (rps)]$

$T 2\pi\ (rps) = E I = 2p K Z_s\ (rps)\ V^2 / [R + 2p K Z_s\ (rps)]^2$

$T = [2p K Z_s V^2 / 2\pi] / [R + 2p K Z_s\ (rps)]^2$

$T = P / [1 + Q n]^2$

Where P and Q are constants, $P = 2p K Z_s V^2 / (2\pi R^2)$ and $Q = 2p K Z_s / R^2$
T is the input torque in newton metres and n is rps

$T [1 + Q n]^2 = P$

This is a hyperbola of $[1 + Q n]^2$ against T

The starting torque ($n = 0$) is $T = 2p K Z_s V^2 / (2\pi R^2)$

On light load, (ie T nearly zero) n is very high.

DC Series Motors have a high torque at low speed which makes them suitable for traction motors or starter motors for petrol and diesel engines. They are not suitable for applications where they may be run without load as they may overspeed.

DC Compound Connected
A combination of Series and Shunt fields can give an alternative Speed/Torque relationship. For example, a shunt motor with a few turns on the field series connected in opposition to the shunt field can give a truly constant speed whatever the torque.

Armature Reaction

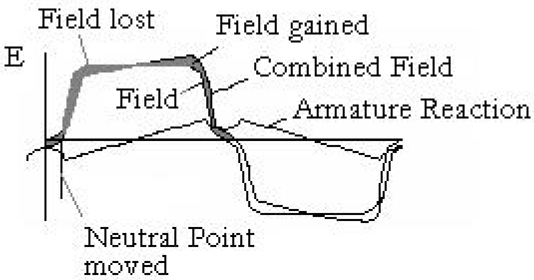


Figure 100; Armature Reaction

The current in the armature of a DC machine causes a magnetising force in a direction between the pole faces. This increases the magnetising force on part of the pole and reduces the magnetising force on the other part by the same amount. Due to saturation of the field magnetic circuit, the increase in flux in part of the pole is less than the decrease in the other part of the pole. Thus the effect of armature reaction is to give an overall reduction in flux. The magnetic field across the pole face is distorted and the neutral point is moved.

The effect of armature reaction can be reduced by;

i) Compensating windings on the stator connected in series with the armature. These are connected to give a field in opposition to the armature reaction. They can be installed on the pole face close to the armature conductors to almost completely eliminate armature reaction.

ii) A deep slot in the pole face that puts an air gap in the field due to armature reaction but not in the main field.

iii) Increased air gap on pole face.

Compensating Winding
Compensating windings are additional windings on the field that are connected in series with the armature and exactly oppose the armature reaction.

<u>Interpoles</u>

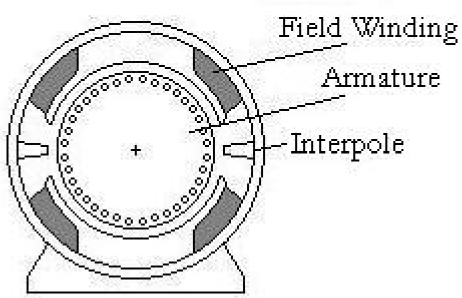


Figure 101; Interpoles

The current reversal is impeded by the self inductance of the winding. Lenz's law means that the change is opposed. Large DC Motors often have Interpoles. These are small poles, connected in series with the armature, sited between the main poles. Their purpose is to induce a voltage in the winding when the current reversal occurs. This voltage is arranged to oppose the self induced emf and assist the current reversal.

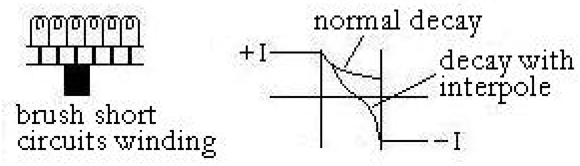


Figure 102; Current reversal in the Armature

The brush short circuits each coil as the commutator passes the brush. Without the interpole, the current decays exponentially. The interpole increases the decay and induces a current in the opposite direction. If L henries is the self inductance of the coil and T seconds is the time it is short circuited, then the interpole must induce an emf of $2IL/T$ volts.

For good commutation, the interpole winding is connected in series with the armature and has a large air gap to reduce the effect of saturation.

The self inductance of the coils can be reduced at the design stage by more commutator bars and more slots and fewer turns. Higher resistance brushes help as do wider brushes which increase the overlap. Without interpoles, moving the brush position so the coil picks up some of the main field can reduce sparking but a different position is needed for a different load.

If the voltage induced by the interpoles is too high, moving the brush position will reduce it. Altering the interpole air gap by packing behind the pole or by machining the face will increase or reduce the induced voltage
The effect of an interpole is not dependent on speed or load or on whether the machine is acting as a motor or generator.

The interpole windings oppose armature reaction, but the interpole introduces a magnetic path for the armature reaction mmf. Thus the interpole does not eliminate armature reaction and may even increase it.

Example
A DC Generator has coils with estimated self inductance L = 8E–6 henries and resistance 0.001 ohms. From the speed of rotation and brush and commutator dimensions, it is calculated that the coil is short circuited for 0.001 secs. Armature current = 180 amps.

$I = 180\ \mathrm{e}^{-R/L\ t} = 180\ \mathrm{e}^{-(1/8)} = 0.882 \cdot 180 = 159$

Average volts required = $L\ \mathrm{d}i/\mathrm{d}t$ = 8E–6 · 2 · 180/ 0.001 = 2.88 volts
Introduce 2.88 volts by interpoles, then $-L\mathrm{d}i/\mathrm{d}t - e = i\mathrm{R}$
$-$ 8E–6 $\mathrm{d}i/\mathrm{d}t - 2.88 = 0.001\ i$
Solving, $i = 180$ at $t = 0$ and $i = -172$ at $t = 0.001$

Example
A 4 pole lap wound generator has 516 conductors and the length of the interpole air gap is 0.4 cms. The maximum flux density under the interpole is to be 0.2 tesla when the armature current is 30 amps. Estimate the turns on each interpole.

Number of armature paths = number of poles = 4
Number of conductors under one main pole is 516/4 = 129

Current in each armature conductor = 30/4 = 7.5

$$\int H\,\mathrm{d}l = (2000 \cdot 0.4) \cdot 2 \text{ poles } + 0$$
$$= (4\pi/10)\ \{ N \cdot 30 \cdot 2 - (516/4) \cdot (30/4)\}$$
$$60\,N = 1273 + 968$$
$$N = 37$$

ie 16 turns to combat armature reaction + 21 turns to give required mmf

Example
6 pole 200 kW 500 volt generator 550 rpm, armature winding 75 cms diameter, active length 20 cms, simple lap winding of full pitch and 880 conductors, 2 turns/commutator bar (ie 220 segments). Calculated reactance volts at full load = 4.5 volts. Find mean flux density under an interpole at full load (assume uniform). Interpole air gap is 0.5 cms. Find turns required on each interpole.

Output amps = 200 · 1000/500 = 400 amps
4.5 volts is to be generated in 2 · (4 conductors of short circuit coil)
But emf = (short circuit turns) · (swept area/sec) · (flux density)

$$E = 4 \cdot (\pi\, D \cdot L \cdot rps) \cdot \mathrm{B}$$
$$= 4 \cdot (\pi\, 75 \cdot 20 \cdot 550/60)/10^4 \cdot B$$
$$B = 0.26 \text{ tesla}$$

$$\int H\,\mathrm{d}l = 2600 \cdot 0.5 \cdot 2$$
$$= (4\pi/10)\ [\text{amps} \cdot N \cdot 2 - (\text{number of conductors/pole}) \cdot (\text{amps/path})]$$

$= (4\pi/10)\ [\,400 \cdot N \cdot 2 - (880/6) \cdot (400/6)]$

$800N = 880 \cdot 400/36 + 2600 \cdot (10/4\pi)$

$N = 12.2 + 2.6 = 15$ turns on each interpole

(12.2 turns to balance armature reaction + 2.6 turns to give voltage)

Armature Windings

Armature windings can be Lap Wound or Wave Wound.

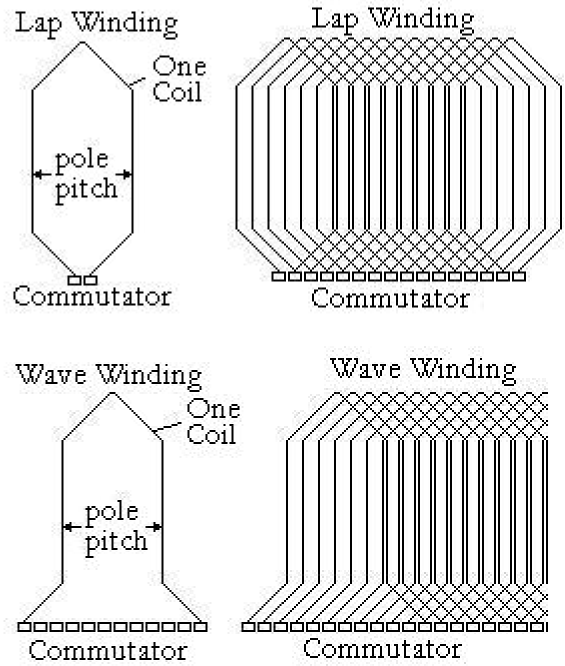


Figure 103; Lap and Wave Windings

With a Lap winding,

Z_s = (total number of conductors) / (number of poles)

With a Wave winding, Z_s = (total number of conductors) / 2

Other typs of winding

i) Gramme Ring

The armature for this type of winding is a tube with spokes to the shaft. The winding is wound round and through the tube by hand secured in shallow slots. One end of the winding is bare and the brushes bear directly on the winding.

ii) Singly re-entrant winding

iii) Doubly re-entrant

iv) Duplex winding

Example

8 pole dynamo has $\Phi = 0.04$ weber and is to generate 220 volt at 250 rpm

Find a suitable number of conductors for a wave winding

Emf equation $E = (2p)\ \Phi\ Z_S \cdot (\text{rps})$

$$220 = 8 \cdot 0.04 \cdot Z_S \cdot 250/60$$

$$Z_S = 165$$

Wave winding, therefore number of conductors $= 2 \cdot Z_S = 330$ conductors

If the coil sides are numbered as usual and pitches (measured in coil sides) are a_1 and a_2 , both a_1 and a_2 must be odd numbers and $(a_1 + a_2)$ must be even.
Hence with 8 poles, $4 (a_1 + a_2) = Z_S \pm 2$

Z_S = (a multiple of 8) ± 2

Nearest value for $Z_S = 170$

Hence total number of conductors = 340

Bearings

The bearings for small motors are in the end covers. Large motors usually have separate pedestal bearings. The motor end covers are then in two halves bolted together. This allows the windings to be inspected without disturbing the rotor.

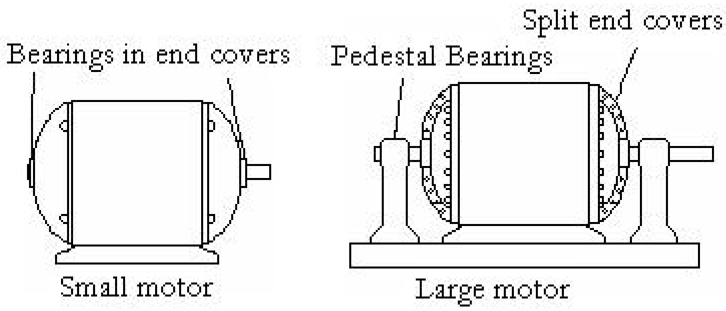


Figure 104; Bearings

Starter for a DC motor

A DC motor will take a very large current if switched directly onto the supply. A series resistance is nearly always required to limit the current until the motor is spinning fast enough to generate back emf.

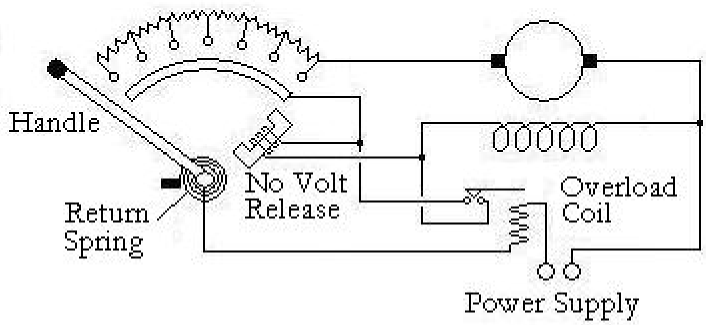


Figure 105; DC Motor Starter

The diagram shows a typical spring loaded starter. The handle is moved manually against the spring over contacts sequentially cutting out the resistance. Finally when all resistance has been cut out, the handle comes up against a solenoid carrying field current. If the supply to the field fails, the handle is released and swings back to switch off the armature current.

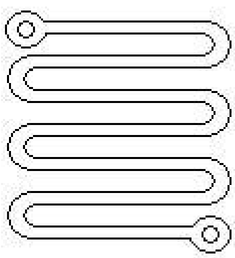


Figure 106; Cast iron Resistor

The resistors typically consist of a iron castings of a zig-zag shape as shown. These are assembled into a stack inside a ventilated metal enclosure.

The maximum current during starting is typically 150% full load current but for large motors may be as low as 110%. The starting resistors are only in use for a short time during starting and only need to be rated for this short time. Repeated attempts to start can overheat and damage the resistors.

Example
Find the number and value of starting resistors for a DC shunt motor rated for 440 volts 100 amps full load. The armature resistance is 0.14 ohms and the current is to be limited to 150 amps during starting.
The initial total resistance of the armature circuit is 440/150 = 2.93 ohms
When the current falls to 100 amps, the volt drop across the resistance of the armature circuit falls to $100 \cdot 2.93 = 293$ volts
As the next stage is switched in, the back emf remains the same so the voltage applied to the armature resistance remains at 293 volts
Total resistance at the next stage = 293 volts / 150 amps = 1.95 ohms.
When the current falls to 100 amps, the *IR* drop falls to 195 volts
Total resistance at next stage = 195 / 150 = 1.30 ohms
Similarly, the total resistance of the following stages are 0.87 ohms, 0.58 ohms, 0.39 ohms, 0.26 ohms, 0.17 ohms and 0.11 ohms.

The armature resistance is 0.14 ohms, therefore no added resistance is needed for the last stage. Subtracting the armature resistance, the starter resistance at each stage becomes 2.79 ohms, 1.81 ohms, 1.16 ohms, 0.73 ohms, 0.44 ohms, 0.25 ohms, 0.12 ohms, 0.03 ohms and finally zero.

Figure 107; Example of Starting Resistors

R1 = 0.98	R2 = 0.65
R3 = 0.43	R4 = 0.29
R5 = 0.19	R6 = 0.13
R7 = 0 .09	R8 = 0.03

Speed control of DC shunt motors.
(i). Field control

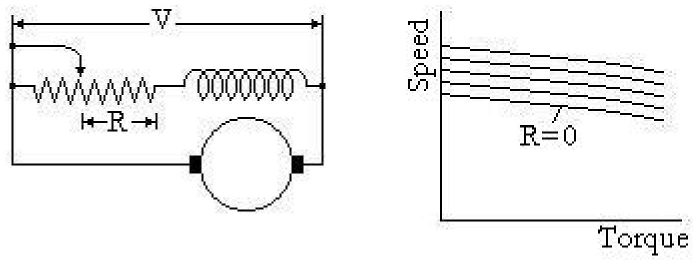


Figure 108: Speed Control for a DC Shunt motor

Speed control of a DC shunt motor is usually achieved by a variable resistance in the field circuit. The resistance increases the motor speed and cannot be used to give a low speed.

(ii). Armature Resistance control

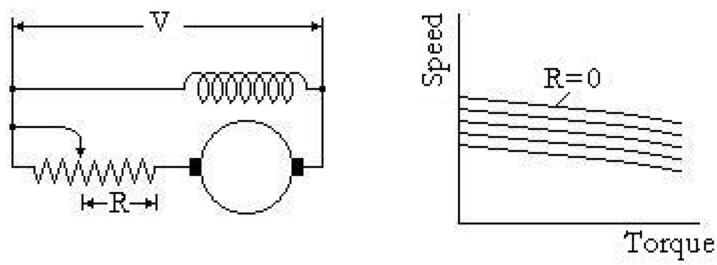


Figure 109: Speed Control lower speed

A resistor in the armature circuit reduces the speed but is very wasteful of energy.

(iii). Armature shunt control

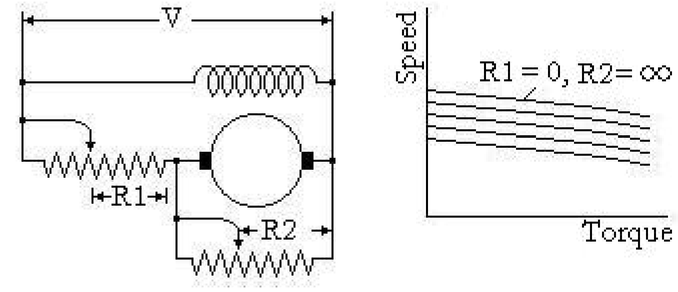


Figure 110: Speed Control lower speed alternative circuit

This is better and gives more stable speed control but is equally wasteful.

(iv). Ward Leonard set

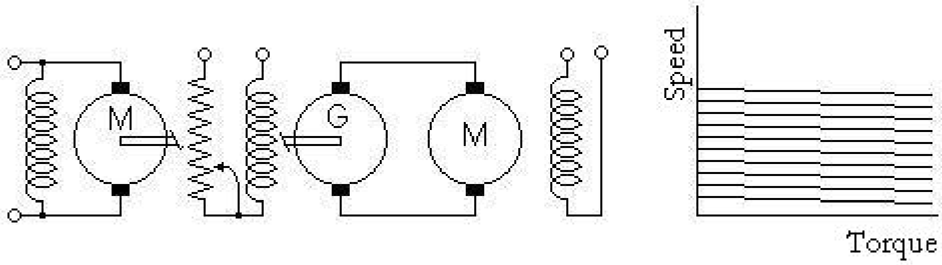


Figure 111: Ward Leonard wide range speed Control

The motor generator set rotates at constant speed but the generator excitation can be varied. The final motor can have a fixed field but the voltage to the armature is variable. Speed control over a 25:1 ratio is possible.

Ship propulsion systems often have a diesel electric drive. The diesel drives a generator directly connected to the DC electric propulsion motor. This avoids the need for a gearbox or mechanical reverse gear and allows the diesel to be installed in the most suitable place on the ship.

Rolling Mill motors are usually Ward Leonard sets. The generator has a very large flywheel which stores up energy till the ingot reaches the rollers. Thus very high power is available for the short period of time that the ingot passes through without taking a high intermittent power from the electricity supply. Furthermore a severe overload can be tripped by a circuit breaker before damage is done to the drive mechanism.

Speed control of a DC series motor

(i) Series resistor

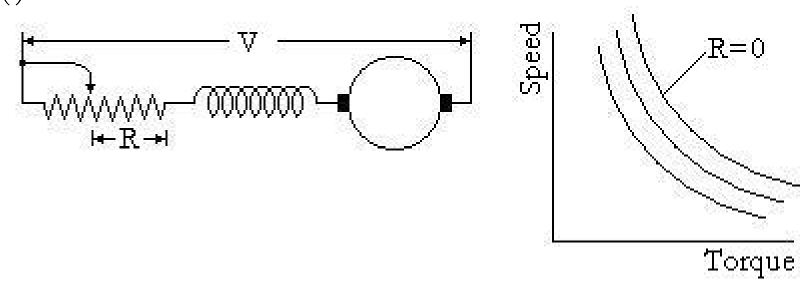


Figure 112; Speed control of a series motor

The speed of a DC series motor can be controlled by switching additional resistance in series. This in effect operates the motor on a reduced voltage.

(ii). Diverter resistor

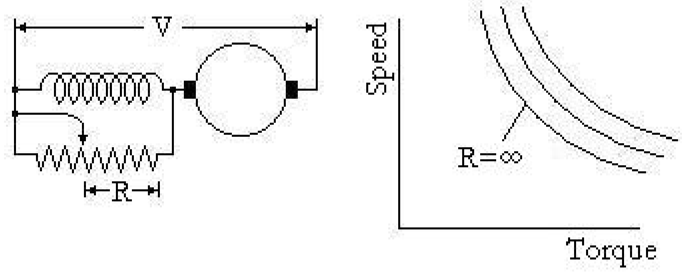


Figure 113; Speed control of a series motor by diverter resistor

A diverter resistor allows higher speeds but is wasteful of energy.

iii. Series / parallel
If two or more SC series motors are in use, then they can be switched between series and parallel connection giving two modes of operation.

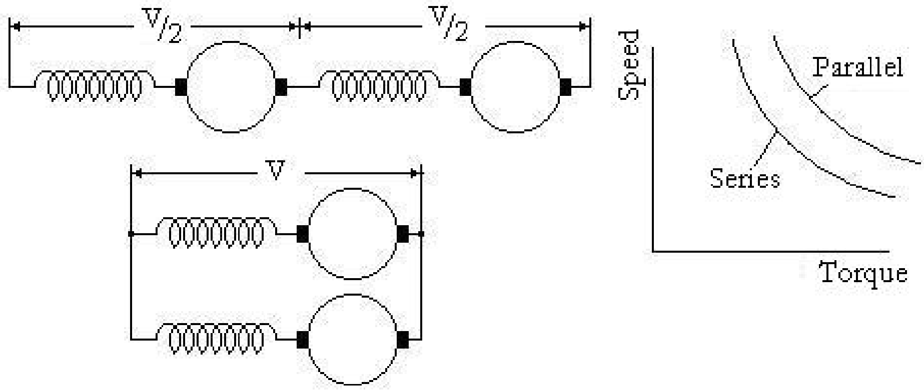


Figure 114; Speed control of a series motors by switching in series or parallel

Drum Controller

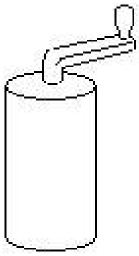


Figure 115; Drum Controller

Electric trams usually have series motors which are controlled by a drum type controller. As a handle on top of the drum is rotated, a wiper moves across the contacts cutting out the series resistance in steps giving speed control.

The controller usually incorporates a blow out coil. This coil carries the motor current giving a powerful magnetic field that acts on any arcing at the contacts drawing out the length of the arc and helping to extinguish it. Speed control resistances must be continuously rated and consume a lot of power when they carry armature current.

Forward/Reverse Control of a small series motor
Forward and reverse control of a conventional series or shunt motor requires a changeover switch to change both connections to the field in addition to the start/stop control.

If however the field winding is made twice the size with the centre connected to the armature, a single switch to either end starts the motor in forward or reverse. This is a popular arrangement for small motors such as the motor controlling the set point of a mechanical governor.

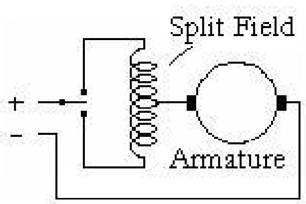


Figure 116; Forward and reverse by a three way switch

If the motor is adjusting a governor via a worm drive, there must be no end play on the motor shaft. If there is end play, the motor armature acts like a solenoid when power is applied and fine adjustment is lost.

DC Shunt shunt generator on no load

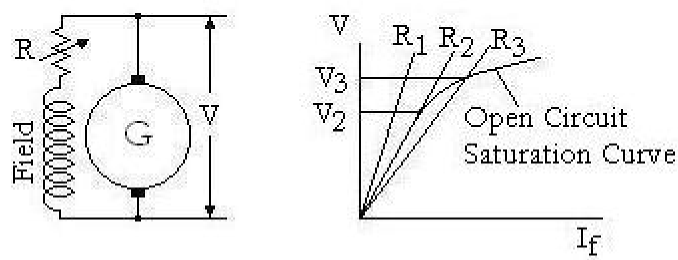


Figure 117; DC shunt generator

The output voltage of a DC shunt generator depends on the field strength. This depends on the output voltage and field resistance. If the field is supplied from another DC source, the voltage will follow the saturation curve for the magnetic circuit since V is proportional to the flux. However, the field is usually supplied by its own armature.

Thus if the field resistance is R_1 in the diagram, then the voltage will be only that due to the residual flux, ie nearly zero. If the resistance is reduced to R_2, the voltage will rise suddenly to V_2. Further reduction in field resistance to R_3 will cause the voltage to rise to V_3. Thus R_2 is the critical resistance, any resistance above this and the output voltage will be nearly zero.

DC Shunt generator on load

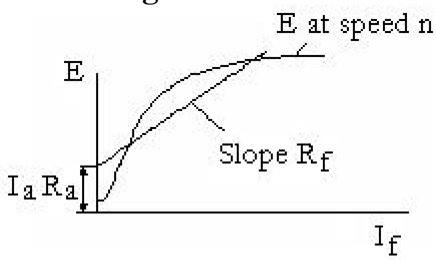


Figure 118; DC shunt generator on load

Neglecting armature reaction and brush drop
$V = E - I_a R_a \;\; = I_f R_f$
$E = I_f R_f + I_a R_a$
Thus for given I_a, the relation between E and I_f is linear.
A shunt generator will fail to excite if;

The field resistance is above the critical value
The speed is too low
There is no residual magnetism
The field connections or rotation are reversed
If the machine is run up with load resistance too low

DC Series generator

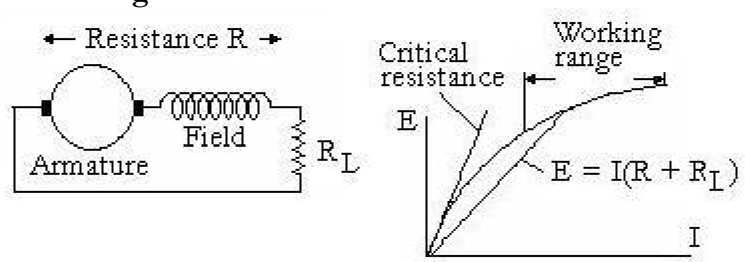


Figure 119; DC Series generator

If R is the resistance of the armature plus field and R_L is the load resistance
Then $E = I\,(R + R_L)$ This can be plotted by reducing R_L from above the critical value.

A series generator will not excite if;

The total resistance is above the critical value
The speed is too low
There is no residual magnetism
The field connections or rotation are reversed

DC Shunt Generators in parallel

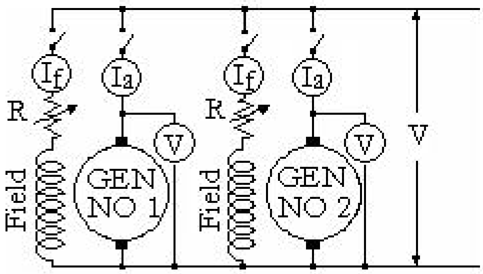


Figure 120; DC generators in parallel

Shunt generators will run in parallel.
With one generator on load, start the second and run up to rated speed. Close the field switch and adjust the voltage to exactly match the voltage on the running set.
Close the armature switch and increase the field current to put the set on load. Reduce the field on the first set as the field is increased on the second keeping the voltage at the correct value.
Repeat step by step till the armature currents are balanced.

The load can be balanced between the generators either by adjusting the field resistors or by adjusting the speed of the prime movers.

DC Series Generators in parallel
DC series generators will not run in parallel unless there is an equalising connection between the fields. The equalising connection must be of low resistance for stable operation.

DC Compound Generators in parallel

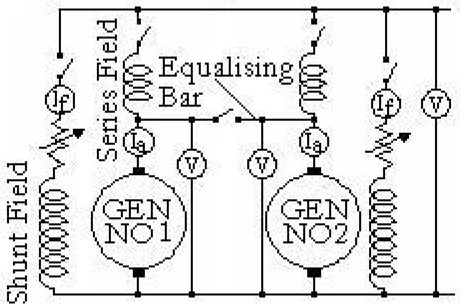


Figure 121; DC Compound generators in parallel

An equalising bar of lower resistance than the series field is required for stable operation. With No 1 machine on load, run up No 2 set to speed and adjust the voltage by the shunt regulator. Close the equalising bar switch and No 2 machine armature switch. Balance the loads by the shunt regulators keeping the voltage constant.

Example
The field of two DC shunt generators are adjusted to give a voltage of 520 volts on no load. The voltage of one generator on its own falls to 500 volts at 400 amps load. The voltage on the other generator falls to 490 volts on 400 amp load. Find the voltage with the generators in parallel and a total load of 500 amps.

At a current I_1 on machine 1, voltage $V_1 = 520 - 20 \cdot I_1/400$
At a current I_2 on machine 2, voltage $V_2 = 520 - 30 \cdot I_2/400$
When the machines are in parallel, $V_1 = V_2$
$520 - 20 \cdot I_1/400 = 520 - 30 \cdot I_2/400$
$20\ I_1 = 30 \cdot I_2$
$I_1 + I_2 = 500$ amps
$I_1 = 300$ amps and $I_2 = 200$ amps
$V = 520 - 20 \cdot I_1/400 = 520 - 20 \cdot 300/400 = 505$ volts

Motor and generator losses

1) Field copper loss is $I_f^2\ R_f$ which depends on field current but not the speed
2) Bearing friction loss is dependent on speed but not the load.
3) Brush friction loss is dependent on speed but not the load
4) Windage losses are dependent on the speed but not the load.
5) Eddy current losses are dependent on the magnetic flux and speed but not the load.
6) Hysteresis losses in the armature is dependent on the magnetic flux and speed but not the load.
7) The armature loss is $I_a^2\ R_a$ which depends on the load.

Swinburne's test to calculate the efficiency

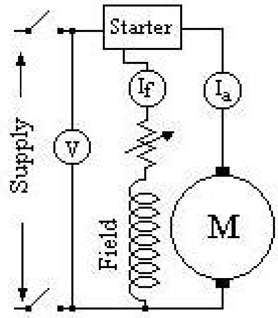


Figure 122; Swinburne's test

This test enables the efficiency of the motor to be calculated without the need to put the motor on load. For this test, the field is supplied separately through a variable resistance.

The armature and field resistances R_a and R_f are measured with the motor at rest.
The motor is started and run up to the design speed on no load.
The Voltage, I_f and I_a are measured.
On no load, the losses equal the input
The total input power, $V\ (I_a + I_f)$ watts.
The friction, windage and iron loss $= V\ I_a - I_a^2\ R_a$ watts
The field loss $= V\ I_f$ watts
The armature copper loss $= I_a^2\ R_a$ watts
Let W = the sum of the friction, windage, iron and field loss
$= V\ I_a - I_a^2\ R_a + V\ I_f$ watts

For a given speed, W is assumed to be constant as load is applied. (This is not exactly true since, at constant field, the speed falls slightly with load.)

At any other armature current I_a, the loss $= W + I_a^2\ R_a$ watts
The power input $= V\ (I_a + I_f)$ watts

The power output = power input – losses

$$= V\,(I_a + I_f) - W - I_a^2\,R_a \text{ watts}$$

Efficiency = (power output) / (power input)

Thus the efficiency can be plotted against power output for this speed.

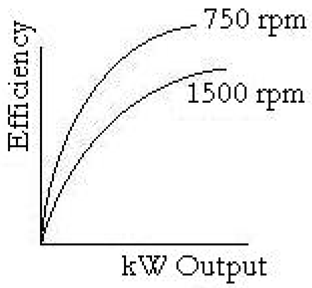


Figure 123; Efficiency curves

The no load test can be repeated at other speeds to obtain a family of curves of efficiency against power output at various speeds.

Swinburne's test allows the efficiency to be calculated without having to measure the mechanical output. The efficiency is plotted against output at given speeds.
It is not exact as the speed and field cannot both be kept constant as the load is applied but is accurate enough for most practical purposes.

Swinburne's test does not test the motor or generator for other possible faults eg inadequate cooling, inadequate mechanical strength.

Hopkinson-Kapp test

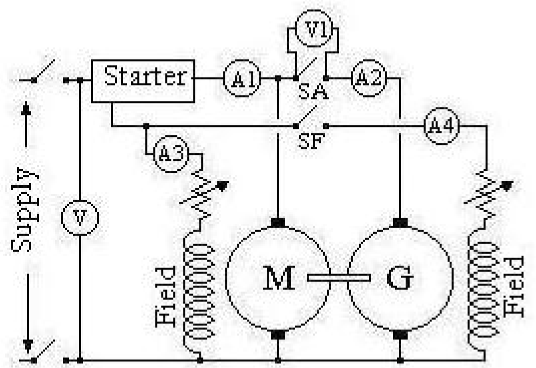


Figure 124; Hopkinson-Knapp test

When a batch of identical DC shunt motors are made, their performance can be found by the Hopkinson test. Two machines are coupled together on a common bedplate so that one machine drives the other which acts as a generator. The generator output is fed back into the motor so the only power taken from the supply is to cover the losses.

The armature resistance R_m and R_g of the motor and generator are measured at standstill. Choose a selection of speeds throughout the range. Start the motor through its starter and run up to each chosen speed with switches SA and SF open.

Tabulate A1 and A3 and $W1 = V \cdot A1$ for each speed.
$W1$ is the friction and windage loss of both machines plus the iron loss of the motor.

Close switch SF.
At each speed, adjust the generator field till $V1$ is zero
and tabulate $A1$, $A3$, $A4$ and $W2 = A1 \cdot V$.
W2 is the friction and windage loss of both machines plus the iron loss of both machines.
Hence at each speed the iron loss for one machine is $(W2 - W1)$ and the friction and windage loss of each machine is $\frac{1}{2}(2\,W1 - W2)$.

The brush friction loss of the generator can be found by measuring the power loss with the brushes in place and repeating the measurement with the generator brushes lifted but all other conditions identical.

Choose a selection of armature currents between zero and full load rating.
At each speed, adjust the generator field till $V1$ is zero and close switch SA.
Adjust the generator field to give the selected armature current.
Tabulate the speed, V, $A1$, $A2$, $A3$, $A4$, $V \cdot A3$ (the field loss of the motor), $VA \cdot 4$ (the field loss of the generator), $V \cdot A1$ (the total friction, windage, iron and armature loss of the two machines), $(A1 + A2)^2\, R_m$ (the armature copper loss of the motor) and $(A2)^2\, R_g$ (the armature copper loss of the generator).
Let $W3$ be any loss unaccounted for (two machines)
$W3$ = total – friction & windage loss (2 machines) − iron loss (2 machines)
− copper loss motor − copper loss generator
$$W3 = V \cdot A1 - (2\,W1 - W2) - 2(W2 - W1) - (A1 + A2)^2\, R_m - (A2)^2\, R_g$$

Motor input = $V \cdot (A1 + A2 + A3)$
Let total motor loss be $W4$
= field loss + friction & windage loss + iron loss + motor copper loss + ½ $W3$
$W4 = (V \cdot A3) + ½\,(2W1 - W2) + (W2 - W1) + (A1 + A2)^2 R_m + ½\,W3$

Motor output = motor input – motor loss = $V \cdot (A1 + A2 + A3) - W4$
Efficiency = motor output / motor input

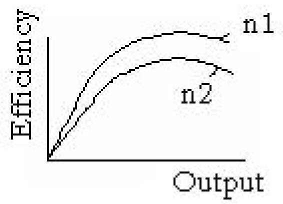


Figure 125; Efficiency curves

A family of curves of efficiency, input and each of the losses can be drawn against output for each speed.

The motor and generator operate up to full load while taking a fraction of this power from the supply. Thus the machines can be tested and run for extended periods on full load in a location where the supply is inadequate to provide full load power, eg to measure the rise in the winding and bearing temperatures.

Decceleration tests
In this test, the weight and dimensions of a heavy flywheel are measured and the moment of inertia is calculated. The flywheel is fitted on the output shaft of a motor and the motor run up to speed. The motor is switched off and the speed is plotted against time as the motor slows down.
The test is repeated with a different flywheel.

At a given speed, the slope $d\omega/dt$ of each curve is measured.
(flywheel inertia + motor inertia) · $(- d\omega/dt)$ = torque due to losses

$(I_1 + \text{motor inertia}) \cdot (- d\omega_1/dt) = (I_2 + \text{motor inertia}) \cdot (- d\omega_2/dt)$
motor inertia = $[I_1 \cdot (d\omega_1/dt) - I_2 \cdot (d\omega_2/dt)] \,/\, [(d\omega_1/dt) - (d\omega_2/dt)]$

Loss in watts = (Torque in newton metres) · 2π · rps
Hence the friction and windage loss can be plotted against speed.
The test is repeated with the field energised during decceleration. The additional loss is the iron loss due to eddy currents and hysteresis.

Example
A torque of 2 ft lb will just keep the armature of a motor turning. With the field fully excited, the power required to keep the motor spinning at 600 rpm is 250 watts. The time taken to stop from 600 rpm is 30 sec with the field fully excited and armature open circuited.
Show the moment of inertia of armature is 38 lb ft^2

At 600 rpm, power = 250 watts
Torque = power / speed = $(250 \cdot 550) / (746 \cdot 20\,\pi) = 2.93$ ft lb
$T = \alpha + \beta\omega$ where $\alpha = 2$ ft lb and $\beta = 0.93/(20\,\pi)$

$K\,\mathrm{d}\omega/\mathrm{d}t \;=\; -\,g\,(\alpha + \beta\omega)$

$\int[\,K\,/\,(\alpha + \beta\omega)]\;\mathrm{d}\omega$ from 20 π to 0 = $-\,g\int \mathrm{d}t$ from 0 to t
$K\,/\,\beta$ [ln($\alpha + \beta\omega$)] from 20 π to 0 = $-\,g\,t$
$K = \beta\,g\,t\,/\,[\,\ln\,\{\,(\alpha + 20\,\pi\,\beta)\,/\,\alpha\,\}] = 37.5$ lb ft^2

Brush Drop
There is a voltage drop across the brushes, which is slightly different at the positive and negative brushes.

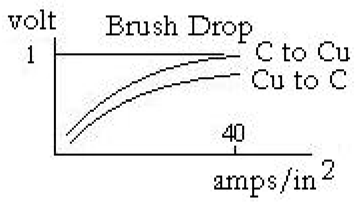


Figure 126; Brush drop

At current densities of 40 amps/sq in or more, the Carbon to Copper drop is about one volt and the Copper to Carbon drop is a little less.   At lower current density, the drop is lower, almost zero at a small current. The drop depends on the type of brush, the state of the commutator and on the brush pressure and to a less extent on speed.

Brush position
The voltage of a DC generator follows a sine wave as the brush angle in moved through the position for the maximum. Thus the exact setting is difficult to find due to the flat response near the maximum. If however an AC voltage is applied to the field, the AC output at the brushes passes through zero as the brush angle is changed.   This allows the neutral position to be set more accurately. In practice, rather than AC, a DC current is used and the kick of a voltmeter noted when the current is switched off. This is called the "kick test". The best position for the brushes may be to one side of this neutral position due to self inductance of the armature which delays the current reversal.

Example
A 4 pole 75 kW, 525 volt, 750 rpm DC Generator is to be designed.
Flux per pole 0.0422 weber.

Armature OD 43.5 cms, ID 16.5 cms, length 22 cms and 3 vent ducts 0.75 cms
Net length of iron = [22 – 3 · 0.75] · 0.89 packing factor = 17.4 cms
Magnetic circuit

Part	Material	Length	Section	Flux	B (tesla)	H	HL
Core	Dynamo sh steel	20 cms	165 cm^2	0.0211	1.28	10	200
2 teeth	Dynamo sh steel	2 nos each 4	102.5	0.0211	2.06	280	2240
Air gaps	Air	2 nos each 0.25	256	0.0211	0.825	8250	4125
2 poles	W.I.	52	157	0.0253	1.61	40	2080
Yoke	Cast steel	84	192	0.0253	1.32	12	1008
						Total	9653

Ampere Turns/pole = (10/4π) · 9653 / 2 = 3840 on each pole

Equalising Connections

The air gaps at the pole faces may change with time, eg due to wear in the bearings. A Lap Wound machine has the same number of brushes as poles. All brushes of the same polarity are connected together on the stator. If the air gaps of the poles are not all the same, then some brushes will carry more current than others. This can be avoided if the commutator segments at the same voltage on the rotor are connected together.

Example

A 6 pole dynamo has a field circuit resistance of 120 ohms and there are 2000 turns/pole. For currents less than 2 amps, the flux/pole is nearly proportional to field current at 0.02 weber/amp. A constant pd of 480 volts is applied, find how long the current takes to reach 2 amps and the energy then stored in the field.

When the current is i amps, $\Phi = 0.02$ i weber/pole

back emf due to self inductance of 6 coils

$$L\,\mathrm{d}i/\mathrm{d}t = 6\ N \cdot \mathrm{d}\Phi/\mathrm{d}t \text{ volts}$$

$$= 6 \cdot 2000 \cdot 0.02\ \mathrm{d}i/\mathrm{d}t = 240\ \mathrm{d}i/\mathrm{d}t \text{ volts}$$

$$V = L\,\mathrm{d}i/\mathrm{d}t + R\,i$$

Hence $480 = 240\ \mathrm{d}i/\mathrm{d}t + 120\,i$

$$4 = 2\ \mathrm{d}i/\mathrm{d}t + i$$

$$4\,e^{0.5t} = 2\,e^{0.5t}\ \mathrm{d}i/\mathrm{d}t + e^{0.5t}\ i = 2\ \mathrm{d}(i\,e^{0.5t})/\mathrm{d}t$$

Integrate $8\,e^{0.5t} = 2\,(i\,e^{0.5t}) + \text{constant}$

$4 = i + K\,e^{-0.5t}$

$i = 0$ when $t = 0$, therefore $K = 4$

$i = 4 \cdot (1 - e^{-0.5t})$

when $i = 2$, $t = 1.38$ secs

Example

A 55 kW 4 pole shunt machine is 440 volt, 125 amp full load I_a, 600 rpm.
$E - I_f$ saturation curve, linear from origin to E=150 at $I_f = 0.3$
and linear from E=400 at $I_f = 1.5$ to E=480 at $I_f = 3.5$
$R_a = 0.14$ ohms Brush drop = 2 volt except at small loads when = 0
Shunt turns = 2000 turns/pole. Neglect armature reaction.
i) Find R_f to give $E = 440$ volts at no load and 600 rpm

Brush drop and $I_a R_a$ are negligible, hence from saturation curve, $I_f = 2.5$ amps
Therefore $R_f = V/I_f = 440/2.5 = 176$ ohms

ii) Find the output voltage as a generator on full load at $R_f = 176$ ohms.
Resistance drop in armature = $I_a R_a = 125 \cdot 0.14 = 17.5$ volts
Brush drop = 2 volts Total drop = 19.5 in armature circuit.
$E = V + 19.5 = 176\,I_f + 19.5$
But from the saturation curve, $E = 400 + (480 - 400) \cdot (I_f - 1.5)/(3.5 - 1.5)$
$176\,I_f + 19.5 = E = 400 + 40\,I_f - 60$
$I_f = 320.5/136 = 2.36$
$V = I_f R_f = 2.36 \cdot 176 = 415$ volts

iii) Find R_f to generate 440 volts on full load
$E = 440 + 19.5 = 459.5$ volts
From saturation curve, $I_f = (459.5 - 340)/40 = 2.99$ amps
$I_f R_f = 440$ volts therefore $R_f = 440/2.99 = 147$ ohms a reduction of 29 ohms.

iiv) With $R_f = 176$ ohms, find the speed to generate $V = 440$ volts on full load.
$E = 440 + 19.5 = 459.5$ volts
$I_f = 440/176 = 2.5$
From the saturation curve, $I_f = 2.5$ gives $E = 440$ volts at 600 rpm
Speed to give $E = 459.5$, $N = 600 \cdot 459.5/440 = 627$ rpm

v) Find the minimum speed for self excitation at $R_f = 176$ ohms
The initial slope of the saturation curve at 600 rpm is $150/0.3 = 500$ ohms
Minimum speed for self excitation = $600 \cdot 176/500 = 211$ rpm

vi) A series coil is wound on each pole,
total resistance of all series coils = 0.1 ohm
Find the turns/pole for the machine to generate $V = 440$ at no load and full load.
On no load, from (i) above, $R_f = 176$ ohms.
On full load, $E = 440 + 125 \cdot (0.1 + 0.14) + 2 = 472$ volts
Hence from saturation curve at 600 rpm,

amp turns = $2000 \cdot (1.5 + 2 \cdot 72/80) = 6600$
shunt field amp turns = $2000 \cdot V/R_f = 2000 \cdot 440/176 = 5000$
series field amp turns = $6600 - 5000 = 1600$
series turns/pole = $1600/I_a = 1600/125 = 12.8$

This must be an integer, therefore series turns/pole = 13

vii) Find series turns/pole to give 450 volts on full load, all other values as (vi)
Full load $E = 450 + 125 \cdot (0.1 + 0.14) + 2 = 482$ volts
amp turns = $2000 \cdot (1.5 + 2 \cdot 82/80) = 7100$
shunt amp turns = $2000 \cdot 450/176 = 5114$
series amp turns/pole = $7100 - 5114 = 1986$
series turns/pole = $1986/125 = 15.9$
nearest integer = 16 turns/pole

viii) The machine is run as a motor. Find the speed on 440 volts with armature load 125 amps and $R_f = 176$ ohms.
$E = 440 - 125 \cdot 0.14 - 2 = 420.5$ volts
$I_f = 440/176 = 2.5$ therefore $E = 440$ volts at 600 rpm
Speed = $(420.5/440) \cdot 600 = 573$ rpm

ix) What value of R_f is required for the machine to run as a motor on full load at 600 rpm on 440 volt supply.
$E = 420.5$ volts as (viii)
From the saturation curve, $I_f = 1.5 + (20.5/40) \cdot 1 = 2.01$ amps
$R_f = 440/20.1 = 219$ ohms

x) Find R_f to run as a motor on full load at 550 rpm on 440 volt supply
As before $E = 420.5$ volts
The field gives $E = 420.5$ volt at 550 rpm.
Therefore field would give $420.5 \cdot 600/550$ at 600 rpm = 459 volts
From saturation curve, for $E = 459$ volts, $I_f = 1.5 + 2 \cdot 59/80 = 2.98$
$R_f = 440/2.98 = 148$ ohms

xi) Find series turns/pole for the machine to run as a motor at 600 rpm on no load and on full load. Assume series winding is 0.04 ohms.
On no load, $R_f = 176$ ohms as before
On full load, $E = 440 - 125 \cdot (0.14 + 0.04) - 2 = 415.5$ volts
From saturation curve, $I_f = 1.5 + (15.5/40) \cdot 1 = 1.89$ amps
Amp turns/pole = $2000 \cdot 1.89 = 3780$
Shunt field provides $2000 \cdot 440/176 = 5000$ amp turns
Series field must provide $5000 - 3780 = 1220$ amp turns
Series field carries 125 amps therefore $1220/125 = 9.8$ turns
Number of turns must be an integer, therefore 10 series turns/pole
The series field is in opposition to the shunt field

xii) Find the number of turns for the machine to run as a motor at 550 rpm on full load, otherwise as (xi).
$E = 415.5$ volts as above. This is obtained at 550 rpm
E at 600 rpm would be $415.5 \cdot 600/550 = 453$ volts
From saturation curve, $I_f = 1.5 + (53/80) \cdot 2 = 2.83$ amps
Amp turns required = $2000 \cdot 2.83 = 5650$
Shunt field provides 5000 amp turns as (xi)
Series field is to provide 650 amp turns with current 125 amps
Series turns = $650/125 = 5$ turns to nearest integer
These series amp turns are cumulative to the shunt amp turns.

AC SYNCHRONOUS MACHINES

AC Generators

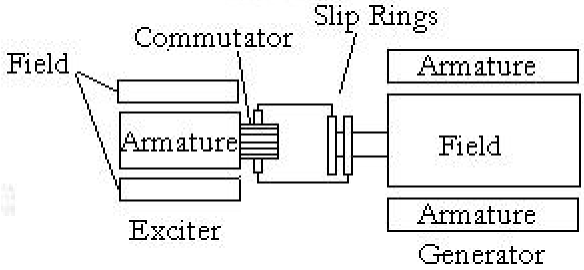


Figure 127; Exciter and Generator

Most AC generators generate three phase EMF on the stator. The generator field is usually on the rotor supplied through slip rings from the exciter. The exciter is a DC generator and it supplies the generator field through its commutator. There are voltage control circuits on the exciter field or the generator field. Large generators have a pilot exciter to supply the main exciter field. The exciter and pilot exciter may be on the same shaft as the generator or may be separately driven.

Brushless generator.

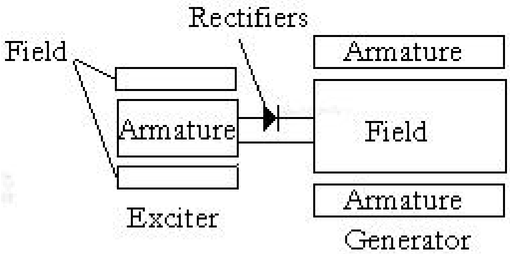


Figure 128; Brushless Generator

A brushless generator does not have the commutator or slip rings. The exciter is a multi phase AC generator on the same shaft as the main generator field. The AC output from the exciter is rectified by high power silicon diodes to supply the DC field current for the generator. The exciter armature is typically six phase AC with six silicon diodes mounted on the rotor..

Generator speed

The generator speed is determined by the supply frequency and the number of magnetic poles. A two pole machine on a 50 Hz system rotates at 3000 rpm (50 cycles/sec times 60 sec/minute). A 4 pole machine rotates at 1500 rpm to generate at 50 Hz..

***EMF* of an AC Generator**

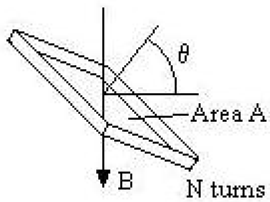


Figure 129; Flux linking a rotating single coil

Consider the Flux through a coil in a uniform magnetic field B

Flux $\Phi = BA \operatorname{Sin} \theta$

Let the coil rotate at a constant angular velocity ω

$\theta = \omega t = 2\pi f t$ where f is the frequency

$d\theta/dt = \omega = 2\pi f$

Generated EMF

$E = -N\ d\Phi/dt$ where N is number of turns on the coil

$= -N\ d[BA \operatorname{Sin}(2\pi f t)] / dt$

$= -N BA (2\pi f) \operatorname{Cos}(2\pi f t)$

But $BA = \Phi_{max}$ weber

$E = -N \Phi_{max} (2\pi f) \operatorname{Cos}(2\pi f t)$

$E_{rms} = (1/\sqrt{2}) N 2\pi f \Phi_{max} = 4.44\ N f \Phi_{max}$ volts

This emf equation is for a single coil with N turns rotating in a uniform magnetic field. A practical generator has many conductors in slots distributed round the machine.

Three phase generation

Advantages are

Generator winding space is fully utilised.
Generation and transmission costs are less
Transformers give a choice of voltage
Motors have a rotating field.
The power and torque are constant, unlike single phase

Voltage and Frequency
Most high voltage machines in the UK generate at 11 kV and in the USA at 13.8 kV. The frequency in the UK is 50 hertz and in the USA is 60 hertz.

Peripheral speed
The peripheral speed of a 34 inch diameter 2 pole 50 hz rotor is nearly 300 mph.

Hydrogen cooling
Hydrogen cooling reduces the windage loss by 90%, eg from 400 kW to 40 kW and the rating could be increased by 20%. First used in the USA in 1930 where the 60 hz system means higher windage loss than a 50 hz system. Used in the UK in the 1950s but the additional cost was enormous. The machine had to be in an explosion proof gas tight enclosure with complicated seals on the shaft. Further costs included expensive gas detection alarms, oil contamination, CO_2 fire protection etc.

Electrical and physical angles
The angle between adjacent pole centres is 180^0 electrical degrees whereas the physical angle is 180^0/(number of pairs of poles).

Machine Rating
Generators, Motors and Transformers are rated in kVA or MVA.
Suppose two machines are identical except that one machine operates on twice the voltage of the other. The higher voltage machine has twice the number of turns on the winding. The machines are the same size so the volume of the windings is the same on both machines. The higher voltage machine therefore has each turn approximately twice the length and half the cross sectional area of the other machine. The resistance of the higher voltage winding is therefore about four times the resistance of the lower voltage machine. For the same I^2R heat loss, the higher voltage machine has half the current.
Therefore the higher voltage machine is rated for twice the voltage and half the current. Thus both machines are the same kVA rating. Thus the kVA or MVA rating is a guide to the physical size of the machine and does not depend on the actual voltage or current rating.

For a three phase machine
kVA = $\sqrt{3}$ · (rated line kV) · (rated amps)
Rated power in kW = (rated kVA) · (Cos ϕ) where Cos ϕ is the design power factor

4 Pole Salient Pole Generator with the winding in slots
The generator armature winding is in slots in the stator.

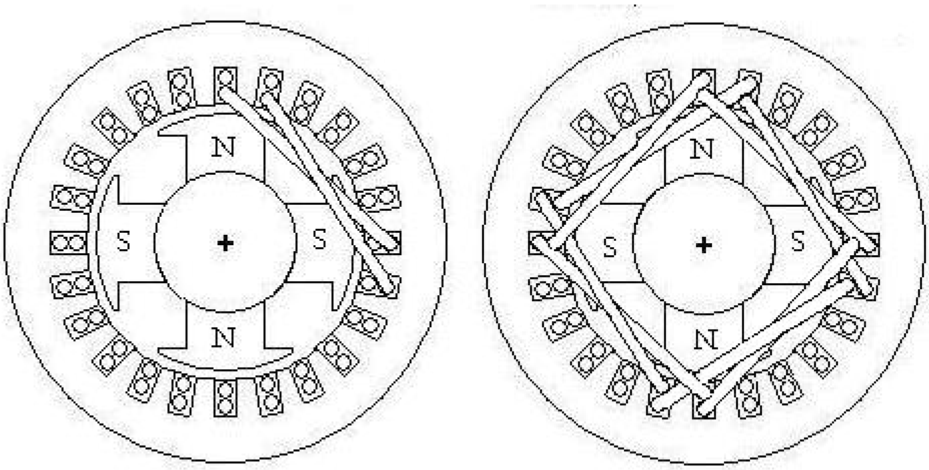


Figure 130; Four Pole Salient Pole Generator

The diagram on the left shows 2 coils with a coil span of 180^0 electrical degrees on a 4 pole salient pole generator. The diagram on the right shows the complete winding for one phase with a coil span of 180^0electrical degrees The coil span can be less than 180^0. All coils on one phase can be connected in series or in parallel. One side of each coil is at the bottom of the slot and the other side is at the top of the slot.

Two pole generator with the winding in slots
Consider a 2 pole three phase generator with 24 slots, 8 coils per phase as shown each with a coil span of 2λ and pole angle of 2α. In this diagram $\lambda = 75^0$ and $\alpha = 30^0$.

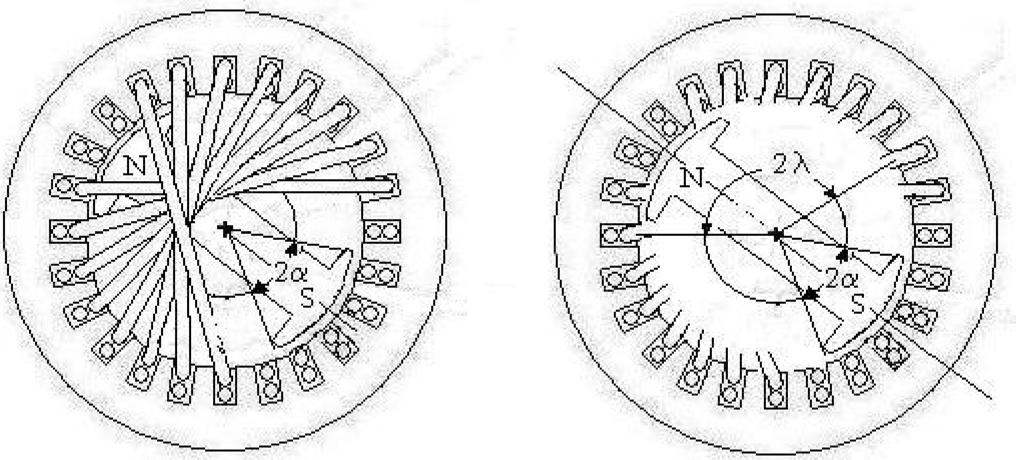


Figure 131; Two pole generator with eight coils/phase, coil span $2\lambda = 150^0$ and pole angle$2\alpha = 60^0$.

The left hand diagram shows all the coils on one phase. The right hand diagram shows the field poles. Let the rotor be rotated in a clock wise direction in steps of 7.5^0 starting in the position shown with $\theta = 0$.. The *EMF* at each step is proportional to the number of conductors opposite a pole face. In the position at $\theta = 0$ as shown, the *EMF*s in the two coils opposite a pole face oppose each other so the total *EMF* = 0.

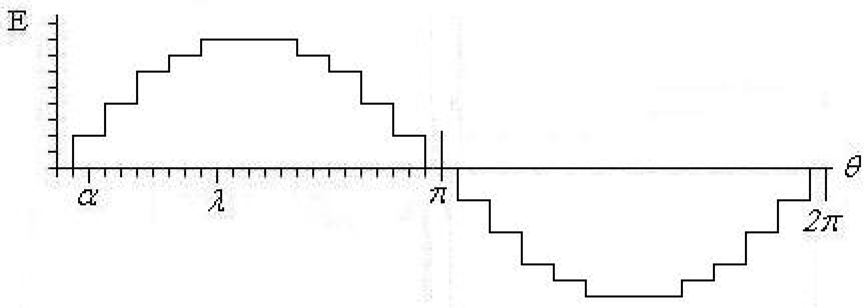


Figure 132; *EMF* curve from $\theta=0$ to $\theta=2\pi$

Complete three phase winding 2 pole generator with 24 slots and coil span $2\lambda = 150^0$.

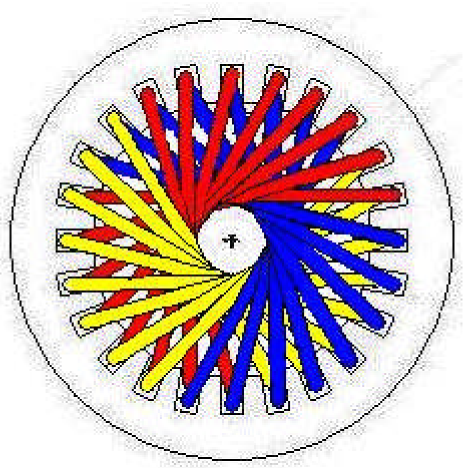


Figure 133; The complete winding for Figure 131 with the three phases coloured red, yellow and blue.

Flux Distribution with the winding in slots

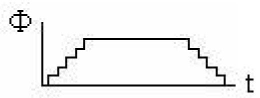


Figure 134; Flux through the winding

The flux rises in steps as the pole face passes each conductor and then falls in steps as the pole face leaves the conductor. The flux Φ follows the same curve as he *EMF*.

With a large number of slots.
If the number of slots is large, the steps become blurred into slopes and the flux follows the pattern shown.

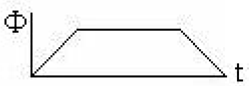


Figure 135; Flux with a large number of slots

Pitch Factor due to the span of the coils

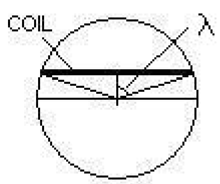


Figure 136; Pitch Factor

Let the coil span be 2λ electrical radians
This reduces the *EMF* by a factor *kp*.

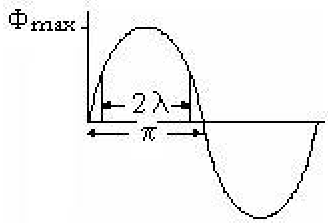


Figure 137; Pitch Factor

Where $kp = \dfrac{\int \sin\theta \; d\theta \text{ from } \pi/2-\lambda \text{ to } \pi/2+\lambda}{\int \sin\theta \; d\theta \text{ from } 0 \text{ to } \pi}$

hence $kp = \dfrac{[-\cos\theta] \text{ from } \pi/2-\lambda \text{ to } \pi/2+\lambda}{[-\cos\theta] \text{ from } 0 \text{ to } \pi}$

$$kp = \frac{-[\cos(\pi/2+\lambda) - \cos(\pi/2-\lambda)]}{-\;[\cos(\pi) - \cos(0)]}$$

$$kp = (2 \sin\lambda)/2 = \sin\lambda$$

$E_{rms} = 4.44\, kp\; N f \Phi_{total}$
where $kp = \sin\lambda$ is the pitch factor

The reduction in coil pitch causes a much larger reduction in the harmonic content.
The pitch factor for the n th harmonic is $\sin[\pi/2 - n(\pi/2 - \lambda)]$.
For n th harmonic where $n = 2m + 1$, pitch factor $knp = (-1)^m \sin(n\lambda)$
Thus for the 3rd harmonic the pitch factor $k3p = -\sin(3\lambda)$
5th harmonic $k5p = \sin(5\lambda)$
7th harmonic $k7p = -\sin(7\lambda)$

Example
6 slots / pole pitch and 5 slots / coil pitch
$2\lambda = (5/6)\pi$
Pitch Factor $kp = \sin\lambda = \sin(75^0) = 0.966$
Hence the smaller coils cause a reduction in emf of only 3.4%
$\lambda = 75^0$ thus the pitch factor for the 3rd harmonic is $-\sin(225^0) = 0.707$
and for the 5th harmonic the pitch factor is 0.259.

Distribution Factor due to phase difference in conductor emfs

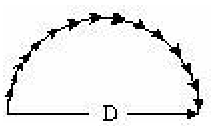


Figure 138; Distribution Factor *kd*

With the conductors evenly distributed round the machine, the voltages induced in each conductor are not in phase. The conductors of a single phase machine generate voltages whose vectors follow the circumference of a circle. The vector sum is proportional to D while the scalar sum is proportional to $\pi D/2$.
Thus the voltage generated with a large number of conductors
is $D/(\pi D/2)$ of the possible maximum.
Distribution Factor $kd = 64\%$.

Distribution Factor for three phase generation

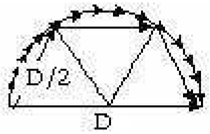


Figure 139; Distribution Factor *kd*

If the conductors are divided into three phases, the vector sum for one coil is $D/2$ and the maximum possible is $\pi D/6$.
Distribution Factor $kd = 3/\pi = 95\%$

Examples of three phase winding in slots.

The EMFs of the three phases are spaced 120 electrical degrees apart.
The diagram shows a typical winding with two conductors in each slot and the coil pitch one slot less than π.

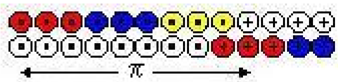


Figure 140; Part of a three phase winding

The completed winding for a 2 pole machine would be like this.

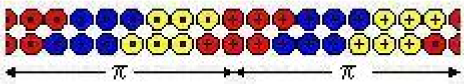


Figure 141; Complete three phase winding

In this example, the coil pitch is (8/9)π
Hence the pitch factor kp = Sin (80^0) = 0.985

But the voltages in each conductor are not in phase. It can be seen that the voltage in each of the three conductors is
$2r$ Sin (10^0) where r is their vector sum.

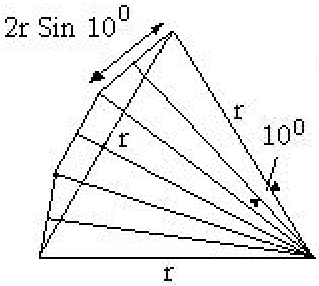


Figure 142; Distribution Factor kd

The distribution factor kd = (vector sum) / (scalar sum)
In this example $kd = r/[3 \cdot 2r\,\mathrm{Sin}(10^0)\,] = 0.96$

In general $kd = 2\,\mathrm{Sin}\,(\theta/2)\,/[2\,c\,\mathrm{Sin}\{\,\theta\,/(2c)\}]$
where θ = π/number of phases and c = slots/phase/pole
If the number of phases = 3, then 2 Sin (θ/2) = 1
and $kd = 1\,/[2\,c\,\mathrm{Sin}\{\pi\,/(6c)\}]$ or $kd = 1\,/[2\,c\,\mathrm{Sin}\,(30^0\,/c)]$
If c = 1, kd = 1
If c is very large $kd = 1\,/[2c\,\pi/(6c)] = 3/\pi = 0.955$

In this example, phases = 3 and c = 3
$kd = 1\,/[6\,\mathrm{Sin}\,(10^0\,)] = 0.960$
Total factor = $kp \cdot kd$ = 0.985 · 0.960 = 0.946

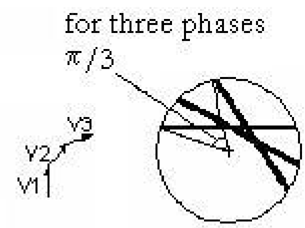


Figure 143; Distribution Factor kd

For the n^{th} harmonic
$knp = \mathrm{Sin}\,(n\lambda)$
$knd = \mathrm{Sin}\,(n\theta/2)\,/[\,c\,\mathrm{Sin}\{\,n\theta\,/2c\}]$

For the 5th harmonic

$k5p$ = Sin (400^0) = 0.642

$k5d$ = Sin (150^0) /[3 Sin{ 300^0 /6}] = 0.5 / (3 Sin 50^0) = 0.218

Total factor = $k5p \cdot k5d$ = 0.642 · 0.218 = 0.140

The factor for the 5th harmonic is only 15% of the fundamental factor.

In the example, suppose there are 30 turns per phase, ie 10 conductors in each layer in each slot
The generated emf would be

$$E_{rms} = 4.44 \cdot 0.946 \cdot 30\, f\, \Phi_{total}$$
$$= 6300 \cdot \Phi_{total} \quad \text{at 50 hertz}$$

If the maximum flux density is 1.2 tesla and the generated emf is 240 volts per phase, the area of the pole face is 317 cm^2.

Fractional Slots per pole per phase

In the above example. There were 3 slots per pole per phase.
It is possible to have fractional slots per pole per phase.
The following arrangement has 3 ½ slots per pole per phase.

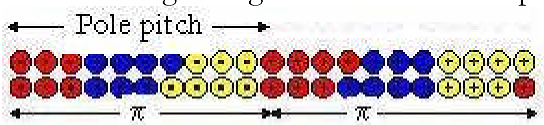


Figure 144; Fractional slot

Fractional slots have a number of advantages.

The same laminations can be used for a different number of poles
There is greater flexibility in the make up of the winding
Certain harmonics are killed

Example

An 8 pole generator has a flux density in the air gap

1/12th pole pitch	0	1	2	3	4	5	6	7
B tesla	0	0.06	0.5	0.76	0.83	0.85	0.86	0.85

Armature diameter 100 cms, conductor length 25 cms speed 750 rpm

Speed of conductor 750 · π · 1 / 60 = 39.3 m/sec

Area swept by conductor = 0.25 · 39.3 = 9.825 m^2 / sec

EMF = 9.825 · B volts

1/12th pole pitch	0	1	2	3	4	5	6
B tesla	0	0.06	0.5	0.76	0.83	0.85	0.86
EMF	0	0.59	4.91	7.47	8.15	8.35	8.45
EMF^2	0	0.35	24.1	55.8	66.4	69.7	71.4

EMF_{rms} per conductor = √[{ Σ(0.35 to 69.7) + 71.4/2}/6] = 6.45 volts

Harmonics

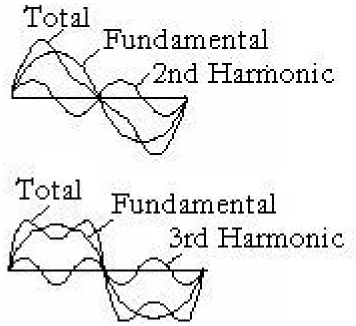


Figure 145; Effects of harmonics on the waveform.

The generated voltage changes as more conductors come into the field. The generated voltage is not therefore a perfect sine wave. Furthermore fluctuations are caused by the slots in the generator rotor and stator. Any harmonics are usually odd harmonics since a wave that is symmetrical about $\pi/2$ can contain odd harmonics but not even harmonics.

The third harmonic in each phase are in phase with each other. Therefore the third harmonic components returning through the neutral do not cancel, they add together. Problems in a neutral are often due to third harmonics. If however a supply containing the third, sixth or ninth harmonic is connected to the delta winding of a transformer, these harmonic voltages are in phase at each end of each delta winding. These harmonic currents do not flow in the primary delta winding and none appear in the secondary.

<u>Summary of harmonics</u>
Every harmonic is multiplied by its distribution factor
Harmonics of the order $6n + 1$ travel in the positive direction ie are +ive sequence
Harmonics of the order $6n - 1$ are negative sequence
Every harmonic travels at $(1/q)$ speed of fundamental
Every harmonic induces an *EMF* of fundamental frequency in the armature
The effect of space harmonics in the flux distribution due to the field current only is to induce *EMF*s of small size and various frequencies which do not appreciably affect the rms value of the resulting voltage.
The effect of rotating harmonics in the gap flux due to the combined armature currents only is to induce *EMFs* all of fundamental frequency but of such small size compared to the fundamental that they can be neglected.
For most practical purposes, ac voltages and currents can be considered to be pure sine waves.

Harmonics in the generated EMF
The mmf due to each coil is of the form;

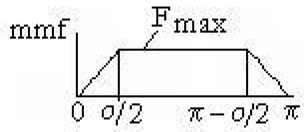


Figure 146; Harmonics

From the Fourier analysis,
$F = F_1 \text{ Sin } \theta + F_3 \text{ Sin } 3\theta + F_5 \text{ Sin } 5\theta + ..$
Where $F_q = (2/\pi) \int [F \text{ Sin } q\theta]\, d\theta$ from 0 to $\sigma/2$
$= (2/\pi) \int [F \text{ Sin } q\theta]\, d\theta$ from $\sigma/2$ to $\pi - \sigma/2$
$= (2/\pi) \int [F \text{ Sin } q\theta]\, d\theta$ from $\pi - \sigma/2$ to π
$=$ Integrals $I_1 + I_2 + I_3 + ...$

$0 < \theta < \sigma/2$ $\quad F = 2F_{max}\, \theta/\sigma$
$\sigma/2 < \theta < \pi - \sigma/2$ $\quad F = F_{max}$
$\pi - \sigma/2 < \theta < \pi$ $\quad F = 2F_{max}\, (\pi - \theta)/\sigma$

Integrating by parts
$I_1 = 4\, F_{max} / (\pi\, \sigma\, q)\, [- \theta \text{ Cos } q\theta + (1/q) \text{ Sin } q\theta\,]$ from 0 to $\sigma/2 = I_3$
$I_2 = - 2/(\pi q)\, F_{max}\, [- \text{Cos } q\theta\,]$ from $\sigma/2$ to $\pi - \sigma/2$
Thus $I_1 + I_2 + I_3$
$= 8\, F_{max} / (\pi\, \sigma\, q)\, [- (\sigma/2) \text{ Cos } (q\sigma/2) + (1/q) \text{ Sin } (q\sigma/2) + (\sigma/2) \text{ Cos } (q\sigma/2)]$
$= 4\, F_{max} / (\pi q)\, [\, \{1/(q\sigma/2)\} \text{ Sin } (q\sigma/2)\,]$

$I_1 + I_2 + I_3 = 4\, F_{max} / (\pi q)\, [\, 1/\{c\, q\sigma/(2c)\} \text{ Sin } (q\sigma/2)\,]$
Since c is large (slots/pole/phase), $\text{Sin}\{q\sigma/(2c)\} = q\sigma/(2c)$
$I_1 + I_2 + I_3 = 4\, F_{max} / (\pi q)\, [\text{ Sin } (q\sigma/2)\,]/[c \text{ Sin}\{q\sigma/(2c)\}]$
$= [4\, F_{max} / (\pi q)]\, knd$

Hence the mmf /phase =
$F1 = (4\, F_{max}/\, \pi) \text{ Cos } (\omega t)\, [\, (k1d \text{ Sin } \theta) + (1/3)(k3d \text{ Sin } 3\theta\,) + (1/5)(k5d \text{ Sin } 5\theta\,) + ..$
$F2 = (4\, F_{max}/\pi) \text{ Cos}(\omega t - 2\pi/3)[k1d \text{ Sin } (\theta\text{-}2\pi/3) + (1/3)k3d \text{ Sin}(3\theta\text{-}2\pi/3) + ..$
$F3 = (4\, F_{max}/\pi) \text{ Cos}(\omega t - 4\pi/3)[k1d \text{ Sin } (\theta\text{-}4\pi/3) + (1/3)k3d \text{ Sin}(3\theta\text{-}4\pi/3) + ..$

Consider order of q (omitting the constants)
$(F1 + F2 + F3)q$
$= \cos \omega t \ \sin q\theta + \cos(\omega t - 2\pi/3)\sin(q\theta - 2\pi/3) + \cos(\omega t - 4\pi/3)\sin(q\theta - 4\pi/3)$
$= ½ [\sin(q\theta + \omega t) + \sin(q\theta - \omega t)$
$+ [\sin\{q\theta + \omega t - (q+1)(2\pi/3)\} + \sin\{q\theta - \omega t - (q-1)(2\pi/3)\}$
$+ [\sin\{q\theta + \omega t - (q+1)(4\pi/3)\} + \sin\{q\theta - \omega t - (q-1)(4\pi/3)\}$

Case 1 $q = 6n + 7$ ie $q = 7, 13$ etc
All terms with $(q + 1)\,)(2\pi/3)$ give $4\pi/3$
All terms with $(q + 1)\,)(4\pi/3)$ give $8\pi/3 = 2\pi/3$

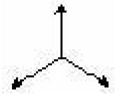


Figure 147;

So we get all the same amplitude so the sum is zero
$(q-1)$ terms all equal $\cos\{(\theta - \omega t)/2\}$ and have the effect $(3/2)\cos(\theta - \omega t)$

Case 2 $q = 3n$ where n is odd
mmfs are all displaced by 120^0
Hence mmf is composed of odd harmonics with the third harmonic missing
$F = (4/\pi)\, F_{max}\, (3/2)[\, k1d \sin(\theta - \omega t) + (k5d/5)\sin(5\theta - \omega t) + (k7d/7)\sin(7\theta - \omega t)$

Assuming no saturation, the emf due to q^{th} harmonic is proportional to $B_q \cdot knd/q$
ie q^{th} harmonic E is proportional to $(kqd/q)^2$
knd/q is small thus the emf is close to a sine wave.

Example
8 pole machine, 3 phase, star winding, 2 layer, 90 slots, coil pitch 1 – 10

Phase A Phase band no	1	4	7	10	13	16	19	22
Number of coil side	4	3	4	4	4	3	4	4
Phase B Phase band no	3	6	9	12	15	18	21	24
Number of coil side	4	4	4	3	4	4	4	3
Phase C Phase band no	5	8	11	14	17	20	23	2
Number of coil side	4	3	4	4	4	3	4	4

Show the emfs are balanced and contain no 3^{rd} or 5^{th} harmonics
Calculate the factors for the fundamental and 7^{th} harmonic
Slots / pole = 90/8 = 45/4
Winding diagram repeats itself after 4 poles and is symmetrical if there are a multiple of 4 poles.
There are 8 poles so the winding is symmetrical.
$\lambda = (9 / 11¼) \cdot 180^0 / 2 = 72^0$
$c = 15$ and $\theta = 60^0$
Fundamental

$kp = \sin(\lambda) = 0.951$
$kd = \sin(\theta/2) / [\, c \sin\{\theta/(2c)\}] = 0.955$

Total factor for the fundamental $= kp \cdot kd = 0.91$

3^{rd} harmonics are in phase in each end of a delta winding. Therefore no third harmonic current flows in a delta primary winding of a transformer. Therefore no third harmonic voltage appears on the star secondary of a transformer.

5^{th} harmonic pitch factor = Sin (5λ)

Sin $(5\,\lambda)$ = Sin $(360^0) = 0$ Hence 5^{th} harmonic is zero

7^{th} harmonic pitch factor = – Sin (504^0) = – Sin(144^0) = – 0.588

$k7d =$ Sin $(7 \cdot \theta/2)$ / [c Sin{ $7 \cdot \theta/(2c)$ }]

= Sin (210^0) / [15 Sin (14^0)] = 0.138

Total factor for the 7^{th} harmonic = $k7p \cdot k7d = 0.081$

7^{th} harmonic is 9% of the fundamental.

In fact there is a further reduction as the 7^{th} harmonic is not generated at the same level as the fundamental before this factor is applied, the fundamental is generated by the main flux whereas the harmonics are only generated by irregularities in the flux.

Example

3 phase 4 pole (non salient) 50 Hz alternator with 42 slots, conductors in 2 layers. Consecutive phase bands in same layer with 4 and 3 slots alternatively. Each phase 28 turns in series. Pitch 10 slots. Rotor dia 60cms, length 100cms, air gap 1cm. Estimated slots and iron reluctance reduce flux wave by 40% of value as calculated for no slots and infinite permeability. Leakage reactance per phase is 10% of synchronous reactance.

Show phases are balanced and calculate the synchronous reactance.

There are 14 slots/phase so there are 2 conductors in each layer in each slot and 56 conductors in series in each phase to give 28 turns in series

c = slots/phase/pole = 42/(3phase · 4 poles) = 3.5
$k1d = 1/\{2 \cdot 3.5 \cdot \mathrm{Sin}(30/3.5)^0\} = 0.9585$
coil pitch is 10 whereas span between poles is 10.5
$k1p = \mathrm{Sin}(90 \cdot 10 / 10.5) = 0.9972$
$k1w = 0.958 \cdot 0.9972 = 0.956$
X = rms volts in one phase due to rms amps in that phase

F = mmf = $(4/\pi)\ (3/2)\ F_m\ k1w\ \mathrm{Sin}(\theta - \omega t)$
where F_m is max mmf due to one phase
The factor (4/π) is due to the Fourier analysis of the *MMF* which is not a sine wave as the winding is in slots. The factor (3/2) is due to the combined effect of the three phases with a cylindrical rotor.

56 conductors in series = 28 turns spread over 2 pairs of poles
hence $F_m = 14\sqrt{2}\ I_{rms}$ ampere turns/pair of poles]
Hence F due to 3 phases is given by
$F = (4/\pi)\ (3/2)\ 14\sqrt{2}\ I_{rms}\ k1w\ \mathrm{Sin}(\theta - \omega t)$
$= 36.1\ I_{rms}\ \mathrm{Sin}(\theta - \omega t)$ ampere turns
Magnetizing Force $H = 1.26 \cdot F/2$ oersted
Hence $B = 60\%\ 1.26 \cdot F/(2 \cdot 10^{-2}) \cdot 10^{-6}$ tesla
$= 0.00136\ I_{rms}\ \mathrm{Sin}(\theta - \omega t)$ tesla
$B_{max} = 0.00136\ I_{rms}$
Max emf = $B_{max}\ l \cdot v \cdot k1w \cdot$ conductors in series
Where B in tesla, l in metres and v in metres/sec
Max emf = $0.00136\ I_{rms} \cdot 1 \cdot \pi \cdot 0.6 \cdot rps \cdot 0.956 \cdot 56$ where $rps = 25$
hence max emf = $3.42\ I_{rms}$
rms value of emf = $3.42/(\sqrt{2}) = 2.42$ volts / amps rms
$X_A = 2.42$ ohms
$X_L / X_A = 1/10$
$(X_S - X_A)/ X_a /= 1/10$
$10\ X_S - 24.2 = X_S$
$X_s = 2.69$ ohms

Effect of harmonics on the rms value of the generated voltage
Let $E = E_1 \mathrm{Cos}(\varphi_1 + \omega t) + E_3 \mathrm{Cos}(\varphi_3 + 3\omega t) + E_5 \mathrm{Cos}(\varphi_5 + 5\omega t) + \ldots$
$E^2 = E_1^2 \mathrm{Cos}^2(\varphi_1 + \omega t) + E_3^2 \mathrm{Cos}^2(\varphi_3 + 3\omega t) + E_5^2 \mathrm{Cos}^2(\varphi_5 + 5\omega t) + \ldots$
$+ E_1 E_3 \mathrm{Cos}(\varphi_1 + \omega t) \mathrm{Cos}(\varphi_3 + 3\omega t) + \ldots$

Average value of $E_q^2 \mathrm{Cos}^2(\varphi_q + q\omega t) = E_q^2/2$
Average value of $E_q E_r \mathrm{Cos}(\varphi_q + q\omega t) \mathrm{Cos}(\varphi_r + r\omega t) = 0$

Hence $E^2 = ½ [E_1^2 + E_3^2 + E_5^2 + E_7^2 + \ldots$
Hence $E_{rms} = 1/\sqrt{2}\ [\sqrt{\{E_1^2 + E_3^2 + E_5^2 + E_7^2 + \ldots\}}]$
$= E_1/\sqrt{2}\ [\sqrt{\{1 + (E_3^2/E_1^2) + (E_5^2/E_1^2) + (E_7^2/E_1^2) + \ldots\}}]$

Suppose 3rd harmonic = 15%, 5th = 7% and 7th = 5% of fundamental
$E_{rms} = E_1/\sqrt{2}\ [\sqrt{(1 + 0.0225 + 0.0049 + 0.0025)}] = 1.015\ E_1/\sqrt{2}$

Harmonics are reduced by winding factors and have little effect on E_{rms}

The effect of slot ripple on the generated voltage.
Suppose the slots exactly line up with the pole face. When the rotor moves ½ slot width, the flux is altered because one tooth is replaced by one air gap.

The flux pulsates with one complete cycle for every time a slot passes.
The fundamental frequency is $f = (rps)\, p$ where p is the pairs of poles.
There are c slots/phase/pole
Therefore in one revolution, c times (3 phases) times $2p$ slots pass any one point
The flux pulsates at a frequency of $6c\,p\,(rps) = 6c\,f$

$$\Phi = \Phi_0\,[\,1 + A\,\mathrm{Sin}\,(\,6c\,2\pi f\,t)]$$

$$E = E_{max}\,[1 + A\,\mathrm{Sin}\,(\,6c\,2\pi f\,t)]\,\mathrm{Sin}\,(2\pi f\,t)$$

$$E = E_{max}\,\mathrm{Sin}\,(2\pi f\,t) + ½E_{max}A[\mathrm{Cos}(6c–1)2\pi f\,t] – ½E_{max}\,A[\mathrm{Cos}(6c+1)2\pi f\,t$$

Slot ripple causes harmonics of a high order that interferes with communication equipment.
There is no easy way to eliminate these harmonics, but they can be reduced by;

Using fractional slot windings
Axial skewing of slots (on the stator or rotor) by one slot pitch.
Axial skewing of poles
Rounding off corners of pole shoes
Use of better ratio of tooth width to slot width
Offsetting of damper bars

Generated harmonics.
The shape of the generated emf E can be plotted out and then analysed by Fourier Analysis.
This can easily be done by computer.

Let the generated emf be represented by

$$E = A_1\,\mathrm{Cos}\,(2\pi\,ft) + A_3\,\mathrm{Cos}\,(6\pi\,ft) + A_5\,\mathrm{Cos}\,(10\pi\,ft) + A_7\,\mathrm{Cos}\,(14\pi\,ft) + \ldots\ldots.$$

$$+ B_1\,\mathrm{Sin}\,(2\pi\,ft) + B_3\,\mathrm{Sin}\,(6\pi\,ft) + B_5\,\mathrm{Sin}\,(10\pi\,ft) + B_7\,\mathrm{Sin}\,(14\pi\,ft) + \ldots\ldots.$$

The value of the n th harmonic is;

$A_n = (1/\,\pi\,)\int E\,\mathrm{Cos}\,(n\,x)\,dx$ from 0 to 2 π
(all zero with a symmetrical wave form).
$B_n = (1/\,\pi\,)\int E\,\mathrm{Sin}\,(n\,x)\,dx$ from 0 to 2 π

With x in degrees, the value of the n th harmonic is;

$A_n = (1/\,180\,)\int E\,\mathrm{Cos}\,\{n\,x(3.14159/180)\}\,dx$ from 0 to 360
$B_n = (1/\,180\,)\int E\,\mathrm{Sin}\,\{n\,x(3.14159/180)\}\,dx$ from 0 to 360

Example

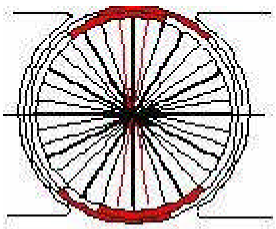


Figure 148; Winding

Pole angle $\alpha = 60^0$, coil pitch angle $\lambda = 80^0$
Find the relative values of the fundamental and the harmonics

Consider one phase. At any angle of the rotor, the *emf* is proportional to the number of conductors under a pole face. This value for *emf* includes *kp* and *kd*.
This is proportional to the total angle of conductors under the pole face

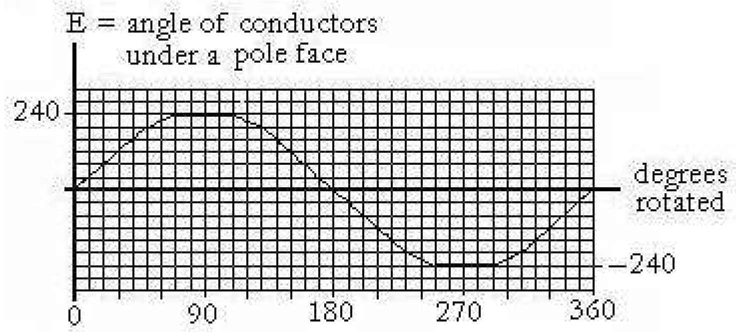


Figure 149; EMF curve

In the rotor position shown, the emf is zero as the conductors under each pole are two halves of the same coil. The total angle of conductors under a pole face is plotted in Figure 149 as the rotor is rotated in steps of 10^0 `

Run the qBASIC program;

```
CLS
FOR q = 1 TO 15 STEP 2
Eq# = 0
FOR x = 1 TO 3600
I = 0
IF x <= 500 THEN E# = .4 * x
IF x > 500 AND x <= 700 THEN E# = 200 + (x - 500) * .2
IF x > 700 AND x <= 1100 THEN E# = 240
IF x > 1100 AND x <= 1300 THEN E# = 240 - (x - 1100) * .2
IF x > 1300 AND x <= 2300 THEN E# = 200 - (x - 1300) * .4
IF x > 2300 AND x <= 2500 THEN E# = -200 - (x - 2300) * .2
IF x > 2500 AND x <= 2900 THEN E# = -240
IF x > 2900 AND x <= 3100 THEN E# = -240 + (x - 2900) * .2
IF x > 3100 AND x <= 3600 THEN E# = -200 + (x - 3100) * .4
I# = E# * SIN(q * x * 3.14159 / 1800)
Eq# = Eq# + I#
NEXT x
Eq# = Eq# / (240 * 1800)
PRINT "E"; q; " = ";Eq#
NEXT q
```

The results rounded to 0.001 are;

Harmonic	Fundamental	3^{rd}	5^{th}	7^{th}	9^{th}	11^{th}	13^{th}	15^{th}
Magnitude	1.037	0	– 0.027	0.007	0	0.003	– 0.004	0

Check up to 15^{th} harmonic

y = 1.037 SIN(x) – 0.027 SIN(5x) + 0.007 SIN(7x) + 0.3 SIN(11x) – 0.0044 SIN(13x)

x	0	10	20	30	40	50	60	70	80	90
y	0	39.8	80.3	119.5	160.9	197.2	221.1	237.0	241.2	239.0

This agrees closely with the plot of E against x

More sophisticated programs can take into account the fringe flux at the edges of the poles.

Instead of a step change as the conductors come under the pole faces, the emf can rise as the conductor passes through the fringe angle f.

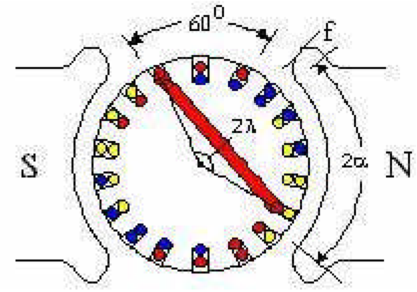


Figure 150; Flux fringe

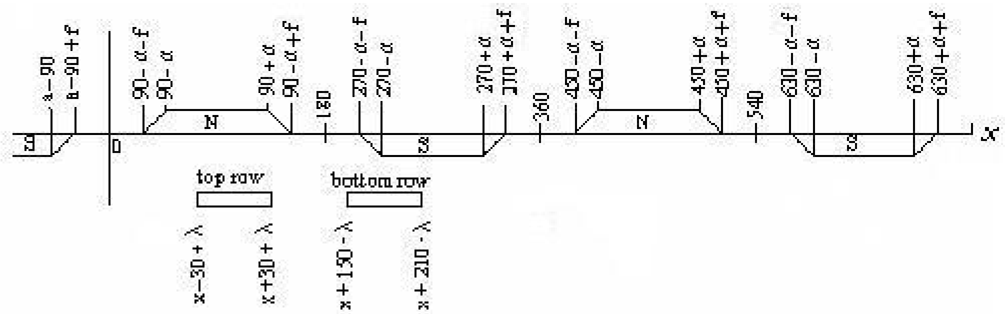


Figure 151; EMF in one coil

Harmonics with Capacitive loading
For resistive or inductive loading, harmonics content is usually small.
However for capacitive loading (eg a long high voltage line with no load), the harmonics will be magnified and can be troublesome.

Damper Winding
Generators usually have a short circuited winding round each pole called the Damper Winding or Amortisseur Winding.

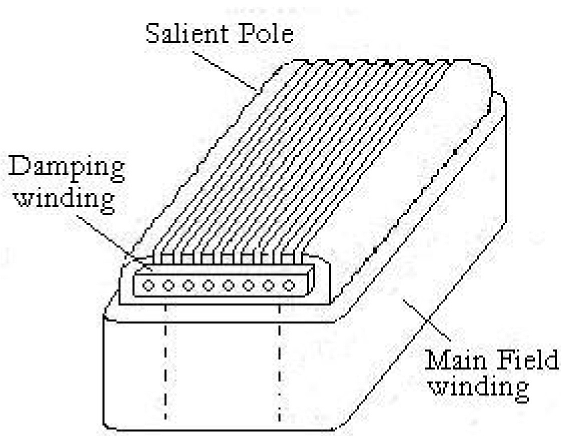


Figure 152; Damper or Amortisseur Winding
Damping windings are heavy copper conductors set in the pole face. They are brazed at the ends to heavy copper strips to short circuit the winding. By Lenz's law, a sudden change in flux causes a current to flow in the short circuited winding opposing the change. The Damping Winding helps to reduce oscillations or sudden changes in the magnetic flux.

Equivalent Circuit for a synchronous machine

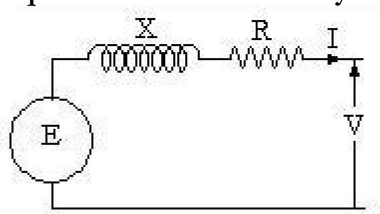


Figure 153; Equivalent Circuit for a generator

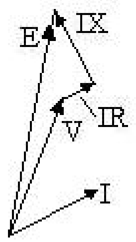


Figure 154; Generator Impedances

The *EMF* and output voltage are related by the vector equation $E = V + IR + jIX$

X is the leakage reactance and R is the resistance. In a full size machine, R is negligible compared to X. The effect of R can be ignored in the impedance but R is required for calculations of copper loss.

With a salient pole machine, the value of X depends on the angle between I and the pole axis. The component of I along the pole axis sees a reactance Xd. The component of I normal to the pole axis sees a reactance Xq. The vector diagram becomes

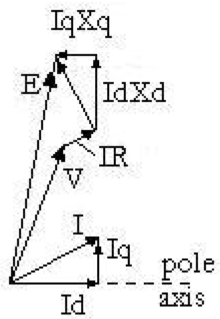


Figure 155; Generator Impedances for a Salient Pole Generator

Leakage reactance and Armature reaction
Armature currents produce a *MMF* in the Main Field magnetic circuit. This is called the armature reaction. The armature reaction that produces a *MMF* that link with the armature windings only is called the leakage reactance.

Leakage Reactance
Leakage reactance is due to flux paths round the conductors in the slots, at the ends of the conductors outside the core and zigzag flux between the armature windings and the pole face.

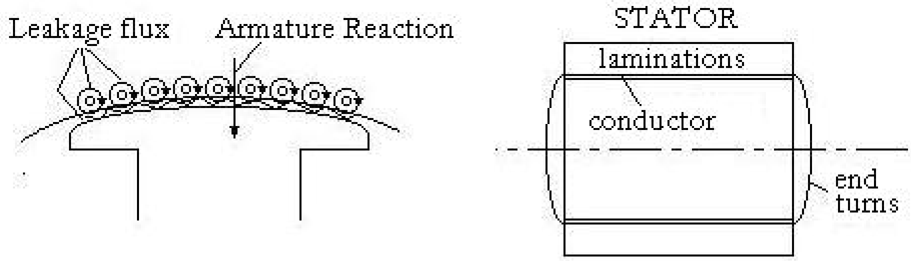


Figure 156; Leakage reactance

All the effects of leakage flux are summed up by X_{Leakage} and can be determined from the dimensions of the machine. In most machines the value is about right, but in turbo-alternators it must be increased, eg by not filling the slots with winding so the slot leakage is increased.

On short circuit, the initial current is E/Xd'' where E is the emf and Xd'' is the subtransient reactance. The leakage reactance is the main limiting factor in the initial value of the short circuit current. Thus Xd'' is usually taken as the leakage reactance.

Saturation
There is a further effect. The leakage reactance depends on the saturation of the magnetic circuit. With no field current the leakage reactance is Xd''. With field current for full volts open circuit, the leakage reactance is the saturated value Xd'' *sat.*

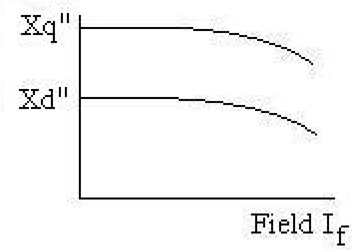


Figure 157; Saturation of Leakage Reactance

Percentage Reactance

Reactances are usually quoted as percentage of the phase voltage of the volt drop across the reactance at rated full load current.

$Xd'' = (\text{Reactance in ohms}) \cdot I \cdot 100\ \% \ / \ V_{ph}$

Armature Reaction

The armature reaction has a big effect on the *MMF* in the Field magnetic circuit. The quadrature axis component is centred between the poles. This causes a field π/2 electrical degrees out of phase with the main field.

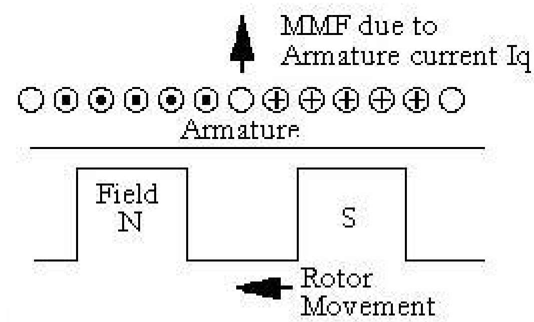


Figure 158; MMF due to Iq

The direct axis component of a lagging current directly opposes the exciter magnetising force and greatly reduces the magnitude of the field. This effect is quantified as the synchronous reactance. It is measured by the open circuit and short circuit tests.
Zero power factor lagging currents are directly demagnetizing while zero power factor leading currents boost the field.

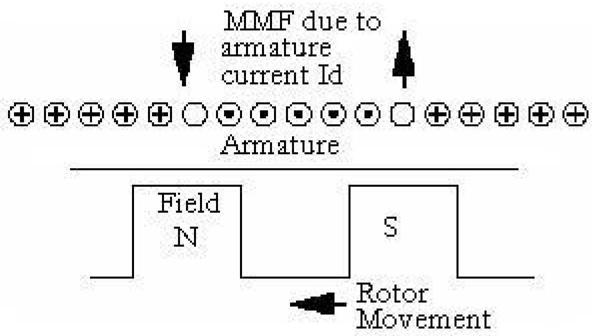


Figure 159; *MMF* due to Id

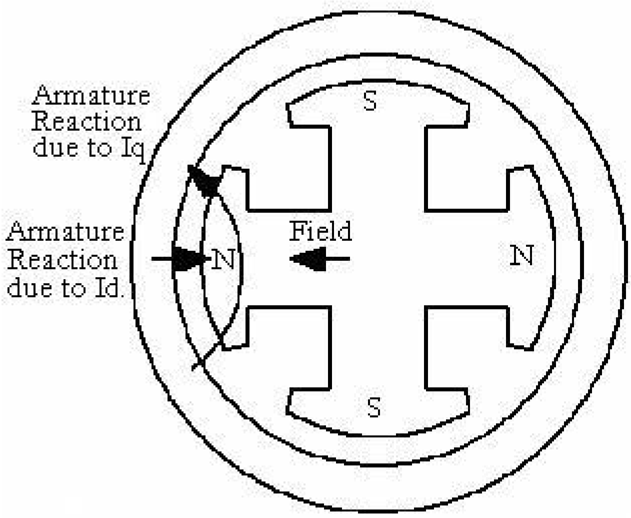


Figure 160; Armature Reaction

The armature reaction due to the quadrature axis component rotates the axis of the flux and this rotates the *EMF* without significantly changing its value. The component of a lagging armature current centred on the pole reduces the main field.

Open circuit characteristic

The machine is run at rated speed and on open circuit. The field current is raised and the phase voltage plotted against field current. The field current is expressed as per unit of the field current for open circuit full volts. The diagram shows the open circuit characteristic. The curve follows the magnetisation curve of the field magnetic circuit.

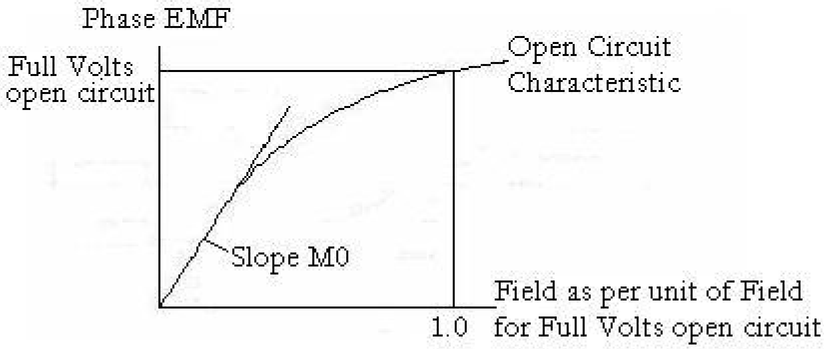


Figure 161; Open Circuit Characteristic

Short circuit characteristic

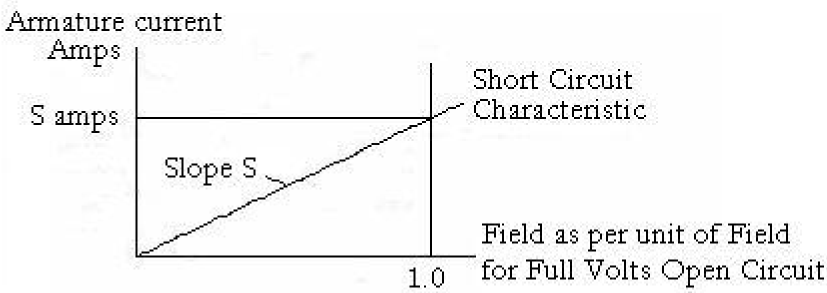


Figure 162; Short Circut Characteristic

The machine is connected to a robust short circuit at the output terminals. It is run up to full speed with no excitation. The excitation is slowly raised and the armature current plotted against field current which is expressed as per unit of the field current for open circuit full volts. It is found to be a straight line. Let the slope of this line be *S*. This diagram shows the short circuit characteristic.

Armature reaction during the short circuit test

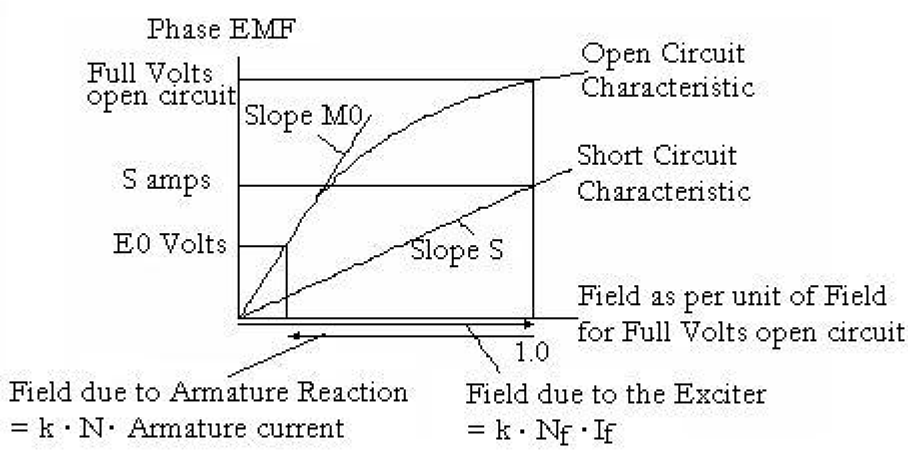


Figure 163; Armature Reaction during the short circuit test

The short circuit characteristic is a line at slope S. Therefore the short circuit current is S amps at unit field current, when expressed as per unit of the field for full volts open circuit..

The field ampere turns due to the exciter are $Nf \cdot If$ where Nf is the field turns and If is the field current due to the exciter for full volts open circuit. Put k equal to the multiplier to convert field ampere turns to per unit value.
Therefore $k \cdot Nf \cdot If = 1$ when armature current = S amps.

The leakage reactance of a large generator is much greater than its resistance. Thus the armature current is near zero power factor lagging. Further more, the generator voltage is centred on the pole axis. Therefore on short circuit, the armature reaction ampere turns of a large generator directly oppose the ampere turns due to the exciter.
The net field on the short circuit characteristic is thus the field due to the exciter minus the field due to armature reaction. The ampere turns due to armature reaction is directly proportional to the armature current. The difference between the ampere turns due to the exciter and the ampere turns due to armature reaction gives the field for the EMF to drive the armature current against circuit reactance Xd''. This value of EMF is small and is on the linear part of the open circuit characteristic. The EMF is proportional to the net Field.
Let $EMF = A \cdot If - B \cdot Ia$
where A and B are constants, If is the Field current and Ia is the armature current..

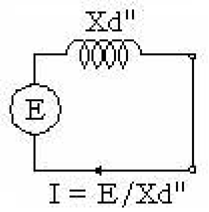


Figure 164; Impedance

But $EMF = Ia \cdot Xd''unsat$

The value of Xd'' depends on the saturation of the magnetic circuit. With the machine on short circuit, the net field is low and the leakage reactance is the unsaturated value Xd'' *unsat.*
$Ia \cdot (B + 1/Xd''\,unsat) = A \cdot If$
$Ia = [A / (B + 1/Xd''\,unsat)] \cdot If$
$[A / (B + 1/Xd''\,unsat)]$ is a constant
Therefore the short circuit characteristic is a straight line

$Ia = S$ when the Ampere turns provided by the Field = 1 per unit value.
When $Ia = S$, the field ampere turns due to armature reaction $= k \cdot N \cdot S$
The winding is actually in slots so N is the number of turns that give equivalent ampere turns
The net Ampere Turns = $(1 - k \cdot N \cdot S)$
$E_0 = M_0 \cdot (1 - k \cdot N \cdot S)$
where M_0 is the slope of the open circuit characteristic near the origin.

The impedance of the armature circuit is the leakage reactance Xd'' unsat.
The current is S amps
Therefore $E_0 = S \cdot Xd''$ unsat
$S \cdot Xd''$ unsat $= M_0 \cdot (1 - k \cdot N \cdot S)$
Divide by $S \cdot M_0$ and rearrange.
$k \cdot N = 1 / S - Xd''$ unsat $/ M_0$

Synchronous Reactance *Xd*

The Synchronous Reactance *Xd* is defined as the ratio of the slope of the open circuit characteristic near the origin to the slope of the short circuit characteristic. This ratio has the dimensions of emf / current so is called reactance.
$Xd = M_0 / S$

Using this definition;
$k \cdot N = (Xd - Xd'' \text{unsat}) / M_0$

On the direct axis;
armature reaction = $Id \cdot (Xd - Xd'' \text{unsat}) / M_0$ ampere turns
This armature reaction directly opposes the field due to the exciter.

Let *Xq* be the synchronous reactance on the quadrature axis. It is expressed as voltage drop on the direct axis due to armature current on the quadrature axis. The large air gap on the quadrature axis of a salient pole machine means the saturation curve on the quadrature axis is approximately linear.
Therefore;
$Ed = Iq \cdot (Xq - Xq'')$ provided the induced currents on the field and damping windings have decayed to zero and steady state has been reached.

Synchronous Reactance curve over pole pitch

The effect of Armature Reaction is called Synchronous Reactance.

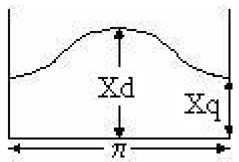


Figure 165; Synchronous reactances

$Xd / Xq = [(4/\pi) \int B_{max} \mathrm{Sin}^2 x \, dx] / [(4/\pi) \int B_{max} \mathrm{Cos}^2 x \, dx]$
both integrals from $\pi/2 - \theta$ to $\pi/2$
$= [\theta + ½ \mathrm{Sin}(2\theta)] / [\theta - ½ \mathrm{Sin}(2\theta)]$
If $\theta = 60^0$ then $Xd/Xq = 2.4$
Experience shows that this figure is not correct due to fringing flux which cannot be analysed.
A better result is obtained by comparing areas.
Xd is proportional to $2\int \mathrm{Sin} x \, dx$ from $\pi/2 - \theta$ to $\pi/2 = \mathrm{Sin}\,\theta$
Xq is proportional to $2\int \mathrm{Cos} x \, dx$ from $\pi/2 - \theta$ to $\pi/2 = 1 - \mathrm{Cos}\,\theta$
$Xd / Xq = [\mathrm{Sin}\,\theta] / [1 - \mathrm{Cos}\,\theta] = 1/ \mathrm{Tan}(\theta/23)$
If $\theta = 60^0$ then $Xd/Xq = 1.73$ which is a reasonable figure.

Vector diagram for the short circuit test

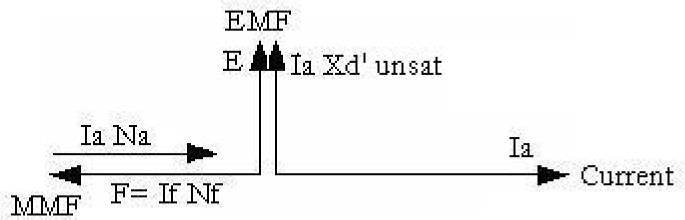


Figure 166; Vector diagram short circuit test

The emf *E* is produced by the ampere turns due to the field current minus the ampere turns due to armature reaction. This drives the armature current *Ia* against the leakage reactance *Xd*" unsat.

Vector Diagram on load
The suffix L will be used to signify the steady state on load condition.

Let the generator be connected to an external load of resistance *RL* and inductive reactance *XL* and let the component of the armature current be *IdL* on the direct axis and *IqL* on the quadrature axis.

The vector diagram of emfs and currents.

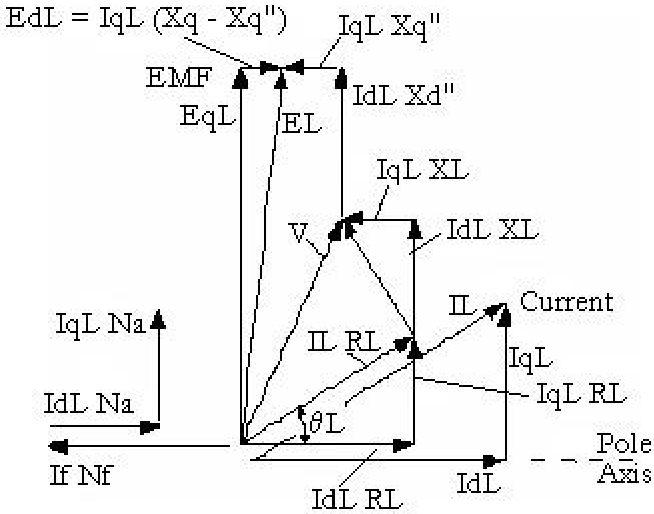


Figure 167; Vector diagram synchronous generator on load

Tan $\theta L = IqL\,RL / [\,(IqL\,(Xq - Xq'') + IqL\,Xq'' + IqL\,XL]$
$\quad = RL/(Xq + XL)$
V = Normal Busbar Volts
$IL = V/\sqrt{(RL^2 + XL^2)}$
$IdL = IL$ Cos θL
$IqL = IL$ Sin θL
Hence the vector diagram can be drawn
Draw vectors *IL RL*, *IdL XL*, *Iql XL* to get vector *V*
Draw vectors *IdL Xd*" *sat* and *Iql Xq*" to get *EL*
EL is the emf generated by the machine. It is rotated from the quadrature axis by the armature reaction on the direct axis which is shown on the diagram as *EdL*

Field current on load
Field *MMF* supplied by the exciter
= Field *MMF* for *EqL* + Field *MMF* to balance armature reaction.
The field to produce *EqL* can be read directly from the open circuit characteristic.
The field to balance the armature reaction = $k \cdot N \cdot IdL = (Xd - Xd'') \cdot IdL / M_0$

Field *MMF* supplied by the exciter (as per unit of field for open circuit full volts)
$= (EqL - U3) / M3 + (Xd - Xd'') \cdot IdL / M_0$

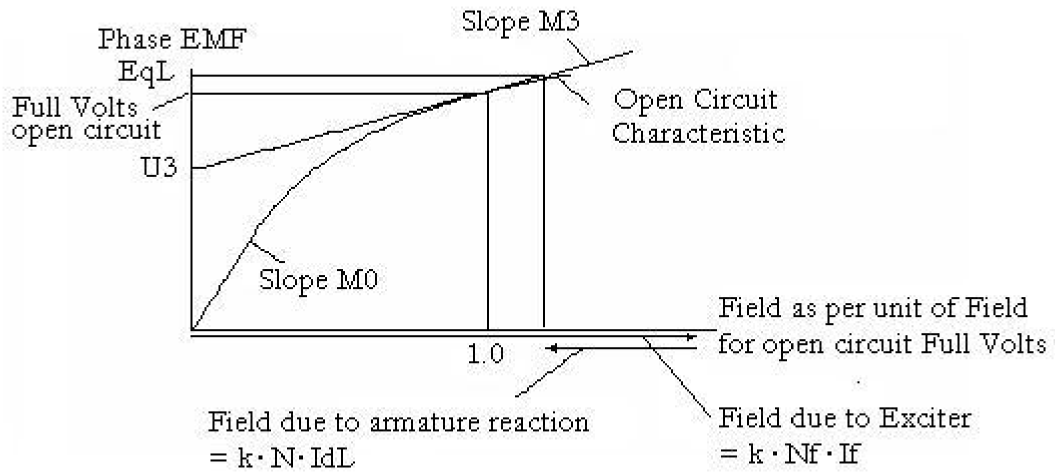


Figure 168; Generator field on load

Example
Find the field required for full load on a 14.5 MVA generator.
Full load is 10.6 MW at 11 kV line voltage
$Xd'' = 18$ %, $Xd' = 27.6$ %, $Xd = 205$ %, $Xq'' = 24$ % and $Xq = 110$ %
Open circuit characteristic as in the diagram.

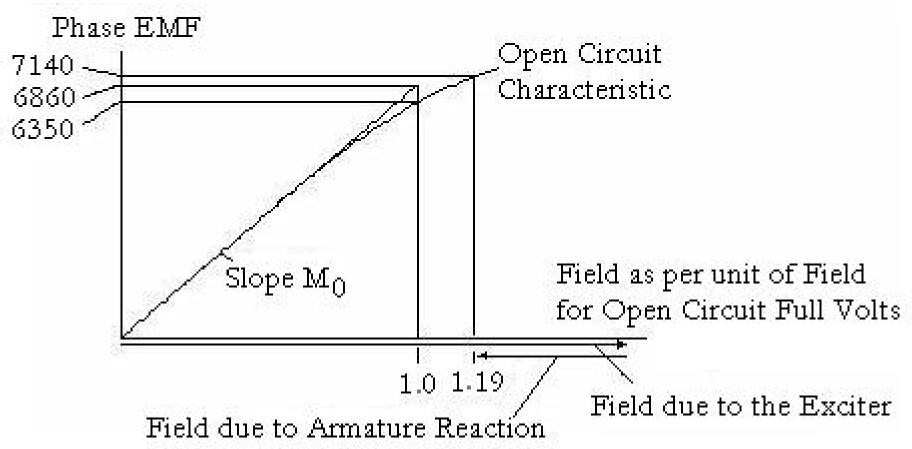


Figure 169; Generator field on load

Phase voltage = 11,000 /√3 = 6,350 volts
Full load current = 14,500,000 /(11,000 · √3) = 761 amps
Power factor = MW/MVA = 10.6 /14.5 = 0.731
Phase angle ϕ = ArcCos (0.731) = 43.0 degrees
Slope of the open circuit characteristic near the origin = 6,860 volts per unit field.
Reactance in ohms · full load current = phase volts · reactance in % / 100
Xd'' = 1.50 ohms, Xd' = 2.30 ohms,
Xd = 17.11 ohms,
Xq'' = 2.00 ohms and Xq = 9.18 ohms
Load Resistance = Power per phase / current^2
$RL = 10.6E6 / (3 \cdot 761^2) = 6.10$ ohms / phase
$XL = RL \operatorname{Tan} \phi = 5.69$ ohms / phase
$\operatorname{Tan} (\theta L) = RL/(Xq + XL)$
$= 6.10/(9.18 + 5.69) = 0.410$
$\theta L = 22.3$ degrees
$IdL = 761 \operatorname{Cos} (22.3) = 704$ amps
$IqL = 761 \operatorname{Sin}(22.3) = 289$ amps

The vector diagram of voltages can be drawn.
$EdL = IqL(Xq - Xq'') = 2070$ volts
$EqL = IqL\ RL + IdL(XL + Xd'') = 6830$ volts

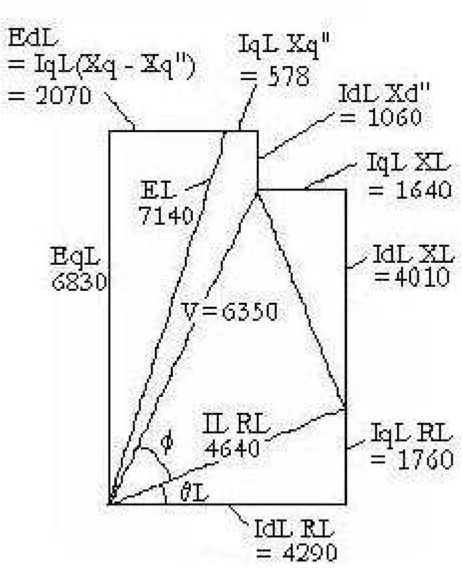


Figure 170; Vector diagram of voltages

The *EMF* is 7,140 volts rotated by the direct axis emf.
From the open circuit characteristic, the field to produce 7,140 volts is 1.19 per unit

The field to balance the armature reaction
$= (Xd - Xd'') \cdot IdL / M_0$
$= (17.11 - 1.50) \cdot 704 / 6{,}860$
= 1.60 per unit
Total field = 1.19 + 1.60 = 2.79 times the field to produce full volts open circuit.

Parallel operation of AC generators

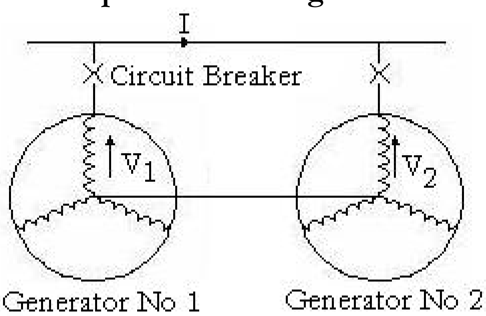


Figure 171; AC generators in parallel

Suppose two AC generators are switched into parallel operation and No 2 generator slows behind No 1.
The voltages are no longer in phase.
A current I will circulate driven by the vector voltage $V_1 - V_2$.
The circuit is highly inductive so the current lags this voltage by almost 90^0.

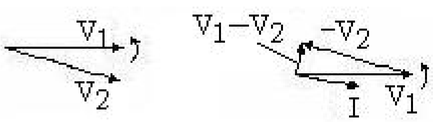


Figure 172; AC generators in parallel

It can be seen that $V_1 \bullet I$ is positive and $V_2 \bullet (-I)$ is negative.

Therefore load on Generator No 1 increases and it slows down. The load on Generator No 2 is reduced and it speeds up bringing the generators back into synchronism.

Thus AC generators will operate in parallel in synchronism. As they are in synchronism, they must run at the same speed.

The governor of the engine driving the generator controls the power output as a function of the speed. Therefore when two AC generators are running in synchronism, the power sharing of sets is controlled by the governor set points. Altering the voltage on either of the sets has no effect on the power sharing but changes the sharing of the wattless component of the current.

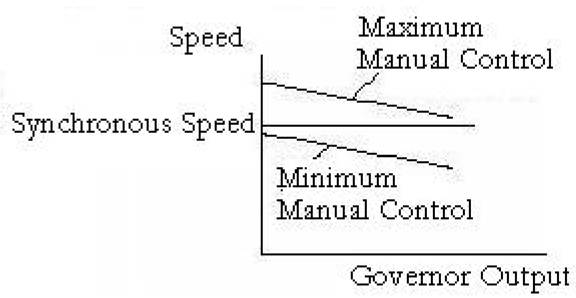


Figure 173; Governor control

For satisfactory power sharing, the governors must have a droop so that as the set is loaded up, the speed falls. Typically the change in speed between no load and full load is about 5%.

The governor has a manual control that adjusts the set point, ie the position of the speed control relative to the synchronous speed.

For unattended operation the manual control is replaced by a slow acting automatic control eg by a DC motor driving through a slow acting worm gear drive. Speed control is through the fast acting governor but the slow acting control in effect moves the governor set point. An even longer term control is also required so that any electric clocks on the system continue to read the correct time. When the clocks read fast or slow, the speed is reduced or increased a fraction respectively. A manned power station would have an electric and a crystal clock side by side on the control panel.

Synchronising an AC generator
Before switching an AC generator into parallel operation;
i) The voltages must be the same
ii) The speeds must be the same
iii) The peak value of the voltages must occur at the same instant

This is achieved by adjusting the field resistance so that the voltages are the same and connecting a synchroscope. One coil of the synchroscope is connected to the "running volts" and the other to the "incoming volts". The synchroscope rotates as the voltages come into and out of synchronism, the speed of rotation depending on the difference in speeds. The speed of the incoming set is adjusted till the synchroscope is rotating slowly in the clockwise direction and the circuit breaker is closed at the instant when the synchroscope needle is in the 12 o'clock position. The sets are now locked in synchronism. The governor of the incoming set is immediately manually raised to load up the incoming set.

Voltage Control of AC generators by the AVR
An AVR, (the Automatic Voltage Regulator), is required to keep the output voltage constant. A lagging current heavily reduces the field *MMF* while a leading current increases the field *MMF* Therefore an automatic voltage regulator controlling the field current is required to keep the output voltage constant.

If two machines are locked in synchronism, the power output of each depends on the governor setting (ie the set point of the governor). Any change in field strength will not change the power output but will change the reactive component (ie the kVAr). Suppose two machines run in parallel both with perfect AVRs, ie AVRs that ensure the output voltage is correct whatever the current or power factor.

If one machine supplies a low power factor lagging current I_1, an AVR with no compounding will raise the field *MMF* to counteract. The other machine will supply a leading current I_2 such that the vector sum of the two machine currents equals the load current I. The AVR of each machine adjusts the field *MMF* to keep the total *MMF* in the field constant for the set point voltage. A change in the reactive component in either machine can avalanche as the AVRs try to correct.

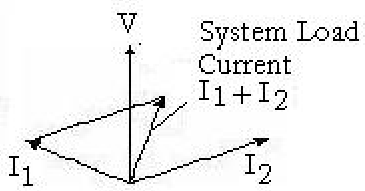


Figure 174; Unequal sharing of wattless current

One of two things will happen;
(1) The lagging reactive current in one set will rise balanced by a rise in the leading reactive current in the other set. The current in both machines will steadily rise till the ammeters go off the scale and the machines trip on overload.
Or (2) the lagging reactive component will swing violently from one set to the other till again the machines trip on overload. One or other of these is a common fault when commissioning a new alternator set and indicates reversed or incorrectly set compounding.

To prevent this happening, a "droop" is introduced in the AVR circuit. A lagging current must slightly reduce the excitation. Thus a lagging current must slightly increase the voltage seen by the AVR which will then reduce the excitation. Conversely a leading current slightly reduces the voltage seen by the AVR which then increases the excitation.

Somewhere between these two extremes of power factor, there is a power factor where the current does not change the voltage. The circuitry is arranged so that this occurs at the design system power factor, ie the expected power factor of the load to be connected to the generator. At this design system power factor, the voltage is correct whatever the current.

If the expected system power factor is close to 0.87 (ie Cos 30^0), then the reference voltage for the AVR could be the vector relation;

$$V_{AVR} = V_{yellow} + I_{red} \cdot R$$

At a power factor of Cos 30^0 lagging, the current vector *Ired* (and hence compounding voltage *Ired* · R) is at right angles to the voltage vector *Vyellow*.

At a power factor of zero lagging, V_{AVR} is greater than *Vyellow*, hence the AVR reduces the field giving the droop. The *Ired* · R term is called the compounding.

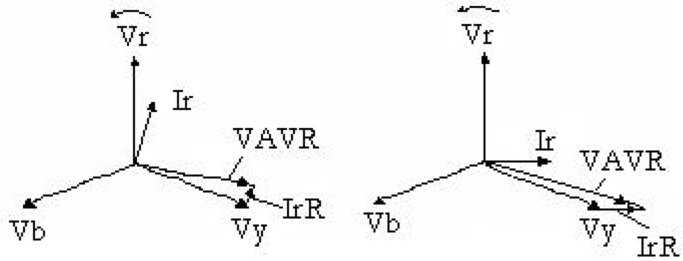


Figure 175; Effect of AVR compounding

Compounding for any other expected system power factor can be arranged by using an AVR that accepts a DC reference voltage. The compounding voltage is added to the reference voltage by vector addition with a 90^0phase shift, (eg blue to yellow phase volts added to a compounding voltage proportional to red phase current. This modified reference voltage is then rectified. An adjustable rectified voltage proportional to the compounding voltage is then subtracted from the reference voltage (ie a vector addition followed by a scalar subtraction) so that at the design system power factor, the reference voltage is the same whatever the current.

For the very best voltage control system, the reference voltage for the AVR is the DC sum of the three phase or line voltages, each with the appropriate compounding. This has the advantage that the alternator response is the same with unbalanced load or single phase fault regardless of which phase carries the heavy current.

Automatic Voltage Regulator

An automatic voltage regulator supplied by one manufacturer is similar to an induction motor with a lightweight drum rotor round a fixed magnetic core. The lightweight drum rotates against a spring. Therefore the drum position depends on the voltage. A copper quadrant controlled by the drum rolls against copper contacts. The resistance between the contacts therefore depends on the voltage and controls the exciter voltage.

Another manufacturer uses a Wheatstone bridge. One arm of the bridge uses a fixed resistor. The other arm uses an incandescent lamp. At the set voltage, the resistance of the incandescent lamp equals the resistance of the fixed resistor. At any other voltage, the resistors are not equal. The difference is amplified and controls the exciter voltage.

Over and under excitation
If the alternator supplies a lagging load, the excitation must be increased to partly overcome the demagnetizing effect of the load current but not enough to completely overcome it. The machine is said to be over excited and the *EMF* is greater than the voltage.

If the machine supplies a leading current, the machine is said to be under excited and the emf is less than the voltage.

Stability
The stability depends on the angle between E and V

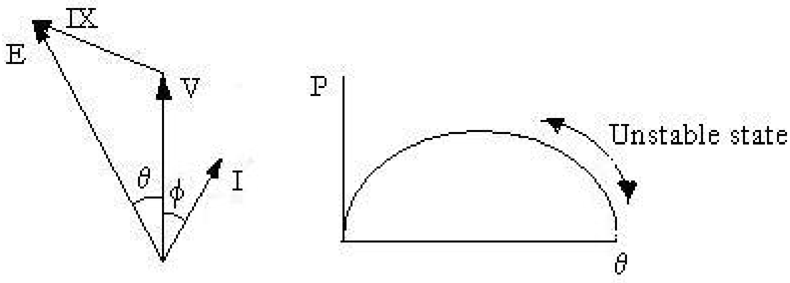


Figure 176; Unstable operation

Power per phase $P = VI \text{ Cos } \varphi$
From Sine formula
$IX / \text{Sin}\theta = E / \text{Sin}(90 + \varphi)$
$I = E \text{ Sin}\theta \; / \; X \text{ Cos}\varphi$
$P = (VE/X) \text{ Sin}\theta$

Salient Pole Alternator

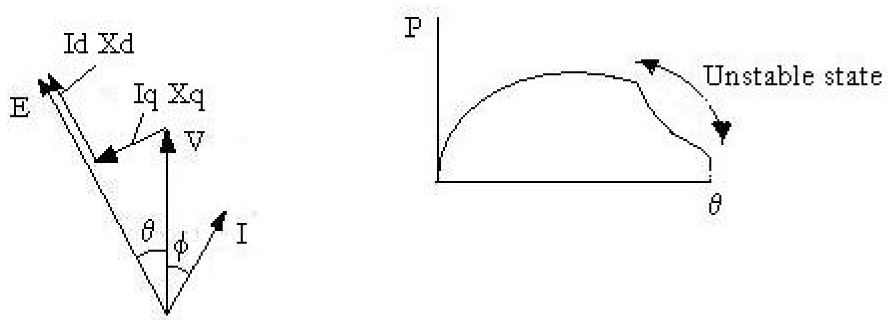


Figure 177; Stability Salient Pole Generator

Power per phase $P = VI \text{ Cos } \varphi = V(Iq \text{ Cos}\theta + Id \text{ Sin}\theta)$
$E = V\cos\theta + IdXd$
$V\sin\theta = IqXq$
$P = (EV/Xd) \text{ Sin}\theta + [V^2 \,(Xd - Xq) \text{ Sin}2\theta] \; / \; [2XdXq)$

The distortion of the curve is due to the second harmonic term.
Maximum power occurs much earlier and the angle between E and V is considerably reduced.
A turbo cannot produce any power by excitation of the armature only. A salient pole machine can produce 10% of full power without excitation. Flux in the air gap is normal to the pole face but fringe flux at the edge of the pole can produce the torque.

Transient Stability
If the machine is operating in the stable region and an additional load is applied, the equilibrium condition may move closer to the unstable region. If this load is applied suddenly, the machine may overshoot into the unstable region and will then cease to operate.

Synchronous Motor
With a synchronous motor, the currents are in the opposite direction.
$V = E + IX$

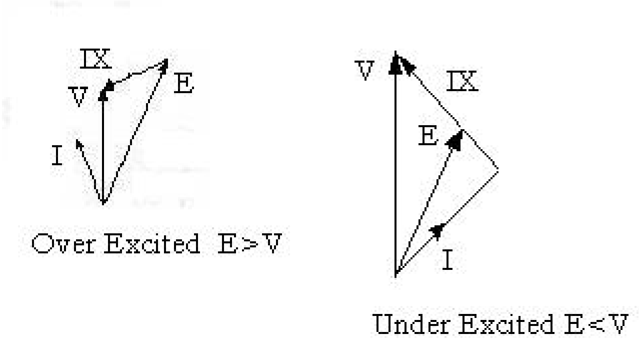


Figure 178; Synchronous Motor

If the *EMF* exceeds the voltage, the current leads the voltage. If the *EMF* is less than the voltage, then the current lags the voltage. This is the opposite of the alternator so it is confusing to refer to the currents only as leading or lagging.

Synchronous Condenser.
The synchronous condenser is neither a generator nor a motor. The current is zero power factor. If the machine is over-excited, it can be considered as an alternator generating a lagging current. Equally it can be considered a motor taking a leading current. If the machine is under-excited, it can be considered as an alternator generating a leading current. Equally it can be considered a motor taking a lagging current.

A synchronous condenser can be used to improve the power factor of an industrial plant. It is described as over or under excited, not as taking a leading or lagging current.

AC INDUCTION MOTORS

Small AC commutator motors

Small AC motors (eg for hand tools) are usually commutator machines with the field connected in series. They are similar to a DC series motor except that the field magnetic circuit is made with stampings of thin sheet metal to limit eddy currents in the AC field. There is a transformer effect between the field and armature which prevents the motor having a shunt connected field.

Rotating Field due to a three phase winding on a cylindrical core

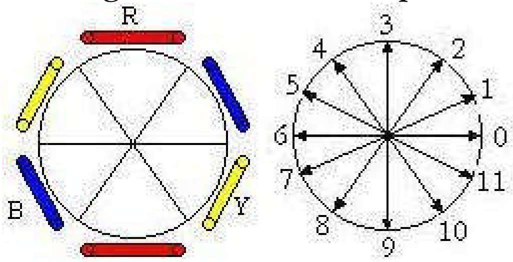


Figure 179; Rotating Field due to a three phase winding

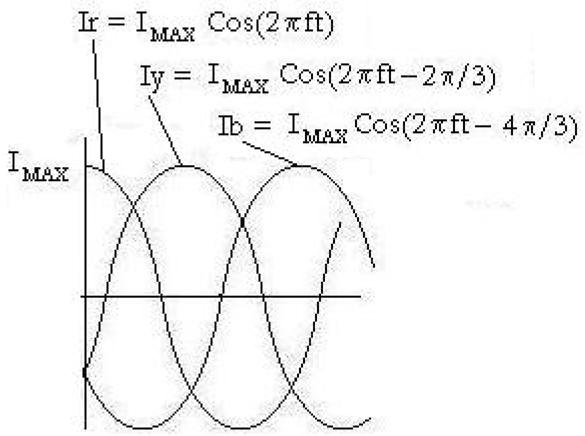


Figure 180; Three phase currents

Let a three phase winding Red, Yellow and Blue spaced at 120^0 have N turns per phase
Let the Red coil cause mmf Fr = Imax N Cos ($2\ \pi ft$) ampere turns in direction 0
Let the Yellow coil cause mmf Fy = Imax N Cos ($2\ \pi ft - 2\ \pi\ /3$) in direction 8
Let the Blue coil cause mmf Fb = Imax N Cos ($2\ \pi ft - 4\ \pi\ /3$) in direction 4

When $t = 0$
Fr = Imax N in direction 0
Fy = Cos ($-2\ \pi\ /3$) Imax N in direction 8 = – 0.5 Imax N in direction 8
= + 0.5 Imax N in direction 2
Fb = Cos ($-4\ \pi\ /3$) Imax N in direction 4 = – 0.5 Imax N in direction 4
= + 0.5 Imax N in direction 10
total mmf = vector sum $Fr + Fy + Fb$ and is in direction 0
Fy has component 0.25 Imax N in direction 0
Fb has component 0.25 Imax N in direction 0
Therefore when $t = 0$, the total mmf = vector sum $Fr + Fy + Fb$
= (1 + 0.25 + 0.25) Imax N = 1.5 Imax N in direction 0

When $t = 1/(12f)$
Fr = Imax N Cos ($2\ \pi ft$) = Imax N Cos ($\pi/6$) = [($\sqrt{3}$)/2] Imax N in direction 0
Fy = Imax N Cos ($2\ \pi ft - 2\ \pi/3$) = Imax N Cos ($\pi/6 - 2\pi/3$) = Imax N Cos ($-\pi/2$) = 0
Fb = Imax N Cos ($2\ \pi ft - 4\pi/3$) = Imax N Cos ($\pi/6 - 4\pi/3$) = Imax N Cos ($\pi/6 - 4\pi/3$)
= – [($\sqrt{3}$)/2] Imax N in direction 4 = [($\sqrt{3}$)/2] Imax N in direction 10
total mmf = vector sum $Fr + Fy + Fb$ and is in direction 11
Fr has component [($\sqrt{3}$)/2] · [($\sqrt{3}$)/2] Imax N in direction 11
= (3/4) Imax N in direction 11
Fb has component [($\sqrt{3}$)/2] · [($\sqrt{3}$)/2] Imax N in direction 11
= (3/4) Imax N in direction 11
total mmf = [(3/4) + (3/4)] Imax N in direction 11 = 1.5 Imax N in direction 11

When $t = 1/(6f)$
Fr = Imax N Cos ($2\ \pi ft$) = Imax N Cos ($\pi/3$) = 0.50 Imax N in direction 0
Fy = Imax N Cos ($2\ \pi ft - 2\ \pi/3$) = Imax N Cos ($\pi/3 - 2\pi/3$)
= 0.50 Imax N in direction 8
Fb = Imax N Cos ($2\ \pi ft - 4\pi/3$) = Imax N Cos ($\pi/3 - 4\pi/3$) = – Imax N

in direction 4 = *I*max *N* in direction 10
total mmf = vector sum *Fr* + *Fy* + *Fb* and is in direction 10
Fr has component 0.5 · 0.5 · *I*max *N* =0.25 *I*max *N* in direction 2
Fb has component 0.5 · 0.5 · *I*max *N* =0.25 *I*max *N* in direction 2
total mmf = (1 + 0.25 + 0.25) *I*max *N* = 1.5 *I*max *N* in direction 2

Therefore in 1/(12*f*) seconds, the mmf rotates from direction 0 to direction 1. This is 1/12 revolution. In the next (1/12*f*) seconds the mmf rotates a further 1/12 revolution. Therefore in one second, the mmf rotates *f* revolutions.

A three phase winding with a cylindrical core, causes an *MMF* that rotates at synchronous speed with a constant value of (3/2) *I*max *N*.

Three phase Induction Motors
The majority of AC motors are three phase induction motors. The stator is wound with three coils spaced equally round the stator. When these coils are connected to a three phase supply, they create a magnetising force that rotates at synchronous speed in the rotor. The synchronous speed is the supply frequency divided by the number of pairs of magnetic poles. Thus the synchronous speed for a 4 pole machine is 1500 rpm on a 50 cps supply. The rotors consist of a cage of bars solidly connected together at each end. Therefore a common name for the rotor is "squirrel cage".

Swapping any two phases causes the rotating field to reverse direction. To change the direction of rotation of a three phase induction motor, swap any two phases.
The rotating field induces an emf in the rotor at "slip" frequency, (ie at a frequency equal to the synchronous speed minus the rotor speed).
Slip $\Sigma = (n_0 - n)/n_0$
where n_0 is the synchronous speed and n is the actual speed.

The emf induced in the rotor is proportional to the slip frequency.
$E = \Sigma E_0$ where E_0 is the emf at slip = 1

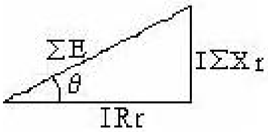


Figure 181; Vector diagram of voltages

Let the Rotor resistance and reactance be R_r and X_r at supply frequency.
Hence the rotor resistance and reactance at slip frequency are R_r and ΣX_r

The rotor current is I shown on the vector diag where;
θ =Arc Sin ($I X_r / E$)
Suppose a resistance ρ is added to the rotor resistance and the load adjusted (if necessary) to maintain the current at *I*. The motor will run more slowly.
With added resistance ρ the slip becomes Σ'. The vector diagram becomes as shown.

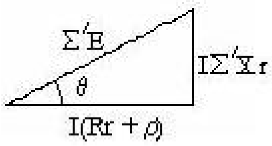


Figure 182; Vector diagram of voltages with added rotor resistance

The phase angle of the current is unchanged at θ =Arc Sin ($I X_r$ / E)

With I and θ unchanged, the magnetic force on each conductor is unchanged, and therefore the torque is unchanged.

Furthermore, since the rotor current magnitude and phase angle are unchanged, the stator current is unchanged in magnitude and phase angle.

Hence by adding resistance to the rotor, nothing is changed except the slip and the speed. Equating values for Cos θ,

$$I R_r \ / \Sigma E \ = \ I (R_r + \rho\ ) \ / \ \Sigma' E$$

Hence $\Sigma ' / \ \Sigma = (R_r + \rho\ ) \ / \ R_r$

Thus Slip is proportional to rotor resistance.

As more resistance is added, the slip increases. We can add resistance till the slip = 1, ie the motor comes to a standstill. Let the resistance for standstill be ρ'

Putting $\Sigma' = 1$ in the value of Cos θ gives

$$1 / \ \Sigma = (R_r + \rho\ ) \ / \ R_r$$

$$\rho' \ = R_r\ (1 - \Sigma) / \ \Sigma$$

where Σ is the slip under normal running conditions.

$$R_r + \rho' = R_r \ / \ \Sigma$$

The only difference between the condition at standstill and at full load is that in one case there is mechanical power output and in the other this same amount of power is consumed by $I^2\rho'$.
Thus $I\rho'$ is equal to the back emf at full load and $I^2\rho'$ is equivalent to the output power per phase.
There are three phases so the total power output is $3\,I^2\rho'$ watts
where $\rho' = R_r\,(1 - \Sigma)/\,\Sigma$ and Σ is the slip at full load.
Power output $= 3\,I^2\,R_r\,(1 - \Sigma)/\,\Sigma$

The motor at standstill has become a transformer. The rotor and stator resistances and reactances can be lumped together as a single resistance and reactance with the appropriate turns ratio. Current is required to magnetise the field and this is added vectorially.

Let R be the total equivalent resistance of the stator and magnetising circuit and X be the total equivalent reactance of the rotor, stator and magnetising circuit.
Then $I = V\,/\,Z = V\,/\sqrt{[(R + R_r + \rho')^2 + X^2]}$
$I = V\,/\sqrt{[(R + R_r\,/\,\Sigma)^2 + X^2]}$
But Σ is small, therefore $R_r\,/\,\Sigma$ is large compared to R
$I \approx V\,/\sqrt{[(R_r\,/\,\Sigma)^2 + X^2]}$

Power output
$P \approx 3\,V^2\,/\,[(R_r\,/\,\Sigma)^2 + X^2]\;\;R_r\,(1 - \Sigma)/\,\Sigma$
$P \approx 3\,V^2\,(1 - \Sigma)\,R_r\,\Sigma\;/\;[\,R_r^{\,2} + X^2\,\Sigma^2\,]$ watts

Let the output torque be T newton metres
$P = 2\pi\,T\,(1 - \Sigma)\,n_0$ watts
$T \approx 3\,V^2\;R_r\,\Sigma\;/\;[2\pi\,n_0\,(R_r^{\,2} + X^2\,\Sigma^2)]$
Plotting T against Σ gives the speed torque curve for an induction motor.

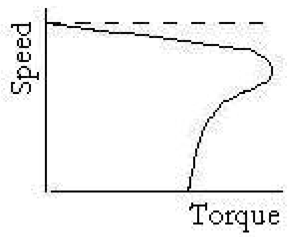


Figure 183; Torque – speed curve

The Speed for maximum torque is given by
$dT/d\Sigma = 0$
$T = A\Sigma/[\,R_r^{\,2} + X^2\,\Sigma^2]$
where $A = 3\,V^2\;R_r\,\Sigma\;/\;[2\pi\,n_0]$
$dT/d\Sigma = A + A\Sigma[\,R_r^{\,2} + X^2\,\Sigma^2]^{-2}(-1)(2\,X^2\,\Sigma\,)$
$= A[\,R_r^{\,2} + X^2\,\Sigma^2]^{-2}[R_r^{\,2} + X^2\,\Sigma^2 - 2\,X^2\,\Sigma^2]$
$= A[\,R_r^{\,2} + X^2\,\Sigma^2]^{-2}[R_r^{\,2} - X^2\,\Sigma^2]$
$dT/d\Sigma = 0$ when $R_r^{\,2} = X^2\,\Sigma^2$ ie when $\Sigma = R/X$
$d^2T/d\Sigma^2 = (-2)\,A[\,R_r^{\,2} + X^2\,\Sigma^2]^{-3}[R_r^{\,2} - X^2\,\Sigma^2] + A[\,R_r^{\,2} + X^2\,\Sigma^2]^{-2}[-\,2X^2\,\Sigma]$
when $R_r^{\,2} = X^2\,\Sigma^2$, $d^2T/d\Sigma^2$ is negative
Therefore T is a maximum when $\Sigma = R_r/X$

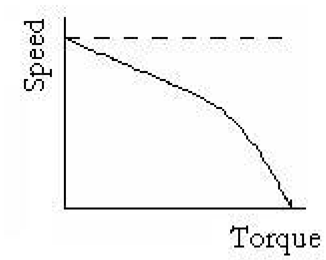


Figure 184; Torque – speed curve

If $R_r = X$ the torque is a maximum when the speed is zero. This makes the motor inefficient at full load, but a wound rotor with slip rings and a variable external resistance are sometimes used where a high starting torque is required.

Induction motors are usually started direct on line, ie the starter switches the supply directly to the winding. Small motors typically take about six times full load current for five or six seconds. Large motors take four or five times full load current for as long as fifteen seconds.

However the starting current is low power factor until the motor is almost up to speed so the current taken from the supply is more likely to be a problem than power taken from the supply. Motors are normally kept on light load till up to speed.

Single phase induction motors

Single phase induction motors start on two phases from a single phase supply. One of the phases has an impedance in series to change its phase. This is often a large capacitance connected through a centrifugal switch which cuts out the capacitance when the motor is up to speed.

Some small motors use a short circuited coil permanently flux linked with one winding. This changes the phase angle of this winding enough for the motor to start and run.

To change the direction of rotation of a single phase induction motor, swap the connections to one of the windings.

Power Factor Improvement

AC Induction motors have inductive coils connected to the supply. The power factor is poor, typically in the range 0.7 to 0.9, usually the lower end. This means that the power loss in the supply to the motor is relatively high. The power factor can be improved by the installation of a bank of capacitors in parallel with the motor. Ideally, the capacitive reactance should be the same as the inductive reactance which would bring the power factor up to unity.

A synchronous motor is an alternative to a bank of capacitors. The synchronous motor can be over excited to take a leading current and can either be used as a motor or run light purely for power factor improvement. The downside is that synchronous motors are more difficult to start and in the event of a system fault may lose synchronism further adding to the disruption.

Three phase winding in slots

If the winding is in a large number of slots distributed equally round the circumference, then each half of each winding covers 60 electrical degrees. Therefore the mmfs of each coil in the winding are not all in the same direction. The mmf of the complete winding is the vector sum divided by the algebraic sum of the emfs of each coil. Thus the mmf of the winding is $3/\pi$ of the sum of the emfs of the coils.

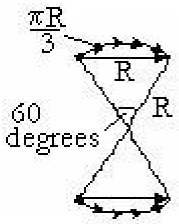


Figure 185; Winding in slots

$F = (3/2)\ (3/\pi)\ N\ I\text{max}\ \text{Sin}(\omega t + \alpha)$

Speed control of ac motors

Speed control of ac motors is possible by thyristor control of the current or voltage. Another method is to rectify the ac into dc and use the dc to power an inverter to give a variable frequency ac supply. Control of motors by semi conductors is the subject of another e-book.

INSULATION

Insulation for electrical machines.
Insulating materials for electrical machines are grouped into several classes.
Class A is organic, eg cotton, linen etc
Class B is inorganic, eg mica, glass, asbestos etc, and is usually impregnated with varnish.

The permitted temperature rise depends on the Class

Class Y	Class A	Class E	Class B	Class F	Class H	Class C
90^0 C	105^0 C	120^0 C	130^0 C	155^0 C	180^0 C	$>180^0$ C

Electrical machines are designed such that the hottest spot in the machine is within the permitted temperature for the Class of insulation when operating on full load in an ambient of 40^0 C.

It follows that at higher ambient temperatures, the rating of the machine is reduced, while at lower ambient temperatures the machine may be operated on a higher load without overheating. At given voltage, power output is approximately proportional to current while copper loss is proportional to the square of the current. Total loss is proportional to the temperature difference between the insulation and ambient temperatures.

Let T_1 be the permitted temperature for the Class of insulation and T_2 the ambient temperature. Let Q be the ratio of the friction, windage and iron loss to the copper loss at full load.
Thus Q = (Watts input on no load)/(I^2 R loss on full load)

Power Output at ambient temp T_2
= Rated Output · $\sqrt{[Q\,(40 - T_2)/(T_1 - 40) + (T_1 - T_2)/(T_1 - 40)]}$

The average winding temperature can be measured by the resistance of the winding. The hot spot temperature will be about 10% higher. If the machine is operated above the permitted temperature, the life of the insulation will be shortened. The effect is logarithmic. For a long life, the machine should be operated well below the permitted temperature.

Class	Class A	Class E	Class B	Class F	Class H
Permitted	105^0 C	120^0 C	130^0 C	155^0 C	180^0 C
10 year life	102^0 C	116^0 C	125^0 C	150^0 C	174^0 C
5 year life	112^0 C	126^0 C	134^0 C	160^0 C	188^0 C
1 year life	128^0 C	145^0 C	152^0 C	178^0 C	206^0 C

Temperature coefficient of insulation materials
Increase in temperature reduces the insulation resistance and the effect is logarithmic. An increase in temperature of 65 ^{0}C reduces the insulation resistance by a factor of 10. The insulation resistance is also dependent on how dry it is. Records of insulation resistance should give the temperature at which the measurement was taken.

TRANSFORMERS

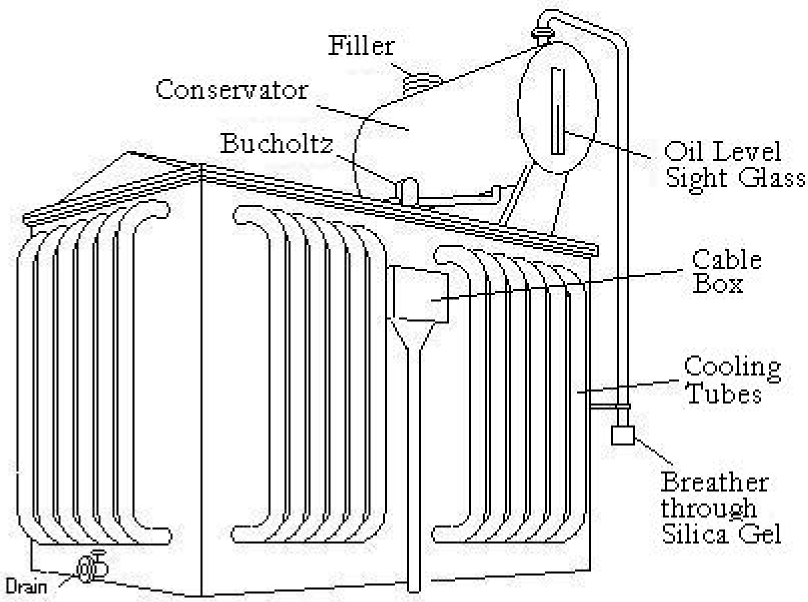


Figure 186; Transformer

Power transformers in electricity supply systems

Transformers are widely used in electrical power supplies. The power loss in the electrical power supply is $I^2 R$, but the power transmitted is $\sqrt{3}\ V\ I$ Cos φ. Thus for a given conductor size, the loss is inversely proportional to V^2.

Electrical power transmission is therefore at high voltage eg 400 kV or 230 kV. The voltage is then reduced to a more manageable value typically 11kV, 33 kV or 66 kV. Finally it is reduced again to about 415 volts to give single phase supplies of about 240 volts.

Construction of power transformers

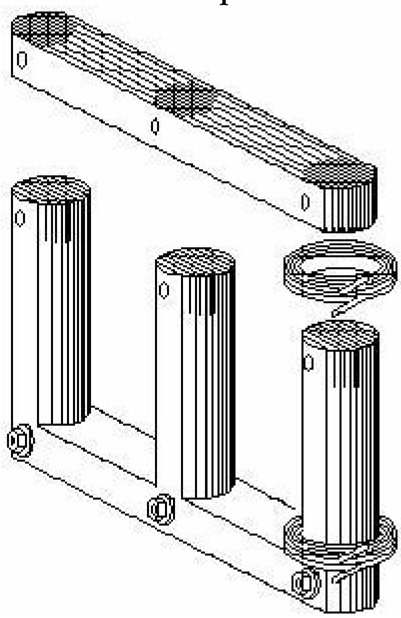


Figure 187; Construction Method

Power transformers are usually oil filled with cooling fins or tubes. The windings are usually wound on a winding machine from strip copper wrapped with paper insulation. The winding on each limb is completed as a number of discs. The core consists of steel laminations, each of which has an insulating material baked on one side. The laminations are assembled with overlapping joints and the top left open.

The winding discs are placed on each limb of the core and the top laminations of the core are then fitted with interleaving joints. The connections between the winding discs are made and the windings are compressed rigid.
The top of the transformer tank is then fitted to the core and bushings are connected to the windings. The assembly is then lowered into the tank and connections are made to cable boxes. The tank top cover is bolted to the tank with an oil tight gasket. When complete, the transformer internals hang from the tank top. The complete transformer is often baked in an oven to thoroughly dry out the paper insulation before filling with oil. The conservator, an oil filled header tank, allows for the expansion and contraction of the oil with a vent to atmosphere through silica gel.

Core Arrangements

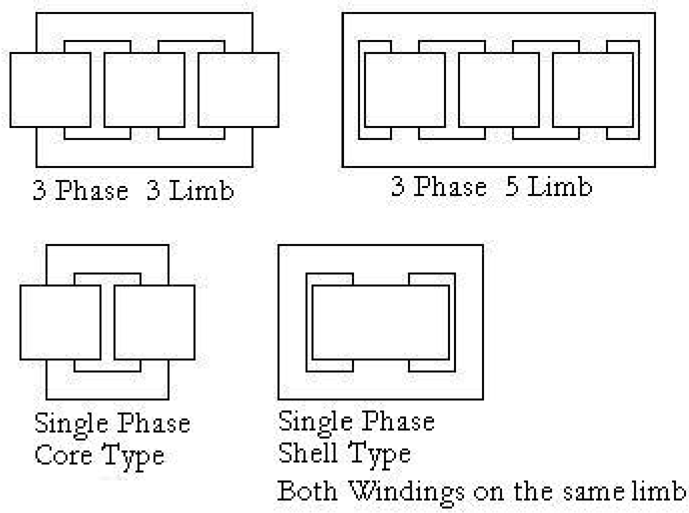


Figure 188; Types of core

3 Phase Power Transformers usually have the Primary connected in Delta and the Secondary connected in Star on a 3 Limb Core. If both windings are connected in Star, then the Core may be 5 Limb.

Single Phase Power Transformers are usually Shell type.

Flux

The flux $\Phi_{max} = [4\,\pi\,\mu\,A\,I_{max}\,N\,/\,L] \cdot 10^{-7}$ weber.
where μ is the permeability of the core, A is the cross sectional area of the core in m^2, I_{max} is the peak value of the magnetizing current in amps, N is the number of turns and L is the length of the magnetic path in the core in metres.

Back emf

The AC supply causes a sinusoidal magnetising force which in turn causes a flux in the iron core.
Let the flux be $\Phi = \Phi_{max}\ \mathrm{Cos}\ (2\pi f t)$

The back emf on a winding of N turns linked with this flux is;

$$E = -N\,\mathrm{d}\Phi/\mathrm{d}t$$

$$E = N\,\Phi_{max}\,2\pi f\,\mathrm{Sin}\,(2\pi f t)$$

Hence $E_{rms} = (1/\sqrt{2})\,N\,\Phi_{max}\,2\pi\,f$

$$E_{rms} = 4.44\,N\,\Phi_{max}\,f$$

where Φ_{max} is in webers, f is in Hz and E_{rms} is in volts

Delta / Star transformation

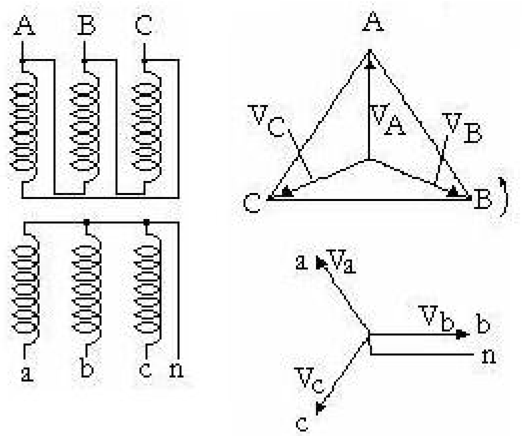


Figure 189; Delta Star transformer

The primary winding of a power transformer is usually connected in delta and the secondary in star with the secondary neutral brought out to a terminal. The secondary ampere turns on each limb of the transformer are balanced by an equal number of ampere turns on the primary. The primary also carries a low power factor lagging current to provide the magnetising force. There are two possible connections. In one the secondary voltage leads the primary by 30^0 and in the other the secondary voltage lags the primary by 30^0.

Star / Star transformation

Very high voltage primaries are sometimes connected in star as all the high voltage connections are then at the top of the transformer. The three phase primary supply does not normally have the neutral connected. This means that there is no circuit on the primary to balance a single phase load on the secondary. A third delta connected winding is usually fitted on a star/star transformer, ie the transformer is star/delta/star. Current circulating in the delta winding allows the transformer to supply a single phase or unbalanced secondary load.

Measurement of losses and efficiency of a power transformer.

The rated primary AC voltage is applied to the primary with the secondary open circuit. A voltmeter, ammeter and wattmeter are fitted to measure the primary input. The instruments measure the magnetising current and iron loss, ie hysteresis and eddy current loss at full volts, as the copper loss is negligible. These give the magnitude and power factor of the magnetising current.

The DC resistance of the primary and secondary windings are measured in another test.

A variable AC voltage is then applied to the primary with the secondary short circuited or alternatively the voltage is applied to the secondary with the primary short circuited. Either way, the current is raised to the rated value. The input voltage, current and power loss are measured. The voltage and current give the impedance and the wattmeter gives the total copper loss as iron loss is negligible. The voltage is the Full Load Impedance Voltage.

The total copper loss is proportioned to the primary and secondary. The AC resistances are assumed to be in the same ratio as their DC resistances to obtain the primary and secondary AC resistances. Hence the primary and secondary copper loss can be obtained, each is $W = I^2R$

The open circuit test establishes the magnitude and power factor of the magnetising current. On full load, the primary winding carries the vector sum of the load current and the magnetising current while the secondary winding carries only the load current. Thus the copper loss in the primary and secondary at full load can be calculated. The total loss = total copper loss plus iron loss.

If the transformer is three phase, the Output Power = $\sqrt{3}\, V\, I$ Cos φ.
If single phase. The Output Power is $V\, I$ Cos φ.

The efficiency at rated current and rated power factor can now be obtained.
The efficiency = Output Power / (Output Power + Total Loss)

The Impedance Volts % = (Full Load Voltage drop) · 100 / (Rated Volts)

If required, the efficiency at part load and/or different power factors can be obtained by the same method. Also, the magnetising current magnitude and power factor can be obtained at lower voltage by the open circuit test carried out at the lower voltage. The curve of AC voltage against magnetising current follows roughly the pattern of the B – H curve for the core.

Inrush of the magnetising current
When the transformer is switched on, the magnetic flux builds up through the hysteresis loops. If the transformer has been recently switched off, there may be very significant residual magnetism in the core. If the transformer is switched on at the instant when the magnetising force is exactly out of phase with the residual magnetism, the core may become very oversaturated. In this case the self inductance of the winding becomes very low and the

magnetising current very high. An initial magnetising current exceeding full load current can occur but will subside to normal within a second. I have seen the ammeter needle bent by the inrush of the magnetising current as the needle hit the stops.

Unbalanced loads
If the load is not balanced three phase, the primary current is related to the secondary by the turns ratio, not by the voltage ratio. Consider each limb of the transformer and convert the secondary current to primary for that limb. Add the primary current vectors on all limbs connected to each phase to get the primary current in that phase.

Example
The diagram shows the size of the stampings for a transformer core. The dimensions are in cms and the core is 2.5 cms thick.

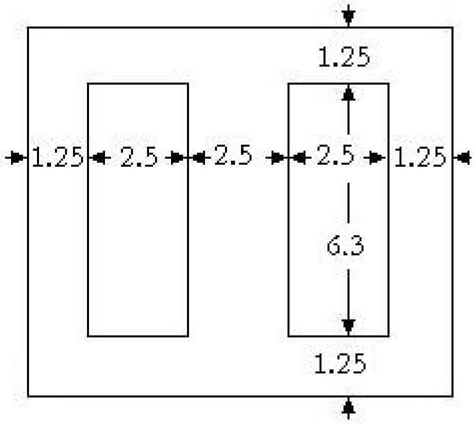


Figure 190; Example core
Stacking factor 0.9
Peak B = 1 Tesla
Space factor for the copper = 0.45
Current density in copper = 2 A/mm^2
Input voltage = 230 volts at 50 Hz Output voltage = 6.3 volts
Calculate the maximum rating and the sizes of wire.

Cross Sectional Area of the flux path = 2.5 · 2.5 · 0.9 = 5.62 cm^2
Peak flux Φ_{max} = 5.62 E–4 weber
$E = 4.44\, N\, \Phi_{max} f = 4.44 \cdot 5.62 \cdot 50 \cdot N / 10000 = 0.125\, N$
Thus the coils have 8 turns/volt

The 230 volt winding has 1840 turns
The 6.3 volt winding has 50.4 turns, say 51

Allow 1.5 mm all round the window.
The window area for the windings is 6.0 · 2.2 = 13.2 cm^2
Net cross section of the copper is 13.2 · 0.45 = 5.94 cm^2
Cross section for each winding is 297 mm^2
Max current in the 230 volt winding is 2 · 297 / 1840 = 0.32 amps
Hence the rating of the transformer = 230 · 0.32 = 74 VA
Wire for the 230 volt winding is 297/1840 = 0.16 mm^2 = 26 SWG
Wire for the 6.3 volt winding is 297 / 51 = 5.82 mm^2
or 4 coils in parallel of 17 SWG enamelled wire

RECTIFIERS

High power rectifiers
Rectifiers convert AC to DC. High power rectifiers provide a high power DC supply, eg for charging a large bank of batteries, for DC traction motors, electric furnaces and similar applications.

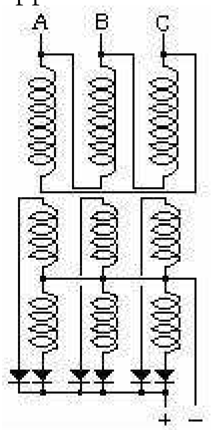


Figure 191; Six Phase High Power Rectifier

The traditional mercury arc rectifier is now largely superceded by semi conductor rectifiers. High power rectifiers are usually supplied from a dedicated transformer that gives a six phase output.

Alternatively the rectifier can be supplied from an interconnected star transformer. Let the turns ratio of the supply be 1:N.

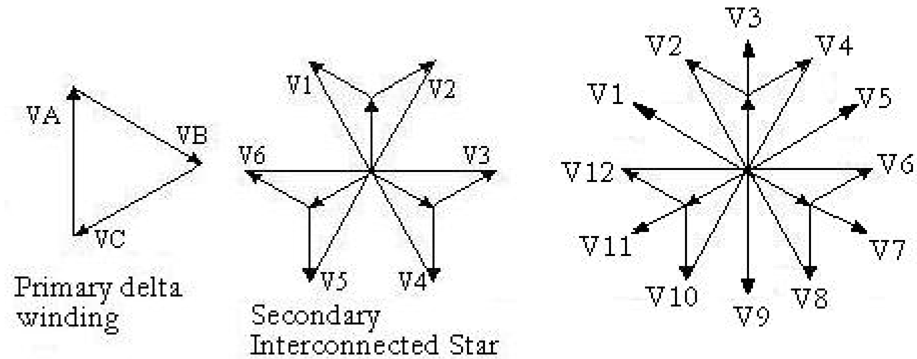


Figure 192; Interconnected Star transformer

An interconnected star transformer has a six phase output.
$V1 = N(VA - VB)$
$V2 = N(VA - VC)$
$V3 = N(VB - VC)$
$V4 = N(VB - VA)$
$V5 = N(VC - VA)$
$V6 = N(VC - VB)$
$V1$ to $V6$ are six phases spaced at 60^0 between phases.

Additional windings can be wound on each limb of the transformer to give a 12 phase output.

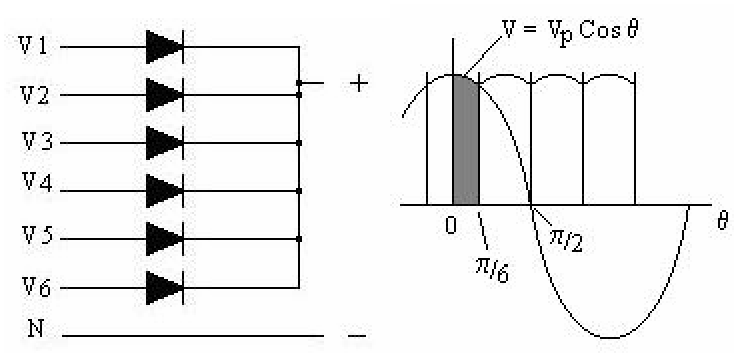


Figure 193; Rectified Voltage

Average DC output voltage = shaded area divided by (π/6)

$$V = (6/\pi) \int V_p \cos\theta \, d\theta \text{ from 0 to } \pi/6$$
$$= (6/\pi) V_p \sin\theta \text{ from 0 to } \pi/6$$
$$= (6/\pi) V_p [\sin \pi/6 - \sin 0]$$
$$= (6/\pi) V_p (1/2)$$
$$= 3 V_p/\pi = 0.955 V_p$$

where V_p is the maximum phase voltage of the six phase supply

Three phase rectification of phase voltages

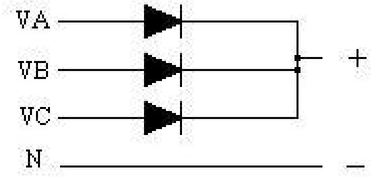


Figure 194; Rectification of phase voltages

Supply from 3 phase voltages spaced 2π/3
Average DC voltage
$V = (3/\pi) \int V_p \cos\theta \, d\theta$ from 0 to π/3
$= (3/\pi) V_p \sin\theta$ from 0 to π/3
$= (3/\pi) V_p [\sin \pi/3 - \sin 0]$
$= (3/\pi) V_p \sqrt{3}/2 = 0.827 V_p$
where V_p is the maximum phase voltage of the three phase supply

Three phase rectification of line voltages

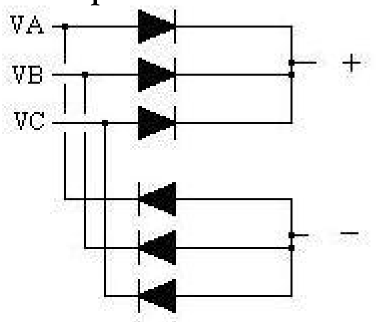


Figure 195; Three Phase Rectification of Line Voltages
Supply from 3 line voltages spaced 2π/3

As above
Average DC voltage
$V = 0.827 V_p$
where V_p is the maximum line voltage of the three phase supply

Single phase full wave rectification

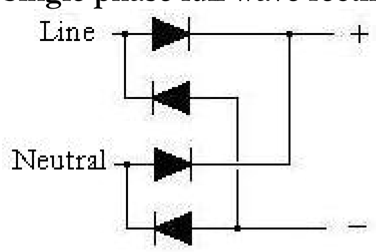


Figure 196; Single phase full wave rectification

Average DC voltage
$V =$ Average value of half a sine wave
$= 0.636 V_p$
where V_p is the maximum voltage of the single phase AC supply

Single phase half wave rectification

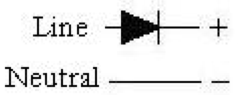


Figure 197; Single phase half wave rectification

Average DC voltage is half full wave value
$V = 0.318\ V_p$
where V_p is the maximum voltage of the AC supply

Capacitance in the DC circuit

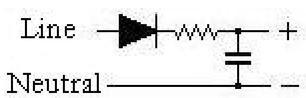


Figure 198; Capacitance in the DC circuit

A capacitance in the DC circuit can raise the DC voltage to the maximum (ie peak) value of the AC voltage. A current limiting device (eg a resistor) is usually fitted between the rectifier and the capacitance otherwise the high initial charging current of the capacitance can damage the rectifier. If there is no DC load, the voltage across the capacitance rises till it reaches the peak value of the supply.

With a DC load, the current flows from the supply only when its voltage exceeds the capacitance voltage. Thus when there is a DC load, the voltage across the capacitance falls to allow current to flow from the supply. The resistor increases the voltage drop even more.

In the diagram, V is the voltage across the capacitance.
Assume the capacitance is large enough for V to be considered constant.
The current flowing from the supply between θ_1 and θ_2 is

$$I = [V_p\ \text{Sin}\ \theta - V] / R$$

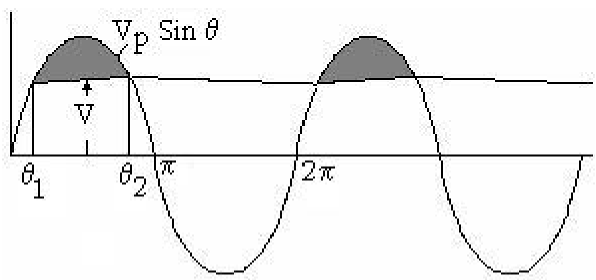


Figure 199; Capacitance in the DC circuit

The total quantity of electricity flowing from the supply in one cycle is proportional to the shaded area.

$$Q = \int [V_p\ \text{Sin}\ \theta - V] / R\ \ d\theta$$

But this is the same as the total quantity of electricity flowing in the load in one complete cycle

$= I_{DC}\ 2\pi$ where I_{DC} is the DC load current.

$I_{DC}\ 2\ \pi = [- V_p\ \text{Cos}\ \theta - V\ \theta\ ] / R$ from θ_1 to θ_2

$= 2\ [- V_p\ \text{Cos}\ \theta - V\ \theta\ ] / R$ from θ_1 to $\pi / 2$

$= 2\ [- V_p\ \text{Cos}\ (\pi / 2) - V\ (\pi / 2) + V_p\ \text{Cos}\ \theta_1 + V\ \theta_1\ ] / R$

$= 2\,[-V(\pi / 2 - \theta_1) + V_p \operatorname{Cos} \theta_1] / R$

But $V_p \operatorname{Sin} \theta_1 = V$

$V_p \operatorname{Cos} \theta_1 - V_p \operatorname{Sin} \theta_1 (\pi / 2 - \theta_1) = \pi R I_{DC}$

Hence θ_1 can be evaluated and hence $V = V_p \operatorname{Sin} \theta_1$

Capacitance in DC output from a multi phase rectifier

Let the supply have Z phases

Then in one cycle, current input = Z times current input of each phase

Other parameters being equal, DC current per phase= I_{DC} / Z

where I_{DC} is the total DC current.

Hence $V_p \operatorname{Cos} \theta_1 - V_p \operatorname{Sin} \theta_1 (\pi / 2 - \theta_1) = \pi R\, I_{DC} / Z$

Evaluate θ_1 and put $V = V_p \operatorname{Sin} \theta_1$

For example

```
CLS
INPUT "Enter the RMS value of the supply "; Vrms
INPUT "Enter number of phases in the supply "; Z
INPUT "Enter the series resistance "; R
INPUT "Enter the average DC current "; Idc
Vp = SQR(2) * Vrms
Xmin = 0 : Xmax = 3.14159/2 : S = 0.1
100 '
Flag = 0
FOR X = Xmin TO X max STEP S
```

```
E = (Vp * COS(X) – Vp * SIN(X)*(3.14159/2 – X)) – (3.14159*R*Idc / Z)
IF X = 0 AND E < 0 THEN PRINT "Resistance too high for output current"
IF X = 0 AND E < 0 THEN END
IF E < 0 AND Flag = 0 THEN NewXmin = X – S : Flag = 1
NEXT X
Xmin = NewXmin: : Xmax = NewXmin + S : S = S/10
IF S > 0.00001 THEN 100
PRINT "X = " ; (Xmin + X max) /2 ;
PRINT "DC voltage = " ; Vp * SIN((Xmin + Xmax)/2)
```

POWER LINES

Transmission lines
Transmission lines are usually on lattice towers and in the UK operate at a voltage of 110 kV, 230 kV or 400 kV three phase. Insulators are nearly always pendant type (ie string insulators). Power lines at these voltages are usually double circuit (ie two independent three phase systems, a total of six power lines and one earth wire).

High Tension Distribution Lines
HT Distribution lines are usually on poles which can be lattice galvanized steel, tubular steel, creosoted wood or reinforced concrete. In the UK, high tension distribution lines operate typically at 11 kV, 33 kV or 66 kV. Lower voltages of 3.3 kV or 6.6 kV have been used but these have no advantage over 11 kV and the power loss is higher.

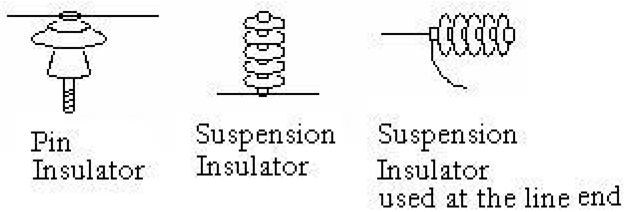


Figure 200; Power line insulators

At voltages up to about 11 kV, line insulators are usually pin type. Above about 11 kV, the insulators are usually suspension type. At the end of the lines and at section poles suspension insulators are used to apply the tension. Section poles and angle poles (where the line deviates from a straight line) are often stronger than intermediate poles. The section and angle poles usually have stay wires to a ground anchor so that the pole can withstand a horizontal force at the top of the pole.

Power line conductors are traditionally hard drawn copper but higher voltage lines now usually use aluminium conductors round a steel central core.

Power lines often have a steel earth wire which is usually above the power lines. This to some extent shields the power line from lightening strikes and prevents a pole becoming "live" in the event of an insulator failure. An earth wire has other advantages but is by no means universal.

Surge diverters
When a power line is struck by lightening, transformers or other equipment connected to the line can be damaged. The risk is reduced by "arcing horns" to provide a spark gap across every insulator and surge diverters at the ends of the line.

Surge diverters (Metrosil or equivalent) are non-linear resistors connected between the lines and earth through a spark gap. The non-linear resistors have a high resistance at low current but the resistance falls by orders of magnitude when the voltage exceeds the rated voltage. A lightening strike is thus short circuited to earth. After the lightening strike, the current falls to a value that should extinguish the arc at the spark gap. The surge diverter will save the switchgear and transformers but may be destroyed by the lightening strike. Doubling the voltage across a surge diverter typically increases the current by a factor of twenty.

Impedance and capacitance

The impedance of power lines is relatively high compared to cables as the self inductance is proportional to ln(d/r). Capacitance however is inversely proportional to ln(d/r) so the capacitance is relatively low compared to cables. Power lines often have a high proportion of steel in their cross section so have a relatively high resistance. Furthermore power lines can have a lower cross section than cables due to the greater cooling of the bare wire in air.

High Tension Underground cables

HT cables are now usually cross linked poly-ethylene (XPLE). These replace the traditional paper insulated cables (PILCSWA&S) ie paper insulated, lead covered, steel wire or steel tape armoured and served.

Buried cable joints are enclosed in a cast iron box in two halves. When the cable cores have been soldered together and insulated with tape, the box halves are united and filled with epoxy resin, or in the case of paper cables, filled with bitumen. XPLE cable joints in air are made watertight by heat shrunk plastic covers.

Cable cores are traditionally copper but aluminium is sometimes used.

Medium Voltage lines and cables

MV cables in the UK operate at about 420 volts three phase four wire or about 240 volt single phase two wire. MV overhead lines are now only used for low cost supplies in rural areas, cables being more normal in urban areas.

MV cables are now usually PVCSWAPVC (ie pvc insulated, steel wire armored and pvc sheathed. Conductors are copper or aluminium. Copper is easier to solder and terminate but aluminium is cheaper.

Large sizes of three or four core cables are difficult to install. A popular arrangement for high current applications is ten single core cables, three each phase and one for the neutral. A single core cable induces a magnetic field. Single core cables therefore can be sheathed in lead but cannot have a magnetic armouring. They are often laid in trefoil, ie in groups of three, one each phase, strapped together to withstand the bursting force of the current. Any magnetic material near a single core cable can be subject to induction heating. Particular care is needed where single core cables pass through a steel bulkhead. If three cables on the same phase pass through a steel bulkhead in line then the impedance of the centre cable is less than the impedance of the cables at each end due to eddy current loss. Thus the middle cable will carry more than its share of the load and may overheat.

Single core PVC cables to cathodic protection ground beds do not last. The chlorine released at the ground bed destroys the PVC. High density polyethylene cables should be used to resist the chlorine.

NEUTRAL EARTHING

Power System Earth connection

The neutral point of generators and transformer secondaries are connected to earth to hold the voltage of each phase constant relative to earth. Failure to do so can result in dangerously high voltages building up relative to earth. All cables on the system have capacitance to earth so an earth fault anywhere on an unearthed AC system will result in current flowing through the fault which may be too small to be detected by the protection system while being large enough to be fatal to humans or animals.

To avoid third harmonic currents circulating between generators, it is usual to earth only one of the neutrals of high voltage generators running in parallel. This is usually done through an earthing resistor to limit the earth fault current to full load. Earthing only one generator neutral can cause problems if there is an earth fault on the system that is cleared at the generator. It leaves the other generators feeding the fault by capacitive current.

Some power stations operate with a reactance in the neutral of each set. However this can resonate with the system capacitance at third harmonic frequency. The problem can suddenly appear when a new feeder is added to the system. If the reactor is iron cored then it can resonate at high current but not at low current. In such cases, the system can be unstable and resonate suddenly without warning. If this happens, there will be a loud 150 hz buzz on all the telephones.

The problem is solved if each generator has a unit transformer, ie each generator is directly connected to a delta/star transformer with the generator switching done on the secondary of the unit transformer. Each generator can have its own neutral connection that is separate from the other generators. The generator third harmonic voltage appears at each end of the transformer delta winding. There is no third harmonic current in the transformer primary so there is no third harmonic voltage on the transformer secondary. The transformer secondaries can all be connected to the same neutral point, either with or without a neutral earthing resistor.

Earth Electrodes

The earth electrodes for a power station or substation are typically steel or copper pipes set vertically in the ground and extending down to the water table. The earth points are usually in pairs with links so that either can be disconnected and tested without disconnecting the station earth. In arid areas, water well drilling equipment may be used to drill and case a pipe down to the water table for use as an earthing electrode.

For safety reasons, the conductors in multicore cables are enclosed in earthed steel armouring. High current single core AC cables may be unarmoured to avoid a magnetic path round the conductor but in that case will have an earthed non-magnetic conducting sheath. The armouring and sheath of multicore cables provide a relatively low impedance connection back to the system neutral.

Power lines often but not always have a separate earth wire above the conductors. This provides a path for earth faults back to the system earth and prevents a pole becoming live. It also provides a measure of protection against a lightening strike, although a lightening strike is likely to break the earth wire while saving more expensive equipment.

Earthing Resistors

Single phase to earth fault currents can exceed the value of any other fault. Therefore to limit the damage done during a fault, a resistor is often installed between the neutral point of high voltage generators and transformers and the system earth. Earthing resistors are usually cast iron in an earthed metal vented enclosure similar to starting resistors for DC motors. Earthing resistors are usually sized to limit the earth fault current to the full load of one generator or transformer. In such an event, the voltage to earth of the faulty phase falls to zero while the other phases rise to line voltage to earth. Earthing resistors are usually rated to carry the full current for 30 seconds only. Protection must be fitted to switch off the faulty circuit within this period.

Neutral earthing of medium voltage power stations

A problem arises with the neutral earthing of medium voltage power stations. If the neutrals of all running generators are earthed, large third harmonic currents will circulate between the generators. The third harmonic voltages on all three phases are in phase. Any difference in the voltage between two generators results in a current of three times the phase current through the neutral. When a set is first synchronized, the emf is much lower on the incoming set than the emf on the running sets. The third harmonic current can prevent the set being synchronized unless the neutral isolator on the incoming set is open until the set is loaded up. This complicates the operating procedure and there is always the risk that the third harmonic current on running sets could overload the sets.

The problem is overcome in some power stations by only earthing one generator neutral, but this risks the much higher danger of losing the neutral connection for the system if this generator trips. In such an event, a consumer's voltage could suddenly rise without warning from 240 volts to 400 volts.

A better arrangement is to have dedicated earthing transformers solidly connected to each section of the busbars. The transformer winding is connected in star and the star point solidly connected to the neutral busbar and to the power station earth. The transformer also has a delta winding. Current circulating through this winding allow a single phase to earth current to flow in the star connected primary winding.

Earthing connections

Earthing connections within a power station or substation are usually made by copper strip with joints riveted and soldered. Generators, transformers and large motors have their metal enclosures connected by copper strip to the station earth. In hazardous areas, the earthing connections may be made by single core insulated cable to prevent the risk of sparking across to adjacent metallic structures.

SWITCHGEAR

Breaking a high power electric current
When the contacts of a switch open, due to the inductance in the circuit, an arc occurs as the current continues to flow. The arc ionises the air and ionised air is a conductor. Thus as the contacts move further apart, the arc is drawn longer and longer causing intense heating. The contacts are quickly destroyed unless there is some mechanism to extinguish the arc. If a circuit breaker tries to clear a fault current higher than its fault MVA rating, the circuit breaker will explode catastrophically.

Air break switchgear
A typical method of extinguishing the arc is the use of an arc chute. The contacts are fitted with horns above the main contacts. The horns are situated in a heat resistant box open at the top and bottom. The heat from the arc causes the air to rise drawing the arc up along the horns and into the heat resistant box. As the hot air rises, fresh air that is not ionised is drawn up between the arcing horns. The box is fitted with vertical partitions greatly extending the path of the arc and ultimately extinguishing it.

Oil break switchgear
The contacts in oil break switchgear are enclosed in a tank filled with oil. The contacts are enclosed in a heat resistant box with passages such that as the oil vaporises the passages direct a blast of oil across the contacts which extinguishes the arc.

Air blast switchgear
The interrupter in air blast switchgear consists of a ball in contact with the end of a tube. When the switch opens, the tube is connected to an air receiver at high pressure. As the ball leaves the end of the tube, the high pressure air is released causing a blast across the contacts of air that is not ionised. This interrupts the current long enough for a separate off load isolator to open. Air blast switchgear is particularly suited to outdoor installations

Vacuum break
Vacuum break switchgear breaks the circuit in a vacuum where there is no air to ionise.

SF6 Switchgear
SF6 switchgear breaks the circuit in a chamber filled with sulphur hexafluoride (SF6).

Closing and tripping mechanism
Circuit Breakers are usually closed by a powerful solenoid. The closing mechanism contains a strut in compression with a knee which can collapse. Normally the knee is "over centre", the strut remains straight and operates in compression. When the circuit breaker is closed, it latches mechanically through this strut. A separate trip coil bends the knee. If the trip coil is energised, the knee bends releasing the circuit breaker. The circuit breaker trips whether or not the closing solenoid is energised or the strutis latched in the closed position. Another method for closing a circuit breaker is to compress a powerful spring by hand or electric motor. The energy in the spring is then used to close the circuit breaker.

Busbars
Circuit breakers connect the circuit to the busbars. The busbars are heavy section copper strips extending the full length of the switchboard. There is one busbar for each phase and for 4 pole circuit breakers a fourth busbar for the neutral. The switchboard is often in two or more

sections with a circuit breaker between each section connecting the busbars. Major switchboards often have duplicate busbars ie a main and a standby busbar for each circuit breaker. In this case a further circuit breaker is required in each section to parallel the busbars. Each circuit breaker then has an off load isolator to select the busbar. This isolator is "make before break" and must be interlocked by a foolproof mechanism so that it can only be operated when the appropriate busbar connecting circuit breaker is closed. Overriding this interlock would cause an explosion.

Plugging type switchboards

Major switchboards are plugging type. Each circuit breaker is plugged into the busbars and circuit. There are two common types, vertical and horizontal plugging. Vertical plugging circuit breakers are on wheels, ie truck type. These are pushed into the switchboard under the busbar and circuit "spouts". The circuit breaker is then jacked up along guides and plugged into the busbar and circuit chamber spouts.

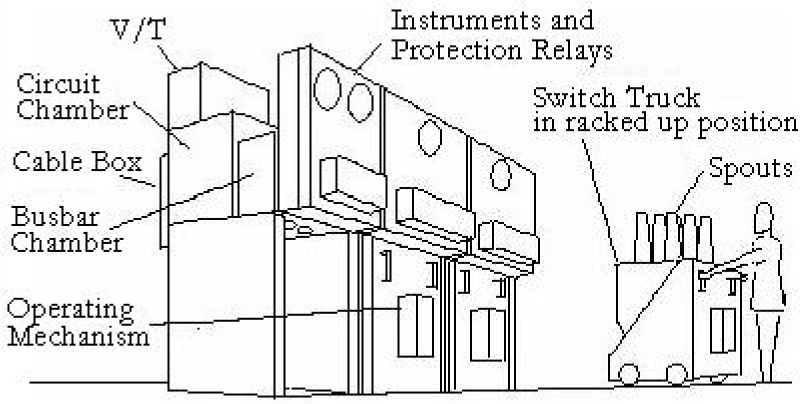


Figure 201; Vertical plugging metalclad switchboard

When maintenance work is to be done on the circuit, the circuit breaker is opened. It is then unplugged. Padlocks are fitted to shutters that close over the busbar and circuit spouts which prevent the circuit breaker being plugged in. The shutter locks play an essential part in the permit to work system.

Horizontal plugging switchgear is racked in or out by a ratchet lever with the circuit breaker running on rails. Again there is provision for locks to be fitted to the busbar and circuit shutters.

Contactors

Medium volt motors are usually switched by contactors. These differ from switchgear in that the contactor is held closed by the contactor coil. If the electricity supply fails, the contactor opens. This is unlike switchgear which latches closed and remains firmly closed until opened by the trip coil. Contactors suffer if the supply voltage falls below a minimum value as the contactor may partially open at the very time that the motor takes a higher current. This can destroy the contacts or in some cases weld them closed.

Instrument Transformers

It is inconvenient to connect ammeters, voltmeters or other instruments directly onto high voltage circuits. They are installed in circuits operating at 110 volts or less and are connected by instrument transformers to the high voltage circuit.

Current Transformer (or CT)

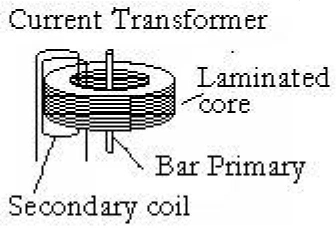


Figure 202; Current Transformer (or CT)

Current Transformers are installed in the Circuit Chamber of the switchgear. These usually have a bar of copper as the primary and a wound secondary. At the design rating, the secondary ampere turns almost exactly equal the primary ampere turns in both magnitude and phase angle. The net magnetising force is small.

The secondary is usually wound for 5 amps at the rated full load but may be other values (eg 1 amp).

The CT is a relatively small device perhaps a 6 inch cube and weighs a few pounds. In metal clad switchgear, they are installed in the circuit chamber, a metal box at ground potential through which the high voltage conductors pass.

If the secondary is open circuited while the primary is on load, the full primary current becomes the magnetising current. This can cause a lethal voltage exceeding 1000 volts on the secondary and can permanently damage the current transformer. Never open a CT secondary circuit when the circuit is on load..

Voltage Transformers (or VT or PT)
Voltage (or Potential) Transformers are connected directly to the high voltage circuit. The VT primary winding is connected across the full circuit voltage. It is usually metal clad with spouts and bushings to plug into the high voltage conductors. The secondary low voltage connection is by plug and socket. The VT can weigh anything from 50 kg to several tons, and its removal from the switchgear usually requires a crane. The VT secondary is usually 110 volts when the primary is at rated voltage.

The primary of a VT can be connected to the three phases in star or in delta. The secondary is also in star or delta to match the primary. However one of the secondary phases, usually the yellow phase, is earthed. The red and blue phases of the secondary are usually at 110 volts to earth at full design primary volts. Thus the secondary red to earth is proportional to the red to yellow primary volts in both magnitude and direction. Similarly the secondary blue to earth volts is proportional to the primary blue to yellow volts in magnitude and direction. Similarly, the secondary red to blue volts is proportional to the primary red to blue volts in magnitude and direction.

Some installations use an "open delta" secondary connection. Here the red to blue winding of the secondary is not connected to the other windings. The three connections to the red-yellow and yellow-blue windings are connected to the red, earth and blue circuits in the normal way, but the red-blue winding is connected to two more secondary terminals allowing this winding to be used for a separated designated purpose, for example a generator automatic voltage regulator.

Circuit breaker close and trip circuits

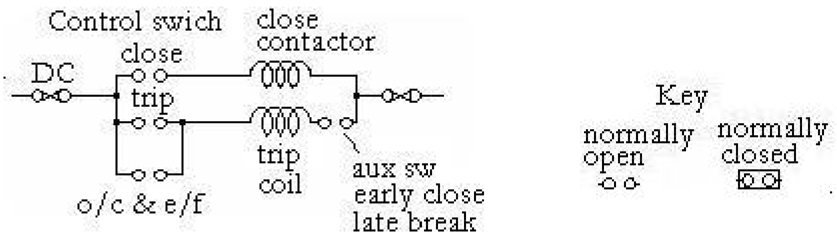


Figure 203; Circuit breaker close and trip circuits

Circuit Breakers are usually closed by a solenoid operated by a contactor and tripped directly by a trip coil. The trip coil and closing contactor operate on DC from a battery but the solenoid may operate on DC from a battery, or on AC or rectified AC. Alternatively, the closing mechanism may be operated by a precharged powerful spring that is released by the closing contactor.

During closing, the auxiliary switch in the trip circuit closes before the main contacts make. During tripping, the auxiliary switch in the tripping circuit opens after the main contacts have opened.

The circuit breaker must not close unless the trip fuses are intact so the closing contactor circuit is usually routed through the trip fuses. There is a slight danger that the closing contactor may be faulty allowing the circuit breaker to close but blowing the trip fuses. Some manufacturers therefore have fuses rated about 15 amps for the trip circuit and additional 4 amp fuses for the closing contactor supplied through the trip fuses.

Circuit breaker indication circuits

The circuit breaker usually has green, red and amber indicator lamps to show if the circuit breaker is open, closed or has tripped on fault.

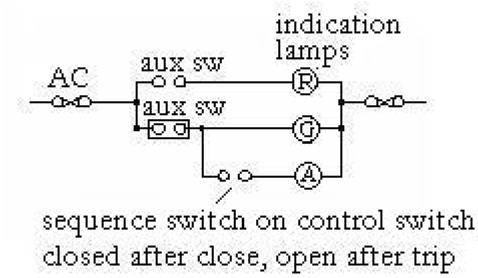


Figure 204; Indication circuits

Current Transformer circuits
The over current and earth fault protection relays and the ammeter are supplied from current transformers on the main circuit conductors. The CT secondary current is proportional to and in phase with the CT primary current.

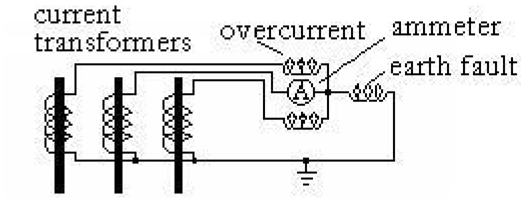


Figure 205; CT circuit

Voltage Transformer circuits
Voltage transformers are usually fitted to incoming circuit breakers. They usually supply a voltmeter and can also supply a kwattmeter, a kwh meter or protection such as directional overcurrent. They can be star/star connected or they can be delta/delta connected.

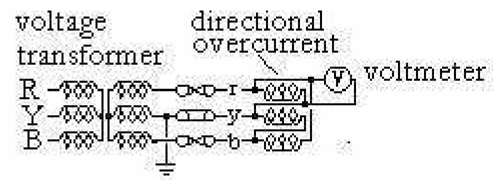


Figure 206; Star connected VT

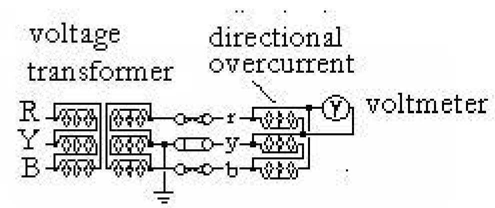


Figure 207; Delta connected VT

With either connection, the secondary r – y, y – b and b – r voltages are proportional to and in phase with the primary R – Y, Y – B and B – R voltages.

Incoming circuit breakers usually have a voltage transformer connected to a voltmeter and the voltage coils of a directional over current relay.

When fitted, a kilowatt meter, a reactive kVA (ie kVAr) meter, a kilowatthour meter, a power factor indicator and a frequency meter each require a supply from the voltage transformer.

A directional earth fault relay requires a special voltage transformer with an open delta secondary. The primary is star connected in the normal way but the secondary winding is connected in open delta. Under normal conditions, there is no output voltage on the open ends of this winding. Under fault conditions, one or more of the primary voltages is reduced giving a voltage on the open delta winding. The phase relation of this voltage depends on the phase or phases suffering the fault as does the phase relation of the fault current.

Alternator circuit breaker circuits
The alternator circuit breaker must not be closed unless;
(i) the voltage is the same as the system voltage
(ii) the frequency is the same as the system frequency

(iii) the voltage is in phase with the system voltage.

Condition (i) above is met by adjacent voltmeters showing the two voltages.
Conditions (ii) and (iii) are met by a synchroscope.

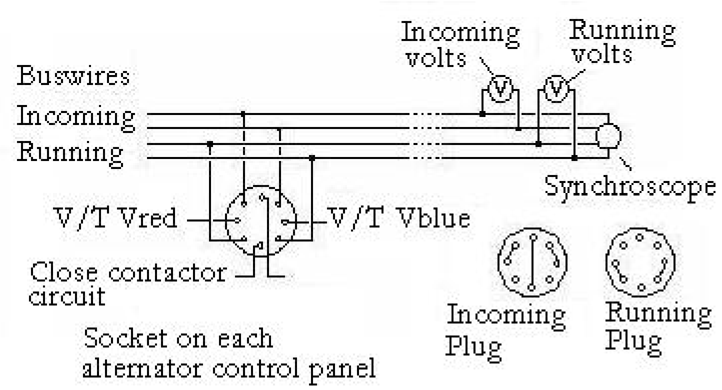


Figure 208; Synchroscope circuit

The diagram shows a well tried system for manually synchronising the sets.
The Running Plug is kept in the socket of a set on load and the Incoming Plug is put in the socket of the incoming set before it is synchronised.

Gas Turbine driven generators often have automatic synchronisation. The gas turbine start button is pressed and the gas turbine goes through the purge cycle driven by the starter motor. When the purge is complete, the starter speed is automatically increased and the fuel is introduced and ignited. The gas turbine runs up to synchronous speed and is automatically synchronised and automatically loaded up till the load is balanced between the generators.

System Time
Two clocks are required, one digital and one electric. The frequency of the system is constantly adjusted to keep the synchronous time in step with the digital time

HV Motor under voltage protection
Motors controlled by a circuit breaker usually have undervoltage protection. When power is restored after a power failure, all motors without undervoltage protection start simultaneously overloading the supply which trips again. With undervoltage protection, the motors can be started in sequence by the operator. Voltage Transformers on busbars are not favoured because of the risk of a fault on the spouts. However, the under voltage can take the voltage supply from the incomer VTs provided it is routed through the circuit breaker auxiliary switches.
The diagram shows a typical arrangement.

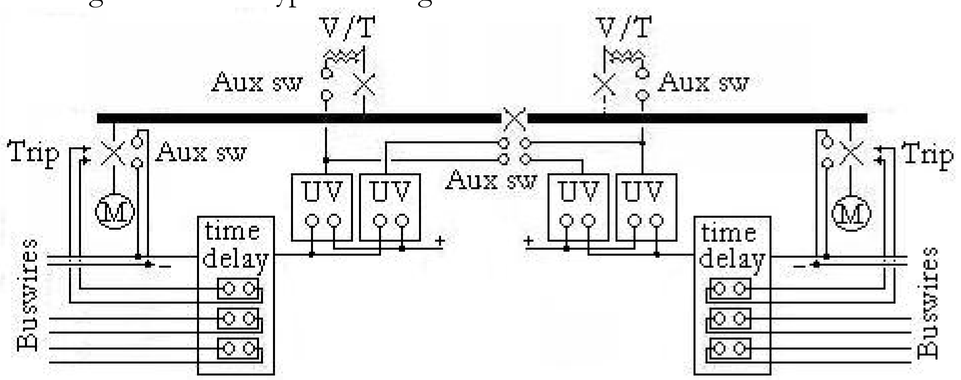


Figure 209; Undervoltage trip circuit for large motors

Contactor circuit

A three phase contactor is usually connected between two phases, ie the coil operates at phase voltage.

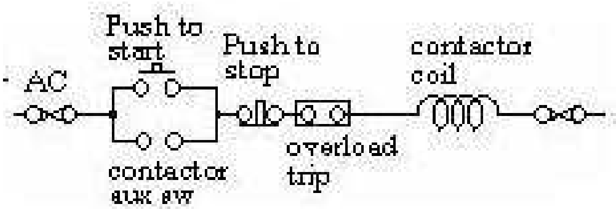


Figure 210; Contactor circuit

When the start button is pressed, the contactor closes and remains closed through an auxiliary contact on the contactor. The contactor is opened either by the stop button or by any normally closed contact in series. The overload is usually thermal (eg bi-metal strips heated by the load current, often with single phase protection). Sometimes there are additional trips (eg earth fault protection). If there is no single phase protection, then a blown fuse may result in the motor burning out. In the event of power failure, the contactor opens. When power is resumed, the contactor remains open until manually restarted.

When the contactor closes, there is initially a large current through the coil limited mainly by the coil resistance. This gives a strong closing force. When the contactor is closed, the air gap in the magnetic circuit is closed, the self inductance of the coil is increased and the coil current is reduced to an economical level.

INSTRUMENTS

Moving Coil Meters

Moving coil instruments measure DC volts and amps or rectified AC volts or amps. They are accurate but are moderately expensive. The scale is linear.

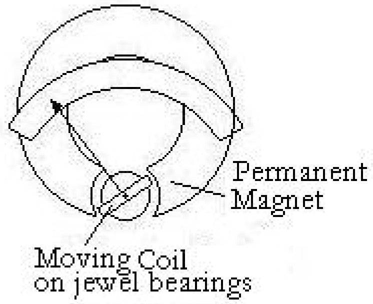


Figure 211; Moving coil instrument

The moving coil is mounted on needle point bearings. It swings between the poles of a permanent magnet and round a cylindrical soft iron core.

The connections to the coil are through spiral springs at the front and back of the spindle which also provide the restraining torque.

The coil is wound with fine wire to give full scale deflection (fsd) with a current which is typically between 50 μA and 1mA and at a voltage typically 75mV.

When used as a voltmeter, it is connected in series with a high resistance.
When used as an ammeter to measure a high DC current, a shunt resistor is connected in the circuit and the meter measures the voltage across the shunt. The connections to the meter are on the shunt between the main connections.

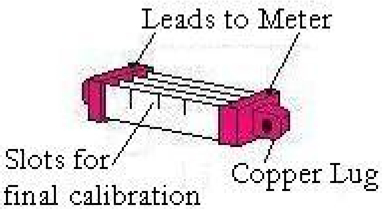


Figure 212; Ammeter shunt

A moving coil meter can be used to measure AC volts or amps when connected to a rectifier, usually a full wave rectifier. The meter measures the average value for a half wave (0.636 of peak value) but is calibrated to display the rms value (0.707 of the peak value). Thus different resistances are needed for readings of DC and AC. The AC reading depends on the form factor of the wave and to some extent on the frequency

A moving coil instrument cannot be used to measure watts.

Moving iron meters

Moving iron instruments contain a fixed and a moving iron element in a fixed coil. The moving iron is restrained by a spring.

Current in the coil magnetises both iron elements similarly
and they repel each other.

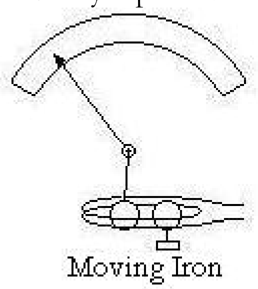


Figure 213; Moving iron

Moving iron instruments measure DC or AC volts and amps.
When calibrated on DC, the meter will read the rms value of mains frequency AC current but due to the inductance of the coil should be calibrated on AC for reading AC volts. The deflection is approximately proportional to the square of the current. On AC the reading depends on the frequency but is not so dependant on form factor.

Moving iron instruments are cheap but not very accurate, typically 5%. Errors arise due to residual magnetism in the moving iron or stray magnetic fields. Instruments on a switchboard are usually moving iron type.

Solenoid type moving iron meter

If the coil is a solenoid pulling the moving iron against a spring, without a fixed iron element, the deflection is more linear. The moving iron can be shaped to give any desired response.

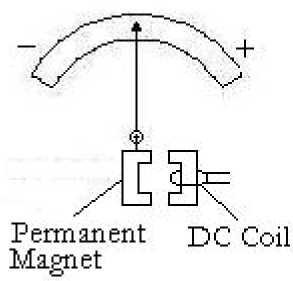


Figure 214; Solenoid type moving iron

Permanent magnet moving iron meter

If the moving iron is a permanent magnet outside the coil, and the coil has a fixed iron core, then the moving iron meter is proportional to the current in the coil and the pointer deflection shows the direction of the current.

Dynamometer instruments

Dynamometer instruments are used for high accuracy measurement of DC or AC volts, amps or watts. The instrument has a fixed coil and a moving coil suspended on a filament. The fixed coil is often in two parts each side of the moving coil. It is intended for laboratory use but can be used with care in the field. Laboratory "sub-standard" voltmeters, ammeters and wattmeters are usually dynamometer type. The moving coil of a dynamometer ammeter is shunted by a low resistance.

For DC or AC voltage measurements, both the fixed and the moving coil are connected in series and in series with a resistance. For accurate measurements of AC volts, the instrument should be calibrated on AC at the same frequency.

For DC or AC current measurements, both coils are connected in series and the instrument measures the volt drop across a shunt. For use on AC, the instrument should be calibrated at the same frequency.

A dynamometer instrument can be used to measure watts. One coil is connected through a series resistance to the voltage and the other coil is connected to a shunt or current transformer in the current circuit. For accurate AC measurements, the instrument should be calibrated on AC at the same frequency. However the self inductance of the coil means that the current in the voltage coil is not in phase with the voltage. There is therefore an unavoidable error in the phase angle between the current in the voltage coil and the current in the current coil unless the instrument has been calibrated at the same power factor.

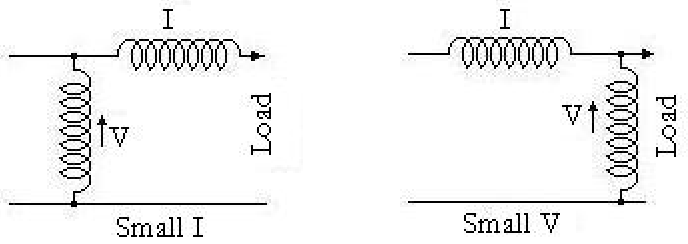


Figure 215; Connection for a dynamometer wattmeter

When used as a wattmeter, the voltage connection can be upstream or downstream of the current coil. If V is small and I is large, the voltage connection should be nearest to the load so that the voltage does not include the drop across the current coil. Conversely, if the voltage is large and the current is low, connect the voltage upstream of the current coil so that the current coil does not include the current in the voltage coil.

A 1st grade voltmeter has an accuracy of 1% of full scale deflection (fsd), ammeter 1½ % and a wattmeter 2½%

Hot wire instruments

Hot wire can measure DC or AC with no frequency or wave form error but have a low overload capacity. They are cheap, not very accurate and are rarely used.

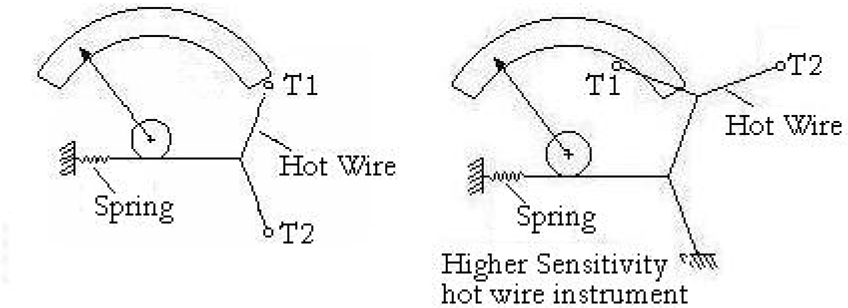


Figure 216; Hot wire instrument

A hot wire instrument contains a resistance wire that is heated by the current. The expansion of the wire is used to deflect the pointer. The long term accuracy is poor and the instrument is easily damaged by overload. There is a delay in the reading as the wire heats up.

However the instrument can be calibrated on DC and will then read *rms* values correctly on AC at any frequency or form factor provided the self inductance is negligible.

Electrostatic Voltmeter

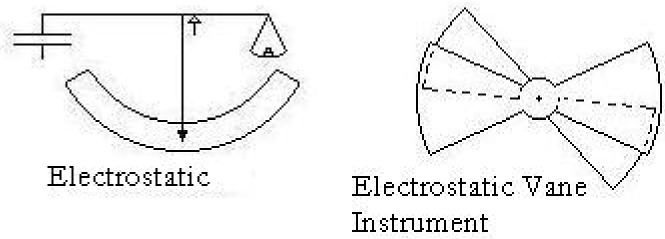


Figure 217; Electrostatic Voltmeter

High voltages can be measured by the electrostatic force between two plates. In the simplest form, the force can be measured by a balance but this is hardly practicable outside a laboratory.

Practical electrostatic voltmeters have a fixed and a moving vane that overlap. The moving vane is restrained by a spring.

When a high voltage, typically tens of kV, is applied between the vanes, they try to increase the overlap.

The voltmeter for a high voltage test set is often electrostatic. On a 0 to 10 kV scale, the markings will not be shown below 5 kV due to insensitivity at low deflections. Electrostatic meters take no current on DC.

DC Amperre Hour or Coulomb meter

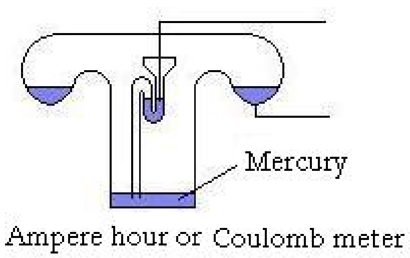


Figure 218; Coulomb meter

DC quantity of electricity can be measured by electrolysis. A typical meter is shown. A glass container is filled with mercury and a liquid mercury salt solution. When current flows, mercury is transferred to the hopper in proportion to the ampere hours. When the tube fills, it siphons out so the quantity in the base is an integral number of the volume in the U tube. Tilt to reset.

Induction meter

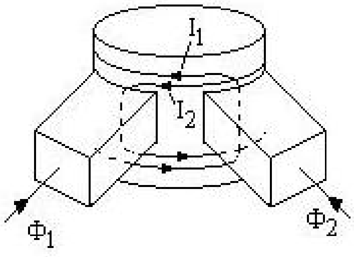


Figure 219; Induction meter

Consider a cylindrical drum of thin metal which is subjected to two AC fluxes Φ_1 and Φ_2 both in the same phase. These induce currents I_1 and I_2 in the drum. By Lens' law, these oppose the ampere turns of the magnetising forces. But due to leakage flux the currents lag the fluxes that induce them.

It can be seen that Φ_1 and I_2 exert an anti-clockwise torque on the drum while Φ_2 and Φ_1 exert a clockwise torque. The drum does not rotate when Φ_1 and Φ_2 are in phase.
The torque due to I_1 is proportional to $I_{1\,rms}\Phi_{2\,rms}\text{Cos}\,\alpha$ where α is the phase angle between I_1 and Φ_2 due to the leakage flux.

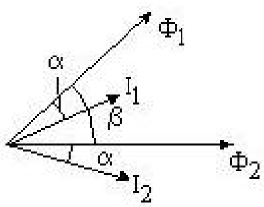


Figure 220; Vector Diagram

Consider now if Φ_1 and Φ_2 are not in phase. Let Φ_1 lead Φ_2 by β.

The torques are proportional to
$\Phi_{1\,rms}\,I_{2\,rms}\,\mathrm{Cos}\,(\alpha+\beta)$
and $\Phi_{2\,rms}\,I_{1\,rms}\,\mathrm{Cos}\,(\beta-\alpha)$ and are no longer equal.

The magnetising forces exert a torque on the drum proportional to

$$\Phi_{2\,rms}\,I_{1\,rms}\,\mathrm{Cos}\,(\beta-\alpha)\; - \Phi_{1\,rms}\,I_{2\,rms}\,\mathrm{Cos}\,(\alpha+\beta)$$

If $\Phi_{1\,rms} = \Phi_{2\,rms}$ then $I_{1\,rms} = I_{2\,rms}$ and

$$\begin{aligned}\text{Torque} &= K\,\Phi_{rms}\,I_{rms}\,[\mathrm{Cos}\,(\beta-\alpha) - \mathrm{Cos}\,(\alpha+\beta)]\\ &= K\,\Phi_{rms}\,I_{rms}\,\mathrm{Sin}\,\alpha\,\mathrm{Sin}\,\beta\end{aligned}$$

Disc Induction Meter

If the drum is rolled out flat, it becomes a disc. The disc will rotate if subjected to the interlinking flux of two coils carrying AC currents which are not in phase.

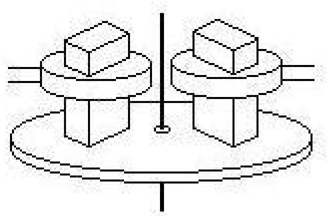


Figure 221; Disc Induction meter

The two coils can be supplied from the same source
if one is supplied through a resistance or capacitance. Alternatively, one of the poles could be a "shaded pole" ie fitted with a thick copper band.

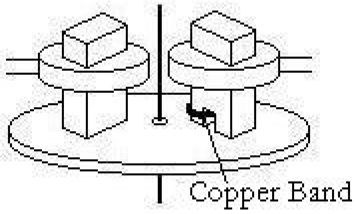


Figure 222; Shaded pole meter

Watt Hour Induction Meter

The Watt Hour meter has one coil connected to the voltage and the current in this coil lags the voltage by 90^0. The other coil is connected to carry the current.

Thus $\beta = 90^0 - \varphi$ and Sin β = Cos φ
The torque is proportional to $V\,I$ Cos φ = watts

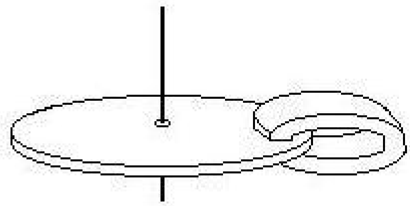


Figure 223; Watt hour meter

A permanent magnet is fitted round the disc. This causes an eddy current loss proportional to (speed) ^{2}.

This loss exerts a braking torque
(braking torque) · (speed) = loss
(braking torque) = K (speed) ^{2}/(speed
= K (speed.)
But the (driving torque) = (braking torque)
Thus (speed) is proportional to (watts)
Hence the (number of rotations) is proportional to (watt hours)

Example

An induction meter has a copper disc. The disc is changed to an aluminium disc of the same size. The speed is to be increased by 50% for the same torque. Find the change in restraining magnet strength.
Specific resistance of aluminium = 1.7 · (specific resistance of copper)

Eddy current loss is $\propto$ $n^2 B_{max}{}^2 t^2 / \rho$
Torque = power/speed $\propto n B_{max}{}^2 t^2 / \rho$
With suffix 2 for new value and suffix 1 for old
$n_1 \; B_{max1}{}^2 t_1{}^2 / \rho_1 \; = n_2 B_{max2}{}^2 t_2{}^2 / \rho_2$
$B_{max2} / B_{max1} = (t_1/t_2) \sqrt{[(n_1/n_2)\,(\rho_2/\rho_1)]} = \sqrt{[(1/1.5)\,(1.7/1)]} = 1.06$
B must be increased by 6%

Fluxmeter

A DC fluxmeter is a moving coil meter without a spring connected to a search coil. When the flux through the coil is changed, a current flows through the meter which is restrained only by its own back emf e.

Meter coil has n turns, area A, flux density B and carries a current i

Search coil has N turns and the flux is Φ, circuit resistance R and inductance L

Torque on meter coil $T = i\,n\,A\,B$

Back emf $e = n\,A\,B\,\mathrm{d}\theta/\mathrm{d}t$ where θ is the deflection.

Let moment of inertia of meter coil be J

$T = J\,\mathrm{d}^2\theta/\mathrm{d}t^2 = J\,\mathrm{d}\omega/\mathrm{d}t$

Eliminate T, $i\,n\,A\,B = J\,\mathrm{d}\omega/\mathrm{d}t$

hence $i = (J/nAB)\,\mathrm{d}\omega/\mathrm{d}t$

Emf in search coil $= N\,\mathrm{d}\Phi/\mathrm{d}t = L\,\mathrm{d}i/\mathrm{d}t + R\,i + e$

Substitute for e and i

$N\,\mathrm{d}\Phi/\mathrm{d}t - nAB\,\mathrm{d}\theta/\mathrm{d}t = L\,\mathrm{d}i/\mathrm{d}t + (RJ/nAB)\,\mathrm{d}\omega/\mathrm{d}t$

Integrate wrt t from 1 to 2

$N(\Phi_2 - \Phi_1) - nAB(\theta_2 - \theta_1) = L(i_2 - i_1) + (RJ/nAB)(\omega_2 - \omega_1)$

At start and finish, $i = 0$ and $\omega = 0$

Thus $N(\Phi_2 - \Phi_1) - nAB(\theta_2 - \theta_1)$

ie the deflection of the meter is proportional to the change in flux

Example

The voltage coil of a wattmeter has inductance 4 mH and is in series with a 2 kΩ resistance.

The meter has been calibrated on DC.

The meter reads 50 watts at a load current of 1 amp at 160 volts and 1 kHz.

Find the % error.

Reactance of coil = $2\pi fL$ = 25.13 ohms

Phase angle of current in voltage coil = arc tan (25.13/2000) = 0.7199 degrees

Cos $\varphi = W/(VI) = 50/(160 \cdot 1)$ therefore φ = 71.790 degrees

True watts = 160 · 1 · Cos (71.790 + 0.720) = 48.09 Hence Error = 4%

PROTECTION

Electrical Protection

Electrical Protection operates in conjunction with switchgear to automatically disconnect faulty equipment. A fault on duplicate feeders is detected by the protection which trips the switches at both ends of the faulty equipment. The supply to consumers is not affected as the other feeder remains in service.

Over Current and Earth Fault

Over current and earth fault protection trips the switch if the current in any phase exceeds the design or if the current in any phase returns through earth. The earth fault current is detected by residual connection of the current transformers. Current in any phase that does not return through another phase or the neutral activates the protection. One method is to route all three phases and the neutral through one current transformer. The secondary then carries the vector sum of these currents.

Many high voltage three wire systems use a resistance in the neutral to limit the earth fault current to no more than full load. In this case sensitive earth fault protection is mandatory, usually operating if an earth fault of 10% full load is detected.

Over current and earth fault protection is usually time lagged so that the switches trip in sequence till the fault is cleared. The most common type is Inverse Definite Minimum Time Lagged (IDMTL O/C & E/F) protection. The relay is induction type and the angle of rotation gives the time lag. The time lag is inversely proportional to the current up to about 20 times the setting. Above this, the time lag is constant and typically can be set at any value between 0.1 and 2 seconds.

In many cases, time lagged over current and earth fault protection on its own is unsatisfactory. Circuit Breakers closest to the power station are the last to trip. Thus faults near the power station, which are the heaviest faults take longest to clear.

HRC Fuses

Medium Voltage Feeders are often protected only by High Rupturing Capacity Fuses. These clear the fault in a fraction of a cycle, before it has risen to its first peak value. Thus faults with a prospective high fault level can be safely cleared.

Instantaneous Over Current and earth fault protection

If the equipment has relatively high impedance (eg a transformer), then over current and earth fault protection, which is set to operate above the maximum through fault value, can be instantaneous. Earth fault protection on the primary side of a delta/star transformer is not operated by an earth fault on the secondary of the transformer. It can have a low time setting that is governed only by the length of time for the initial inrush to decay.

Circulating Current Protection

This protection compares the current flowing into the protected equipment with the current leaving. If they are not the same, the protection operates.

Restricted Earth Fault Protection

Restricted Earth Fault protection is a form of circulating current protection. Current transformers are fitted to each phase and the neutral and the vector sum is measured. Any current leaving a phase that does not return through another phase or the neutral trips the protection. Restricted Earth Fault Protection only detects earth faults within the protected zone.

Distance Protection
This protection measures both current and voltage. The voltage divided by the current gives the impedance to the fault. The time lag of the protection is related to the impedance. If the impedance is low, the protection clears quickly. If the impedance is high, the time delay is longer. Thus switches nearest the fault clear the fault first. Heaviest faults are cleared quickly.

Generator Protection
AC generators are usually provided with circulating current or restricted earth fault protection. Circulating current protection applied to each winding protects against all faults on the winding except inter turn faults on the same winding. This protection has current transformers in the neutral pit and in the main output circuit breaker. The protection operates if current entering a winding from the neutral does not leave through the main circuit breaker. If the star point of the generator is within the windings, then circulating current protection must be of restricted earth fault type. Restricted earth fault protection does not protect against faults between windings. However inter turn faults and faults between windings will quickly develop into earth faults.

If the current transformers are not exactly balanced, a high value through fault may operate the circulating current protection incorrectly. To avoid this, some relays have a "bias" restraining coil of a few turns which carries the through current. Alternatively, a series resistance of the right value in the relay circuit can be used and the protection is then called Unbiased Circulating Current Protection.

Simple time lagged overcurrent protection is unsatisfactory for generators. Over current protection without voltage restraint may not operate due to the demagnetizing effect of a heavy low power factor current. The over current protection must be time lagged to allow feeders to clear the fault before the power station is shut down. By this time, the fault current may have fallen below the overload setting of a relay that does not have voltage restraint. Generators are therefore usually provided with voltage restrained time lagged overcurrent protection. At full voltage, the current setting is above full load, but at lower voltage, the current setting is lower.

Reverse power protection is often provided. If the generator prime mover shuts down, the generator may act as a motor and keep the machine rotating. Reverse power protection prevents this.

Negative Phase Sequence protection is sometimes provided for large generators. This operates if the currents in each phase are so different that the uneven heating in the generator will damage the generator.

Over and Under Voltage Protection is sometimes provided to shut down the machine if the AVR has become faulty causing the voltage to go out of range. However this may not operate if several generators are running in parallel.

Cables and Power Lines
Cables and Distribution lines are usually provided with IDMT Over current and Earth Fault protection. Transmission lines use more sophisticated protection eg Distance Protection.

Circulating current protection can be applied to cables or power lines but then a pilot cable or carrier wave is needed to transmit the information to the far end for comparison. One pilot cable system converts the three phase current to a single phase voltage with different conversion factors for each phase. The protection system at each end compares the two voltages and trips the local circuit breaker if they are not the same. Transmission lines often use a high frequency carrier wave on the power line to transmit the information and it can also be used for telecommunication between the substations.

Directional overcurrent and earth fault protection is usually fitted at the receiving end of duplicate power lines or cables unless circulating current protection is fitted. This is to prevent a feed back to the fault. It is usually time lagged to trip before the overcurrent on the healthy feeder.

Transformer Protection
Instantaneous earth fault protection with a low current setting can be provided on the primary side of a delta/star transformer. An earth fault on the secondary side is seen by the primary as a phase to phase fault. Instantaneous over current protection can be fitted on the primary provided it is set at a current above the through fault value.

Time lagged over current protection is required on the primary to protect against overload and through faults that are not cleared elsewhere. This should be on all three phases. The traditional two over current and earth fault relay is unsatisfactory as phase to phase faults on the secondary of a delta/star transformer are seen as one high current and two lower currents on the primary.

Thus fault clearance times would depend on which phases the fault occurs. A three phase overcurrent relay and a separate earth fault relay are preferable.

Directional overcurrent and earth fault protection is usually provided on the secondary side of duplicate transformers to prevent the secondary feeding back to a fault on the primary side. As this current may be at nearly zero power factor, appropriate phase displacement of the voltage connection is needed.

The secondary windings of a transformer are usually protected by Restricted Earth Fault Protection. This protection operates if the vector sum of the three phases and the neutral on the secondary is above the setting, typically 10% full load. The setting is low to protect against faults near the star point.

Circulating current protection can be used with appropriate current transformer ratios. For a delta/star transformer, the CTs are connected in star on the primary and delta on the secondary. The protection must be time lagged to allow the magnetizing current to stabilize. The magnetizing current is seen as an in zone fault and this current can exceed full load current for several cycles when the transformer is switched on. Furthermore, many transformers have a tap changer to allow the turns ratio to be changed. The current setting of circulating current protection must be high enough to prevent spurious operation with an out of zone through fault on any tap setting.

Buchholtz Protection detects faults in oil filled transformers. It consists of a chamber in the oil pipe between the transformer and its expansion tank. It contains a float switch which sounds an alarm if air or vapour collects in the chamber. This is often an indication that there is a fault between turns in the winding or a fault between laminations in the core and provides early warning that maintenance is needed.

A separate trip switch is operated by a flap in the Buchholtz chamber. In the event of a more severe fault, there is a rush of oil or vapour through the Buchholtz chamber which operates this flap tripping the switches on the primary and secondary of the transformer.

Large transformers are often fitted with an oil temperature trip and a "hot spot" temperature trip. This is a temperature trip in a chamber in the oil with a self contained heater carrying a current proportional to the load current.

Motor Protection

Overload Protection and Single Phase Protection is normally provided for large motors. The overload protection is usually thermal, ie operated by bi-metal strips. The single phase protection operates if the thermal response of each phase is not the same on all phases.

Instantaneous over current and earth fault protection is also usually provided. The instantaneous over current protection is set above the starting current value. Current can only exceed this value when there is a fault so operation can be instantaneous. The earth fault protection is usually set at about 10% full load. If the motor is star connected, a fault near the star point may not be detected until it has developed into a more serious fault.

Time delayed Under Voltage protection is usually provided for large motors. If the supply fails all motors stop. If the supply is then restored, all motors try to start simultaneously overloading the supply which will then trip again. Time lagged under voltage protection trips the motor when the supply fails and the motors can then be started in sequence manually when power is restored.

Small motors are usually provided with thermal overload and single phasing protection. High Rupturing Capacity Fuses (HRC Fuses) give protection for short circuits. Small motors are usually supplied through contactors which trip in the event of a temporary power failure allowing them to be started in sequence.

POWER SYSTEMS

Underlying philosophy
All essential supplies are through duplicate feeders. The protection is arranged to automatically isolate any faulty equipment from both the supply and from feedback through the duplicate (healthy) equipment.

Typical System supplied by two generators

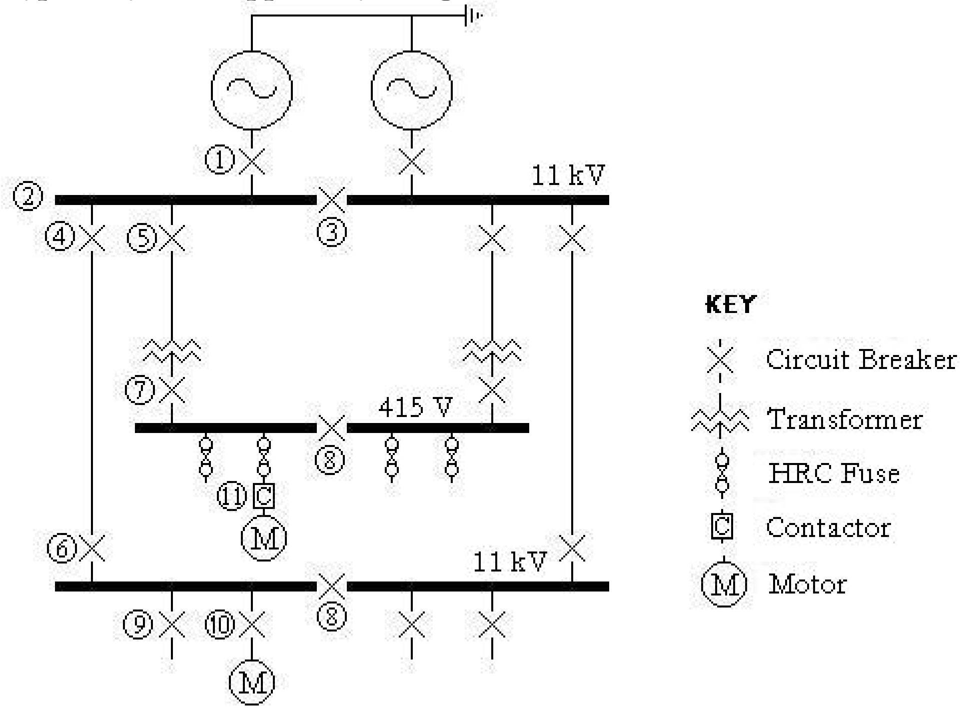


Figure 224; Typical Power System

Protection, typical settings and typical instrumentation
(1) Alternator
Circulating Current or Restricted Earth Fault set at 10% full load to trip the circuit breaker and the field suppression switch.
(If the Star Point is inside the generator winding then use Restricted E/F)
Voltage Restrained Over current set at 100% full load 1.3 sec
Earth Fault set at 10% 1.3 sec
Reverse Power to prevent the generator being driven as a motor.
Negative Phase Sequence Protection if required

Typical instruments are ammeter(s), kW meter, kVAr meter, power factor indicator, field voltage and field current.

The incoming voltmeter, running voltmeter, incoming frequency meter, running frequency meter, system time clock, master time clock and synchroscope are on the synchronising panel

2) **Busbars**
Circulating Current protection is sometimes fitted to each side of the power station busbars. The total net current flowing into each phase of the busbars is measured and if not nearly zero all circuit breakers on the bus section and the bus section circuit breaker are tripped after a short

time lag. Provision is provided to switch off the protection to allow injection tests on any circuit breaker.

3) **Bus Section Circuit Breaker**
Over current and Earth Fault Protection set at 100% and 10% 0.9 sec. This is not required if there is busbar circulating current protection. Even without Circulating Current protection, it is often not fitted. Busbar faults are very rare and when they do occur, usually result in total shutdown of the generating station.

4) **Feeder**
Over current and Earth Fault Protection set at 100% 0.5 sec
Typically the only instrument would be an ammeter unless
kW or kWh are required in which case a VT will be needed.

5) **Transformer Feeder**
Over current, set at 100% 0.5 sec
High set instantaneous over current protection (set above the through fault level)
Earth Fault, 10% full load 0.1 sec
Circulating Current (eg Mag Balance) is sometimes fitted
Buchholtz protection
Inter tripping with LV circuit breaker
Typically the only instrument would be an ammeter.

6) **Incomer**
Directional Overcurrent set at 50% 0.1 sec and Directional Earth Fault set at 10% 0.1 sec sensitive to lagging currents in reverse direction to normal.
Typically a voltmeter and ammeter would be fitted which requires a voltage transformer..

7) **Transformer Incomer**
Either Circulating Current protection or Restricted Earth Fault protection for the LV windings set at 10% 0.1 sec

Directional Overcurrent set at 50% 0.1 sec and Directional Earth Fault set at 10% 0.1 sec sensitive to zero power factor lagging currents in reverse direction to normal.
Inter tripping with HV circuit breaker
Typically a voltmeter and ammeter would be provided.

8) **Bus Section**
Over current and Earth Fault protection is sometimes fitted but serves little useful purpose. It could be time lagged with the feeders to isolate one side of the switchboard in event of a very rare busbar fault. However to get discrimination, the feeder time lags must be increased causing excessive delay in clearing the much more common feeder cable faults.
An ammeter is sometimes provided, but is rarely necessary.

9) **Feeder**
Overcurrent protection set at 100% 0.1 sec and Earth Fault protection set at 10% 0.1 sec.
Typically, the only instrument would be an ammeter.

10) **HV Motor feeder**
High set instantaneous over current protection set above starting current level.
Instantaneous Earth Fault protection
Thermal overload set at 100% CMR (continuous maximum rating) of the motor.
Single phasing protection with thermal delay
Under voltage with time delay of 5 sec
Trips for mechanical reasons (eg bearing temp)
Trips for driven machine faults (eg pump low suction)
An ammeter is required both on the circuit breaker and on the remote stop/start station.

11) **MV Motor feeder**
Over current fault protection is provided by HRC Fuses.
The motor is controlled by a contactor with thermal overload and single phasing protection.
Earth Fault protection is often provided
Under voltage protection is inherent in the contactor.
Trips for mechanical reasons can be incorporated.
An ammeter is required both on the contactor and on the remote stop/start station.

Typical Supply to several substations on an HV ring main where interruptions to the supply are to be kept to a minimum

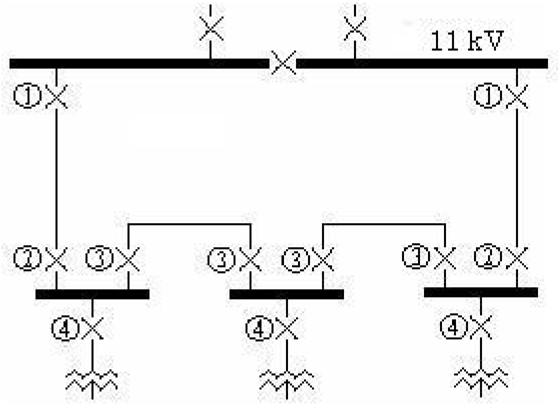


Figure 225; Ring Main

1) **Feeders**
Over current and Earth Fault Protection set at 100%, 0.5 sec

2) **Incomers**
Directional Overcurrent protection set at 50% 0.1 sec and
Directional Earth Fault protection set at 10% 0.1 sec. This provides fault protection for the incoming cable and also prevents feedback via the ring main if the bus section switch in the main substation is opened.

3) **Inter Connectors**
Circulating Current protection on the interconnecting cable set at 10% 0.1 sec. This requires a pilot wire cable to be laid with the main power cable

4) **Transformer Feeder**
Over current Protection set at 100%, 0.1 sec
Earth Fault Protection set at 10%, 0.1 sec

A fault on any cable is cleared without interrupting the supply to any consumer. Any number of substations can be added to the ring main without increasing the fault clearance times. But the system requires pilot wire cables to be laid with the interconnector cables and this adds to the cost.

Low cost Ring Main System

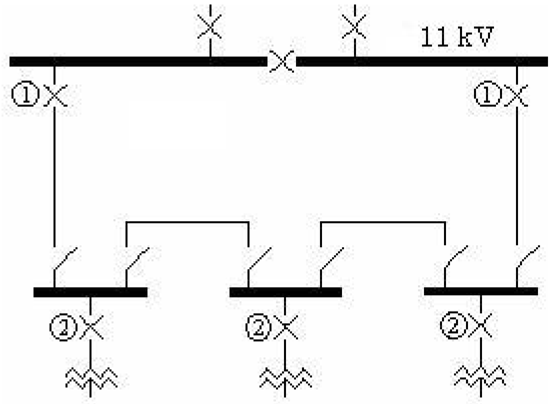


Figure 226; Low cost Ring Main

1) Feeders
Over current Protection set 100%, 0.5 sec
Earth Fault Protection set at 10%, 0.5 sec
Directional Over current set at 50% 0.1 sec and Directional Earth Fault Protection set at 10% 0.1 sec. (To prevent feedback via the ring main if the bus section switch is opened)

2) Transformer Feeder
Over current Protection, 100%, 0.1 sec
Earth Fault Protection, 10%, 0.1 sec

This system uses Ring Main Units each containing one circuit breaker and two isolators. The system is operated with one of the isolators open. If a fault occurs on any interconnecting cable, then all consumers on the same part of the ring main loose their supply. However the faulty cable can be quickly isolated and power restored to all consumers by opening or closing the appropriate isolators.

The transformer in each substation is protected by the circuit breaker. Some ring main units use the system current to trip the circuit breaker. The circuit breaker has three or four trip coils each connected directly to a current transformer. A fuse is fitted across each trip coil. When the current exceeds the rated value, the fuse blows and the circuit breaker is tripped. Thus no tripping battery or charger is required. Earth fault protection is obtained from a "core balance" current transformer, ie the CT primary contains all three phases.

GENERATOR RESPONSE TO SYSTEM FAULTS

Generator size.

The reactance of a coil is proportional to the cross sectional area of the core and inversely proportional to the length of the flux path. The reactance is therefore proportional to the size. The resistance of a coil is proportional to the length of wire and inversely proportional to the cross sectional area of the wire. The resistance of a coil is therefore inversely proportional to the size. The ratio L/R is therefore proportional to the square of the size.

In large generators, (10MW or larger) the leakage reactance per phase is two orders of magnitude larger than the winding resistance per phase. Therefore, if there is a short circuit on the generator output, the current is at near zero power factor. If the generator is on load, the short circuit lowers the voltage reducing the load while the fault at near zero power factor adds very little load so the generator speeds up. For small generators, (10kW or less) the leakage reactance is of similar order to the resistance. The power factor of a short circuit is larger and the fault current adds load. The generator slows up and a short circuit can stall the generator driver.

Armature reaction.
A balanced three phase current in the armature winding causes a rotating magnetic flux of constant value equal to (3/2) times the peak flux due to one phase. In generators, the *MMF* due to the armature reaction of a balanced three phase current rotates at synchronous speed and is proportional to the armature current. The *MMF* due to armature reaction therefore gives a flux in the main field magnetic circuit. If the armature current has a lagging power factor, armature reaction reduces the field. If the armature current has a leading power factor, armature reaction boosts the field.

The rotating *MMF* due to a three phase armature current is $(3/2)\sqrt{2}\, I_a N$ ampere turns where I_a is the rms value of the armature current and N is the "equivalent number of turns per phase of the armature winding". As the winding is actually conductors in slots, the "equivalent number of turns per phase" has the value of number of turns in a coil to give the same ampere turns as the armature with the same current.

The *MMF* due to armature reaction causes a magnetic flux in the main field magnetic circuit. This flux links with the main field winding and the damper winding. By Lenz's law, the magnetic flux cannot suddenly change. Therefore if a short circuit occurs on the generator output, currents are induced in these windings to initially prevent the change in flux. As the flux does not instantly change, the ac *EMF* does not instantly change.

The induced currents in the main field coil and damping winding decay exponentially. As these currents decay, the flux and ac *EMF* decay with the same time constants.

A heavy fault current on a large generator is low power factor lagging so the final steady state current (if it were allowed to persist) can fall below the full load value. Even with maximum excitation, the current can fall below full load current due to the large demagnetization of the low power factor fault current.

Time constants of Damper Winding and Main Field Winding
Let the available space for the coils be divided so that a cross sectional area of copper A_1 is used for the main field and a cross sectional area A_2 is used for the damper winding, see Figure 227.

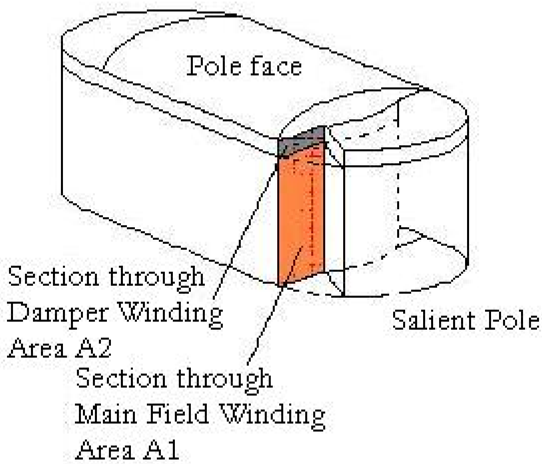


Figure 227; Time constants of windings on a salient pole

Let there be N_1 turns on the main field and N_2 turns on the damper winding. Let the length of a single turn be s.

The resistance of the main field winding is given by;
$R_1 = \sigma N_1 s / (A_1 / N_1) = \sigma N_1^2 s / A_1$
where σ is the resistance of a unit cube.

Similarly, the resistance of the damper winding is given by $R_2 = \sigma N_2^2 s / A_2$

Let the self inductance of one turn be L
The self inductance of the main field is $L_1 = N_1^2 L$
The self inductance of the damper winding is $L_2 = N_2^2 L$

Thus the time constant of the main field is;
$L_1 / R_1 = N_1^2 L / [\sigma N_1^2 s / A_1] = L A_1 / (\sigma s)$
Similarly the time constant of the damper winding is;
$L_2 / R_2 = L A_2 / (\sigma s)$

Thus the time constants are in the ratio A_1 / A_2 and the number of turns does not affect the circuit time constant. The time constants are proportional to the volume of the winding. The time constant of the damper winding is therefore much less than the time constant of the main field winding.

Works short circuit test

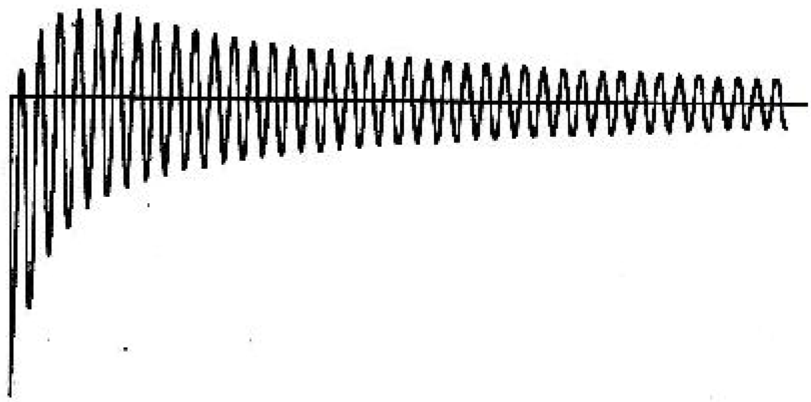


Figure 228; Short circuit current

The manufacturers carry out tests on their generators. The generator is driven at full speed and full voltage open circuit. The generator is then switched onto a zero impedance short circuit. The current is recorded and analyzed. The current recording of each phase of a three phase short circuit test on constant excitation would decay as the armature current reduces the field.

The initial value of the fault current is determined by "the *EMF* due to the flux crossing the air gap" acting on the leakage reactance of the generator. The flux, the *EMF* and the armature current all decay with the same two time constants.

The current is zero when the switch is closed. Therefore the ac current is displaced above or below zero to bring the initial value to zero. This causes a dc component which decays in a few cycles. The magnitude of the dc component on each phase depends on when, in the ac cycle of that phase, the switch is closed. Figure 228 shows an extreme case with the maximum dc component. The ac current combined with the dc component is called the asymmetric value.

If there is risk of damage due to the mechanical forces of the fault current, then the dc component must be considered. The maximum asymmetric peak value of the fault current is often taken as 1.8 times the calculated peak ac value. The figure of 1.8 is the theoretical two times reduced by the approximate decay over a quarter cycle to the first peak.

Ignoring the dc component, the rms value of the fault current can be measured from the recording and will be found to be of this form;

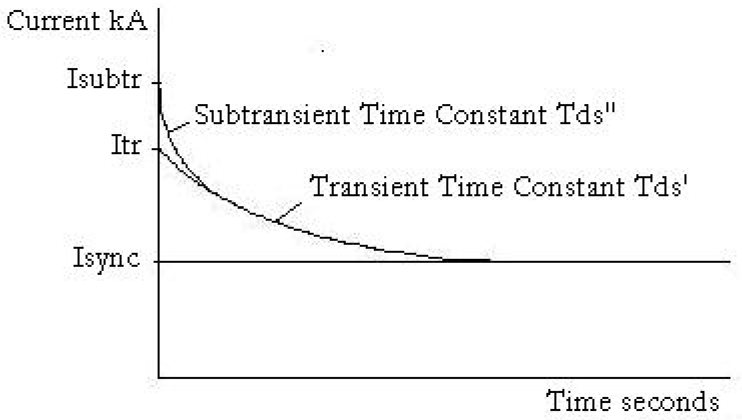


Figure 229; RMS value of the short circuit current

The initial value of the ac current, *Isubtr*, is called the subtransient value.
Extrapolating back along the transient curve gives the transient current *Itr*

The final steady state current, *Isync*, is called the synchronous value.
The recorded current can be analysed to evaluate the subtransient current *Isubtr*, transient current *Itr* and synchronous current *Isync*. The subtransient and transient direct axis short circuit time constants Tds'' and Td' respectively can also be evaluated from the recording.

On a full size machine, the resistance is negligible compared to the reactance. The subtransient reactance Xd'', the transient reactance Xd' and the synchronous reactance Xd are defined as;
$Xd'' = E_0 / Isubtr$
$Xd' = E_0 / Itr$
$Xd = E_0 / Isync$ where E_0 is the open circuit phase to neutral voltage.
The initial subtransient voltage E_0 is the emf before the short circuit. Thus Xd'' is approximately the leakage reactance with the field saturated for full volts open circuit.

The open circuit characteristic

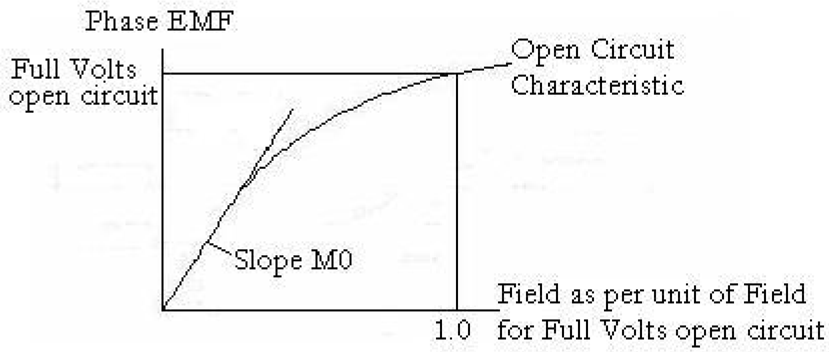


Figure 230: Open Circuit Characteristic

The short circuit characteristic

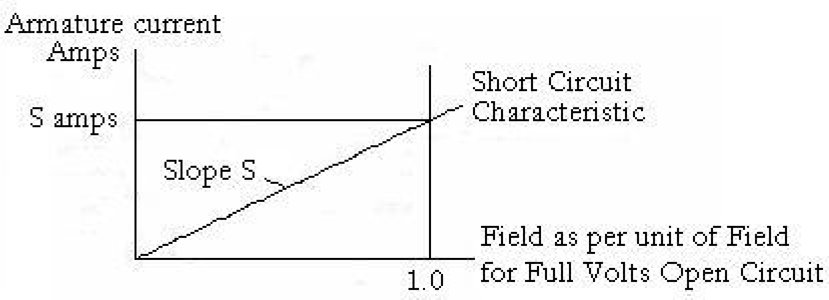


Figure 231: Short Circuit Characteristic

Armature leakage reactance
The armature leakage reactance can be measured with the rotor locked in one position. On a salient pole machine, the reactance depends on the rotor position. When the field poles are under the winding, the armature leakage reactance with Field 1.0 is the subtransient direct axis reactance Xd''. Between poles, the reactance is the subtransient quadrature axis reactance Xq''.

The effect of iron saturation on the leakage reactances.

The values of Xd'' and Xq'' depend on the gradient of the magnetisation curve. They are higher at low magnetic flux than at high magnetic flux. The values at the magnetic flux for full volts open circuit are quoted as "saturated reactances". The values at zero magnetic flux are quoted as "unsaturated reactances".

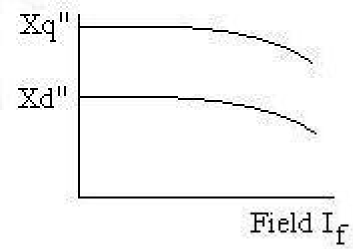


Figure 232; Xd'' and Xq''

The initial value of the short circuit current is at full field.
Therefore Xd'' is the saturated value and the initial current *Isubt* is given by;
$Isubt = E_0/Xd''\,sat$
where $Xd''\,sat$ is the saturated value of the leakage reactance.

The final value of the fault current is at near zero field as the fault current has demagnetised the field. The leakage reactances in the final synchronous state approach the unsaturated values.

The flux due to armature reaction is on the same magnetic circuit as the main field.
The magnetic flux = $\Phi - \Delta\Phi$
where Φ is due to the field current If and $\Delta\Phi$ is due to the armature current Ia
Ia provides the *MMF* to give $\Delta\Phi$ in the air gap plus the *MMF* to give $\Delta\Phi$ in the iron
$\Delta\Phi = K\, Ia\, /\, [\, L_1 + L_2/\, \mu]$
where K is a constant, L_1 is the length of the air gap, L_2 s the length of path through the iron and μ is the permeability of the iron which is proportional to the slope of the saturation curve.
The reactance Xd'' is proportional to $\Delta\Phi\, /\, Ia$
$Xd'' = 1/[A + B/(\mathrm{d}V/\mathrm{d}If)]$
where A and B are constants and $\mathrm{d}V/\mathrm{d}If$ is the slope of the saturation curve.

The flux due to Xq'' is at a right angle to the main flux. Therefore Xq'' is constant for low values of field current.

Approximate values for Xd'' and Xq'' can be obtained by applying an external voltage to one phase of the armature and rotating the armature manually. As the armature is rotated, the value of the armature reactance changes. The minimum value is Xd'' and the maximum value is Xq''. Both values depend on *If*, Figure 232..

Evaluation of the short circuit armature currents and the armature reaction constant *N*

Consider a generator with negligible resistance on full volts open circuit switched onto a zero impedance short circuit.

The resistance of a full size machine is negligible compared to the reactive impedance.
Let I'' be the part of the current that decays with the time constant Td'' and I' be the part that decays with the time constant Td'.

The armature current $Ia(t)$ at time t seconds after closing the switch is given by;
$Ia(t) = I'' \exp(-t / Td'') + I' \exp(-t / Td') + I\,sync$
Where $I'' = Isubtr - Itr = E_0 / Xd'' - E_0 / Xd'$
And $I' = Itr - Isync = E_0 / Xd' - E_0 / Xd)$
And $I'' + I' = (Isubtr - Isync) = E_0 / Xd'' - E_0 / Xd$
$E_0 = (Isubtr - Isync) / (1/Xd'' - 1/Xd)$.
Thus $I'' = (Isubtr - Isync)(1/Xd'' - 1/Xd') / (1/Xd'' - 1/Xd)$.
And $I' = (Isubtr - Isync)(1/Xd' - 1/Xd) / (1/Xd'' - 1/Xd)$.
where Xd'' is the saturated value.

Consider the initial subtransient current.
With negligible resistance, the initial value of the current is;
$Isubtr = E_0 / Xd''sat$ where $Xd''sat$ is the saturated value of Xd''.

Consider the final current after the decay. This is the short circuit characteristic test.
The field before the short circuit is the field for open circuit full voltage (per unit value = 1)
The final current is S amps, where S is the slope of the short circuit test.
The reduction in field due to armature reaction is proportional to id the direct axis component of armature current. With negligible resistance, the final current S is all direct axis current.
Let N be the equivalent number of turns per phase of the armature winding such that N times the armature current is the ampere turns due to armature reaction.
In the final synchronous state, the reduction in field due to armature reaction = $N\,S$
The final net field MMF is therefore $(1 - N\,S)$ ampere turns.
Let $Esync$ be the value of E in the final synchronous state when demagnetisation is complete.

Esync is small and is near the origin on the open circuit characteristic where the slope is M_0.
Esync = $M_0 \cdot$ (net field ampere turns) = $M_0 \cdot (1 - N\,S)$
But armature current $S = Esync\ /\ Xd''unsat$
where $Xd''unsat$ is the value of the direct axis leakage reactance with the flux to give *Esync*.
$M_0 \cdot (1 - N\,S) = S \cdot Xd''unsat$
$N = (1/S) - Xd''unsat\ /\ M_0$
One definition of *Xd* is the ratio of the slope of the open circuit curve near the origin to the slope of the short circuit curve
$Xd = M_0\ /\ S$
With this definition of *Xd*,
$N = (Xd - Xd''unsat)\ /\ M_0$
This equation is only valid if there is negligible resistance in the armature winding.

Similarly, on the quadrature axis, the quadrature *EMF* in steady state conditions is;
$Eq = iq\ (Xq - Xq'')$

Consider now the short circuit current for a generator with armature resistance *R* ohms/phase.
The generator is run at full volts open circuit and switched onto a zero impedance short circuit.
If the generator is laboratory size, the resistance is not negligible. The short circuit is not zero power factor.
The volt drop due to *id*, the direct axis component of the armature current is $id\ Xd''$. The volt drop due to *iq*, the quadrature component of current, is $iq\ Xq''$.

When the switch is closed, the flux does not immediately change. Currents induced in the damper winding and main field winding prevent the change. The *EMF* is proportional to the flux. Therefore the *EMF*s do not immediately change when the switch is closed. Thereafter, the flux and *EMF*s fall as the induced currents decay.

Evaluate the initial (subtransient) short circuit current Isubtr.
The subtransient current *Isubtr* can be found in terms of the open circuit *EMF* E_0, the reactances Xd'' and Xq'' and the resistance *R*. On short circuit, $V = 0$.
The vector diagram of voltages is shown in Figure 233.
On open circuit, E_0 leads the pole axis by 90 degrees.
In the subtransient state, the reactances are the saturated values.

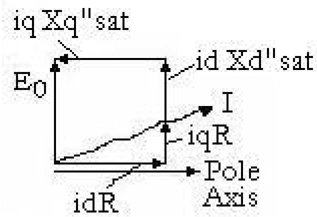


Figure 233: *Isubtr*

$iq\ Xq''sat = id\ \mathrm{R}$
$E_0 = iq\ \mathrm{R} + id\ Xd''sat = id\ (\ \mathrm{R}^2\ /\ Xq''sat + Xd''sat)$
where E_0 is the phase emf at full volt open circuit voltage leading the pole axis by 90 electrical degrees.
$id = E_0\ /\ (\ \mathrm{R}^2\ /\ Xq''sat + Xd''sat)$
$iq = id\ \mathrm{R}\ /\ Xq''sat$
$Isubtr = \sqrt{(id^2 + iq^2)}$

Consider the final (synchronous) short circuit current.
The final current is S amps where S is the current at per unit field on the short circuit characteristic.

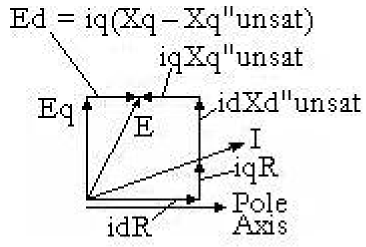


Figure 234; *Isync*

Let *Idsync* and *Iqsync* be the components of S on the direct and quadrature axes.
Let $Xd''unsat$ and $Xq''unsat$ be the unsaturated values of Xd'' and Xq''.
$Ed = Iqsync\ (Xq - Xq''unsat)$ is the *EMF* due to armature reaction on the quadrature axis.
where Xq is a measure of the armature reaction on the quadrature axis.
$Iqsync\ Xq = Idsync\ \mathrm{R}$
$S = \sqrt{(Idsync^2 + Iqsync^2)} = Idsync\ \sqrt{(1 + \mathrm{R}^2 / Xq^2)}$
Therefore;
$Idsync = S / \sqrt{(1 + \mathrm{R}^2 / Xq^2)}$
$Iqsync = Idsync\ \mathrm{R} / Xq$
$Eq = Iqsync\ \mathrm{R} + Idsync\ Xd''unsat$
$Ed = Iqsync\ (Xq - Xq''unsat)$
$E = \sqrt{(Ed^2 + Eq^2)}$
$N\ Idsync$ is the reduction in field due to armature reaction
N = (Field amps – field amps to give Ed) / *Idsync*

Now consider the armature current when the generator on full volts open circuit is switched onto an external impedance resistance *Re* and reactance *Xe*.

The Field due to the exciter is kept constant at 1.0 per unit value.

Evaluate the subtransient short circuit current.
By Lenz's law, the initial value of the flux and therefore the ac *EMF*s are unchanged. Currents are induced in the coils linking with the flux opposing the change.

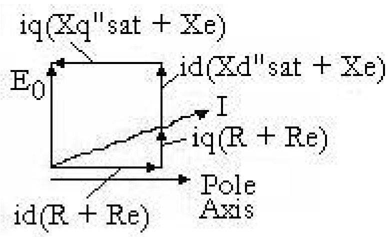


Figure 235: *Isubtr* with R , *Re* and *Xe*

E_0 is the full volt open circuit voltage leading the pole axis by 90 electrical degrees.
$Eq = E_0$ and $Ed = 0$

Let *id* and *iq* be the direct and quadrature axis components of the subtransient current *Isubt*.
Figure 235 shows the vector diagram of the initial subtransient voltages.
$iq\ (Xq''sat + Xe) = id\ (\mathrm{R} + Re)$

$E_0 = iq\,(R + Re) + id\,(Xd''sat + Xe)$
$\quad = id\,[\,(R+ Re)^2 / (\,Xq''sat + Xe) + Xd''sat + Xe]$
Therefore $id = E_0\, /\, [\,(R+ Re)^2 / (\,Xq''sat + Xe) + Xd''sat + Xe]$
And $\quad iq = id\,(R + Re)\, /\, (Xq''sat + Xe)$
$Isubtr = \sqrt{(id^2 + iq^2)}$

Evaluate the synchronous short circuit current.

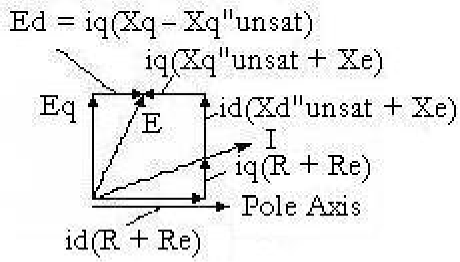


Figure 236: Isync with *R* and *X*e

Let *id* and *iq* be the direct and quadrature axis components of the synchronous current.
$Xd''unsat$ and $Xq''unsat$ are the unsaturated values.
$iq\,(Xq + Xe) = id\,(R + Re)$
$Eq = id\,(Xd''unsat + Xe) + iq\,(R + Re)$
$\quad = id\,(Xd''unsat + Xe) + id\,(R + Re)^2\, /\, (Xq + Xe)$
Eq is the voltage on the open circuit characteristic at net field of (1.0 – *N id*)
where *N* has the value obtained in the short circuit test.
$id = Eq\, /\, [(Xd''unsat + Xe) + (R + Re)^2\, /\, (Xq + Xe)]$
$iq = id\,(R + Re)\, /\, (Xq + Xe)$
$Idsync = \sqrt{(id^2 + iq^2)}$

Consider now a generator on full load with a load resistance *RL* and reactance *XL*.

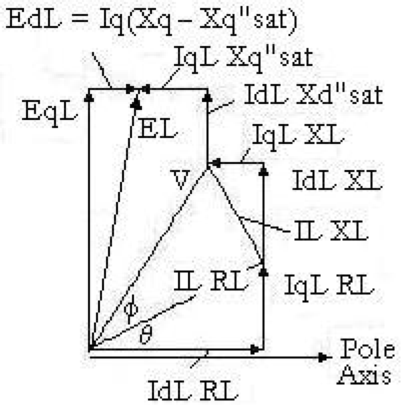


Figure 237; Full Load steady state

Use the suffix L for the synchronous on load condition.
Let the load be represented by a resistance *RL* and reactance *XL* per phase.
The vector diagram shows the full load steady state.

Let full load current be *IL* and full load be *W* watts
Let busbar voltage be *V* and full load power factor be Cos ϕ
$3\ IL^2\,RL = W$
$RL = W\, /\, (3\ IL^2)$
$XL = RL\,\mathrm{Tan}\,\phi$

The vector diagram shows the phase voltages on full load.
Tan $\theta = RL / (Xq + XL)$
$IdL = IL \text{ Cos } \theta$
$IqL = IL \text{ Sin } \theta$
The condition is steady state $EdL = IqL\,(Xq - Xq''sat)$
$EqL = IdL\,(XL + Xd''sat) + IqL\,RL$

Let the generator on full load with a load resistance *RL* and reactance *XL* be subjected to a fault resistance *RF* and fault reactance *XF*.
Let the combined load and fault resistance be R and the combined reactance be X.

Evaluate the subtransient current and busbar voltage with combined load and fault
When the fault occurs, EdL and EqL are initially unchanged.
Give the subtransient parameters the suffix 0 (ie at time $t = 0$).
The quadrature axis *EMF* is no longer $Iq(Xq - Xq'')$. Iq changes immediately but, due to induced currents on the quadrature axis, the quadrature axis *EMF* is initially unchanged at EqL.

The fault impedance RF and XF is in parallel with the load RL and XL.
The fault impedance $ZF = \sqrt{(RF^2 + XF^2)}$
The load impedance $ZL = \sqrt{(RL^2 + XL^2)}$
Put $A = RL/ZL^2 + RF/ZF^2$ and $B = XL/ZL^2 + XF/ZF^2$
Then $R = A / (A^2 + B^2)$ and $X = B / (A^2 + B^2)$

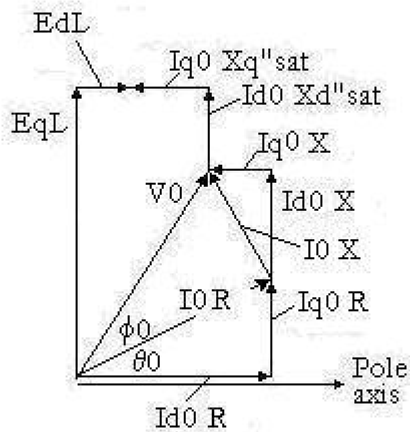


Figure 238: Subtransient Vector diagram

The vector diagram of subtransient voltages are shown in Figure 238..
$Iq0\ (Xq''sat + X) = Id0\ R - EdL$
$Iq0 = (Id0\ R - EdL)\ /\ (Xq''sat + X)$

$EqL = Iq0\ R + Id0\ (Xd''sat + X)$
$EqL = (Id0\ R^2 - EdL\ R)\ /\ (Xq''sat + X) + Id0\ (Xd''sat + X)$

Therefore the subtransient parameters can be evaluated
$Id0 = [EqL(Xq''sat + X) + EdL\ R\ ]\ /\ [R^2 + (Xd''sat + X)\ (Xq''sat + X)]$
$Iq0 = (Id0\ R - EqL)\ /\ (Xq''sat + X)$
$I0 = \sqrt{(Id0^2 + Iq0^2)}$
The busbar voltage $V0 = I0\ \sqrt{(R^2 + X^2)}$

Evaluate the synchronous current with combined resistance *R* and reactance *X*.
The final synchronous state is the same as has been investigated above. The combined load and fault resistance is *R* and the combined load and fault reactance is *X*.

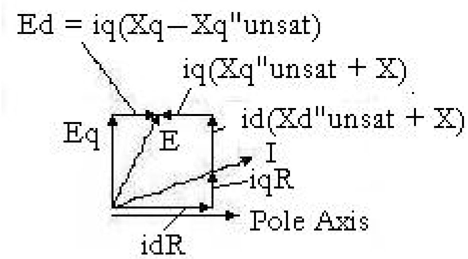


Figure 239: Synchronous current with *R* and *X*

The condition is final steady synchronous state
$Edsync = Iqsync\ (Xq - Xq''unsat)$
$Iqsync\ (Xq + X) = Idsync\ \ R$
$Eqsync = Iqsync\ R + Idsync\ (Xd''unsat + X)$
$\qquad = Idsync\ R^2/(Xq + X) + Idsync\ (Xd''unsat + X)$
$Idsync\ \ = Edsync\ /[R^2\ /\ (Xq + X) + (Xd''unsat + X)]$

Let *FeL* be the field for full load.
Keeping the field current constant, the field in the synchronous state with fault current
Net field = *FeL* – *N Idsync*

The Open Circuit Characteristic can be represented approximately by four straight lines, slopes *M1*, *M2*, *M3* and *M4* with intercepts *U2*, *U3* and *U4* which meet at points (*F1*,*E1*), (*F2*,*E2*) and (*F3*,*E3*).

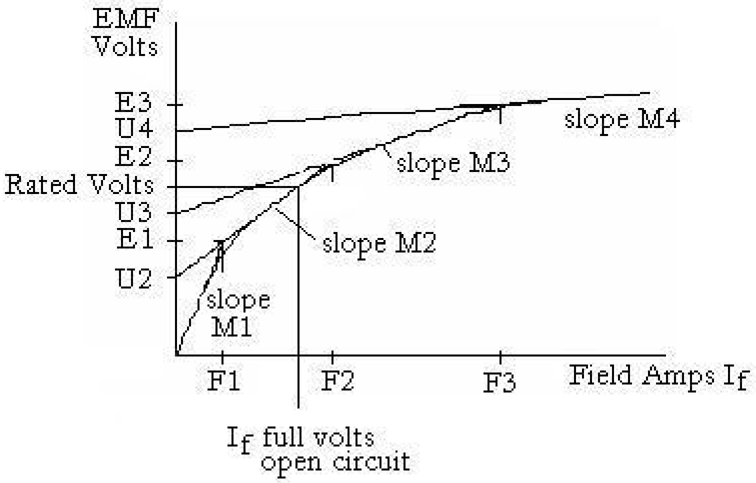


Figure 240: Open circuit characteristic

Let the open circuit curve be represented by a series of tangents each expressed by values for intercept *U* and slope *M*.
The tangents intersect at points (*F1*,*E1*), (*F2*,*E2*) and (*F3*,*E3*)

$Eqsync = U + M\,(FeL - Idsync\ N)$
Where *U* and *M* have values appropriate to *Eqsync*.

Therefore;
$Idsync = [U + M\,(FeL - Idsync\ N)] / [X + Xd''unsat + R^2 / (Xq + X)]$
$Idsync = [U + M\ FeL] / [X + Xd''unsat + R^2 / (Xq + X) + M\ N]$
This evaluates *Idsync*

$Iqsync = Idsync\ R / (Xq + X)$
$Isync = \sqrt{(Idsync^2 + Iqsync^2)}$

Current at time *t* seconds after the switch is closed with constant excitation
The direct axis current *id* changes from the subtransient to the synchronous value with two time constants *Td''* and *Td'*. The quadrature current *iq* changes from the subtransient to the synchronous value with the time constant *Tq''*.

Let the transient value of the current be *Itr* with component *Idtr* on the direct axis.

Let the current at time *t* be (*it*) with components *idt* and *iqt* on the direct and quadrature axes.
$idt = Idsync + (Idtr - Idsync) \exp(-t/Td') + (Idsubtr - Idtr) \exp(-t/Td'')$
$iqt = (Iqsubtr - Iqsync) \exp(-t/Tq'')$
$it = \sqrt{(idt^2 + iqt^2)}$
These equations give the short circuit current at any time t with the excitation kept constant provided we can evaluate the transient current *Idtr*.

Theoretical evaluation of transient current Itr.
Theory (1).
Assume the subtransient component is due to the current induced in the damper winding and the transient component is due to the current induced in the field winding. The current in the damping winding decays quickly. As it decays, some of its ampere turns are transferred to the field coil due to their flux linkage. Both coils are linked with the same rate of change in flux. Therefore the initial value of the currents induced in the damper winding and field coil are always in the same ratio. Their relative values depend on their number of turns and their impedances which are constant.

Let the initial value of the current induced in the main field winding be proportional to *It0*. As the current in the damper winding decays, some of the ampere turns of the damper current are transferred to the field current due to their flux linkage. When the damper current has decayed to zero, the fault current is on the transient path

The figure shows how the fault current decays during the subtransient period. The current initially induced in the field is proportional to $(It0 - Isync)$ and the current initially induced in the damper coil is proportional to $(Isubtr - It0)$.
Let $(It0 - Isync) = C1\ (Isubtr - It0)$.

The current in the damper coil decays quickly to zero. Some of the ampere turns of the damper coil are transferred to the field coil, the amount is proportional to $(Isubtr - It0)$.
Let $(Itr - It0) = C2\ (Isubtr - It0)$

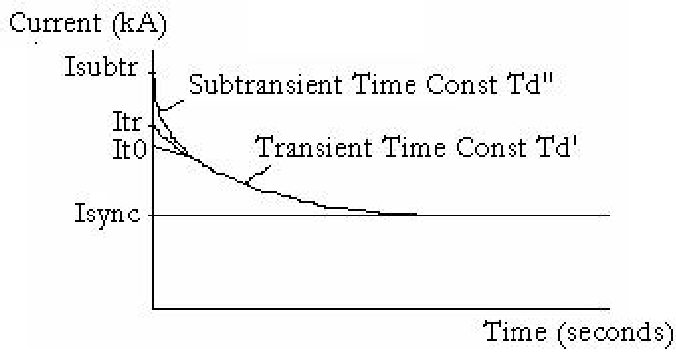


Figure 241: Current decay with R and X

Therefore $Itr - Isync = [(C1 + C2) / (1 - C2)]\ (Isubtr - Itr)$
$C1$ and $C2$ have the same values for all magnitude of fault.

Put $Factorsubtr = (Isubtr - Itr) / (Isubtr - Isync)$
And $Factortr = (Itr - Isync) / (Isubtr - Isync)$
$Factorsubtr$ and $Factortr$ have the same values for all magnitudes of fault.
When $Xe = 0$ and $Re = 0$, $Factortr = Xd''sat\,(Xd - Xd') / [Xd'\ (Xd - Xd''sat)]$
Therefore for all values of Xe and Re;
$Itr = Isync + Factortr\,(Isubtr - Isync)$

By this theory;
$Itr = Isync + (Isubtr - Isync)\ Xd''sat\,(Xd - Xd' / [Xd'\,(Xd - Xd''sat)]$

Theory (2)
If however $C2 = 1$, then all the ampere turns of the damper winding are transferred to the field coil. When the current in the damper winding has decayed, the current in the main field is the same as it would have been with no damper winding.
The current decays with the single time constant of the field coil. However as the emf falls, the circuit reactance changes from $(Xd''sat + Xe)$ to $(Xd''unsat + Xe)$ where $Xd''sat$ is the saturated value and $Xd''unsat$ is the unsaturated value. The current begins on the decay curve value applicable to the saturated values and changes to the decay curve applicable to the unsaturated values. The subtransient current can be calculated using the saturated values. The current ends up on the decay curve from an initial value calculated on the unsaturated values.
With zero impedance short circuit, $Itr = E / Xd'$
where Xd' = unsaturated value of Xd''

Itr and component $Idtr$ can be calculated exactly as for $Isubt$ and $Idsubt$ but with Xd'' replaced by Xd'.

Theory (2) is used by definitive texts on the subject including IEEE STD-551.
This theory means that $Xd'' = Xd''sat$ and $Xd' = Xd''unsat$.
With external impedance Z,
the initial circuit impedance = $Xd''sat + Z$ and the final circuit impedance = $Xd' + Z$
Both methods give a result of sufficient accuracy for practical purposes.

The Time Constants
When the fault occurs, there is a sudden change in the armature current. The change in MMF due to the armature current is exactly balanced by the MMF due to currents induced in the

damping winding and main field winding. Thereafter, the dc currents in the damping winding and main field winding decay with two time constants. Their *MMF*s decay with the same two time constants and hence the direct axis flux and ac *EMF* also decay with the same two time constants.

If the armature current remained constant during the fault, the currents in the damping winding and main field winding would decay with time constants that are the same whatever the initial value of the fault current. If the fault impedance is very high, the fault current is low and the change in armature current is negligible. These time constants are quoted as *Tdo*" and *Tdo*', the open circuit time constants. Therefore these would be the time constants for all faults if the armature current remained constant during the fault.

But the armature current does not remain constant during the fault. It decays as the flux and ac *EMF* decay. This acts like a positive feedback hastening the decay of the dc currents in the damping and main field windings. The time constants are reduced by an amount that depends on the initial value of the fault current.
Equations for the time constants were derived many decades ago.
$Td'' = \quad Tdo'' (Xd'' + X) / (Xd' + X)$
$Td' = \quad Tdo' (Xd' + X) / (Xd + X)]$
$Tq'' = \quad Tqo'' (Xq'' + X) / Xq + X)$
Where;
Td" is the subtransient time constant with fault impedance *X* external to the generator.
Tdo" is the subtransient time constant on open circuit.
Xd" is the saturated subtransient reactance of the generator.
Td', *Tdo* ' and *Xd*' are the transient values of these parameters.
Xd is the synchronous reactance of the generator.
Tq", *Tqo*", *Xq*" and *Xq* are the corresponding parameters on the quadrature axis.

Putting $X = 0$ in these equations gives the short circuit time constants,
$Tds'' = \quad Tdo''\, Xd'' / Xd'$
$Tds' = \quad Tdo'\, Xd' / Xd$
$Tqs'' = \quad Tqo''\, Xq'' / Xq$

However all these parameters can be measured and the measured values do not satisfy these relationships, perhaps due to saturation. Moreover any resistance in the fault path must have some effect on the time constants. According to these equations, resistance in the fault path has no effect on the time constants.
Time constants calculated by these equations can lead to errors if *R* is significent..

Consider the case of a generator on no load with a single coil (the damping winding) linking the flux as shown in the Figure.

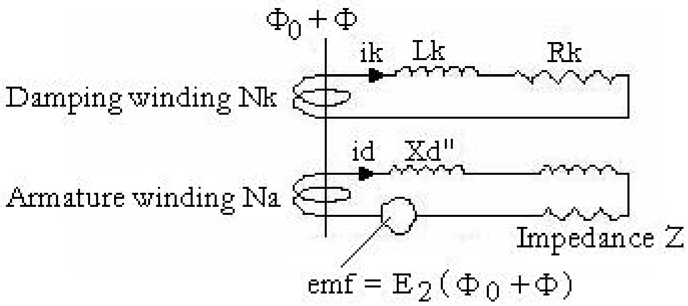


Figure 242; Flux linkage

At time t seconds after the fault occurs, let the current in the damping winding be ik and the direct axis component of the ac fault current be id. Let the flux before the fault be Φo. Let Φ be the additional flux due to armature winding and damper winding.

Thus the flux at time t is $\Phi + \Phi o$. The ampere turns of $(Na\ id)$ due to armature reaction act to reduce the flux and induce a current ik in the linked coil. The ampere turns $(Nk\ ik)$ increase the flux.

$\Phi = M2\ (Nk\ \ ik - Na\ id)$

where $M2$ is the slope of the magnetisation curve at open circuit full volts (flux against ampere turns)

Let the armature ac EMF at time t be $E = E_2$ times the flux $= E_2\ (\Phi o + \Phi)$

The direct axis component of the armature current at time t is

$id = E_2\ (\Phi o + \Phi)\ \text{Cos}\ \psi\ /\ (Z + Xd'')$

where Cos ψ is the angle between the armature current and pole axis.

Substitute for id and differentiate, assume Cos ψ remains unchanged

$\mathrm{d}\Phi/\mathrm{d}t = P\ \mathrm{d}(ik)\ /\mathrm{d}t$

where $P = M2\ Nk\ /\ [1 + \ M2\ Na\ E_2\ \text{Cos}\ \psi\ \ /\ (Z + Xd''\ )]$

Let the EMF induced in the damping winding be $M3$ times $\mathrm{d}\Phi/\mathrm{d}t$. A falling flux creates a positive ik

$-\ M3\ \mathrm{d}\Phi/\mathrm{d}t = Rk\ ik + Lk\ \mathrm{d}(\ ik)\ /\ dt$

where Rk and Lk are the resistance and leakage reactance of the damping winding.

$ik = Ik0 \exp(-t / T)$
where $Ik0$ = initial induced current in the damper winding and $T = (M3\,P + Lk) / Rk$
Put $T = T1 + T2$
where $T1 = Lk / Rk$ and $T2 = M3\,P / Rk$
$T2 = (M3 / Rk)\, M\, Nk / [1 + M2\, Na\, E2 \cos\psi / (Z + Xd'')]$
$T1$ is the time constant of the coil due to resistance and leakage reactance.
$T2$ is the time constant of the coil due to resistance and flux linkage with the field magnetic circuit.

Thus $T2$ is very much larger than $T1$, as a first approximation, $T1$ can be ignored.
Put $\alpha = M3\, M2\, \mathrm{Nk} / Rk$
And $\beta = M2\, Na\, E2 / Xd''$
And $k1 = Xd'' \cos\psi / (Z + Xd'')$
$T2 = \alpha / (1 + \beta\, k1)$
When Z is very large (ie open circuit), $k1 = 0$, $T2 = Tdo$, $\alpha = Tdo$
When $Z = 0$, (ie short circuit), $k1 = 1$, $T2 = Tds$
$Tds = Tdo / (1 + \beta)$
$\beta = (Tdo - Tds) / Tds$
For other values of Z, $T2 = Td$
$Td = Tdo\, Tds / [Tds + (Tdo - Tds)\, k1]$
where $k1 = Xd'' \cos\psi / (Z + Xd'')$
The subtransient time constant is therefore
$Td'' = Tdo''\, Tds'' / [Tds'' + (Tdo'' - Tds'')\, k1]$
where $k1 = Xd'' \cos\psi / (Z + Xd'')$ and ψ = angle between armature current and pole axis.

If equation $Tds'' = Tdo''\, Xd'' / Xd'$ is satisfied, then both methods give the same result.
Applying the same principle to the other time constants.
$Td' = Tdo'\, Tds' / [Tds' + (Tdo' - Tds')\, k2]$
where $k2 = Xd' \cos\psi / (Z + Xd')$ and ψ = angle between armature current and pole axis.
$Tq'' = Tqo''\, Tqs'' / [Tqs'' + (Tqo'' - Tqs'')\, k3]$
where $k3 = Xq'' \sin\psi / (Z + Xq'')$ and ψ = angle between armature current and pole axis.

These equations give the same values as the traditional equations if;
$Tds'' = Tdo''\, Xd'' / Xd'$
$Tds' = Tdo'\, Xd' / Xd$
$Tqs'' = Tqo''\, Xq'' / Xq$

The response of the automatic voltage regulator and governor.
These equations have been derived to give the short circuit current through an impedance with the excitation kept constant. However the excitation is not kept constant when a fault occurs on a power system. The excitation rises exponentially under the control of the automatic voltage regulator.

When the terminal voltage is below the system voltage, the automatic voltage regulator boosts the field due to the exciter. The voltage rises towards the maximum with a time constant dictated by the exciter field circuit and the main field coil. The time constant for the exciter is quoted as Te. The exciter voltage rises with this time constant but acts on a circuit with a time constant Td'. The target voltage for the automatic voltage regulator is influenced by the compounding circuit. At a lagging armature current, the target voltage is reduced to reduce the response. This is required for parallel operation of the generator. Without the compounding, the generators running in parallel can become unstable with the wattless current swinging wildly from one machine to the other. Fault currents can be large at very low power factor lagging. This can significantly reduce the target voltage of the automatic voltage regulator.

The reduction in target voltage is proportional to armature current and the sine of the phase angle with a zero value at the system design power factor.

The target voltage (*vavr*) with fault current (*it*) is therefore;
$vavr = 1 - it$ / (full load current) $[(X / \sqrt{(R^2 + X^2)} - \sin\phi]$ (*Compounding*%)/ 100
where $\cos\phi$ is the design system power factor.
And (*Compounding*%) is the compounding voltage as per cent full volts with full load current.
The voltage rises exponentially to *vavr*.

Fault calculations on a power system.
The impedance of the fault path is calculated. The subtransient, transient and synchronous values of the armature current are calculated. The time constants are calculated and hence the value of the armature current at any time t after fault incidence can be calculated.

The manufacturer can usually provide figures for the machine reactances Xd'', Xq'', Xd', Xd, Xq, the negative sequence reactance X_1 and the zero sequence reactance X_0. Similarly, the manufacturer can usually provide figures for the time constants Tdo'', Tdo', Tds'', Tds' and Te. If these figures are not available, practical tests on the machine may enable approximate values to be obtained.

The response of the exciter and pilot exciter must also be considered together with the compounding and response of the automatic voltage regulator. The change in speed of the generator can also be considered. Fault currents are usually low power factor. The reduction in voltage reduces the resistive load. Therefore generators larger than 10MW usually speed up during a heavy fault. If however most of the power station load is induction motors, these motors can contribute to the fault. It is widely assumed that induction motors will contribute a current equal to their starting current during a heavy fault. It is true that induction motors can generate if the frequency falls below the motor slip frequency. However this is most unlikely to occur with a low power factor system fault as the generators may speed up during the fault. If the fault is near the power station busbars, then the voltage at the power station busbars can fall to zero. In this case any induction motors on the system may generate but at slip frequency. In this case, the total fault current can rise and fall as the generated voltage goes into and out of phase with the motor generated current.

Example

Tests were carried out on a laboratory sized generator with significant armature resistance. The machine was given artificially increased armature reaction. A voltage proportional to the armature current was input to a three channel amplifier, one phase to each channel. The output from each channel was rectified. The sum of the three outputs was connected into the field circuit to reduce the net field by an amount proportional to the armature current. The machine then behaved like a full size generator.

Test results.

Field current	0.00	0.50	1.00	1.50	2.00	2.50	2.80
Open circuit volts	00.4	04.0	07.6	10.1	11.4	12.0	12.3
Xd''	0.235	0.208	0.192	0.183	0.173	0.166	0.162
Xq''	0.313	0.313	0.312	0.283	0.259	0.240	0.229

Armature resistance = 0.1016 ohms/phase

Armature short circuit current S = 13.3 amps at 2.80 amps field current

$M_0 = 2.8\ (4.0 - 0.4)\ /\ 0.5 = 20.2$ volts/unit field

$Xd = M_0\ /\ S = 20.2\ /\ 13.3 = 1.52$ ohms / phase

Put $Xq = Xd\ /2 = 0.76$ ohms/phase.

Xq is a very approximate value but the actual value has very little effect on the calculated results. The minimum value of Xq is Xq'' which implies no quadrature armature reaction. The maximum value is very large which implies negligible quadrature component of current.

Short circuit test
The field current was set at 2.80 amps and the armature current on a short circuit was recorded.

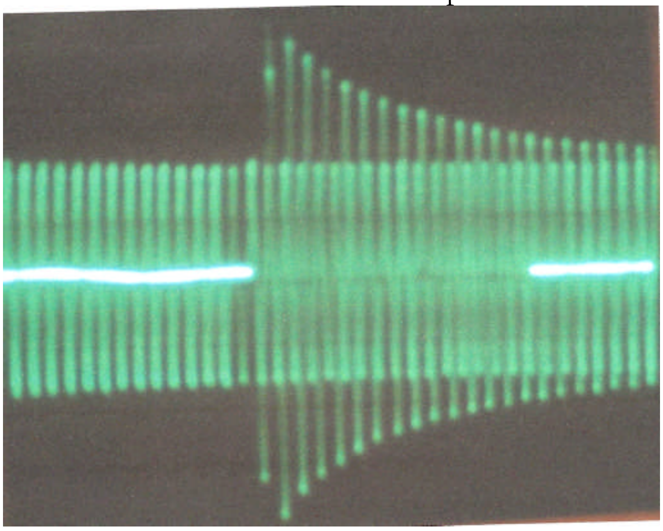


Figure 243; Zero impedance short circuit.
The subtransient current was calculated
Isubt, the initial value of fault current, is with saturated reactances.
$id = E_0 / (R^2 / Xq''sat + Xd''sat) = 12.3 / (0.1016^2 / 0.229 + 0.162) = 59.4$ amps
$iq = id\, R / Xq''sat = 59.4 (0.1016 /0.229) = 26.3$ amps
$Isubtr = \surd(id^2 + iq^2) = 65.0$ amps

The synchronous current was calculated
Isync, the value of fault current, is with unsaturated reactances.
$id = S / \surd(1 + R^2 / Xq^2) = 13.3 / \surd(1 + 0.1016^2 / 0.76^2) =13.2$ amps
$iq = id\, R / Xq = 13.2 (0.1016 / 0.76) = 1.8$amps
$Isync = \surd(13.2^2 + 1.8^2) = 13.3$ amps
$Eq = iq\, R + id\, Xd''unsat = 1.8 \cdot 0.1016 + 13.2 \cdot 0.230 = 3.2$ volts
$Xd''unsat = 0.230$ ohms/phase at 3.2 volts.
$Xq''unsat = 0.313$ ohms/phase at 3.2 volts
$Ed = iq (Xq - Xq''unsat) = 1.8 (0.76 - 0.313) = 0.8$ volts
$E = \surd(Eq^2 + Ed^2) = 3.3$ volts

Field for 3.3 volts is 0.40 amps
Armature reaction is equivalent to 2.80 – 0.40 = 2.40 field amps
Let armature reaction be equivalent to $k \cdot id$ field amps
The direct axis armature reaction is proportional to the direct axis current *id.*
However the artificial armature reaction is proportional to *ia* not *id*
$N = 2.40 / 13.3 = 0.180$

The recorded current was analysed from 3rd half cycle
Isubtr = 65.0 and *Isync* = 13.3 amps;
Armature current = 13.3 + 18.2 exp(–n / 17) + 33.4 exp(–n / 1.0) amps
Tds' = 17 and Tds'' = 1.0 where *n* is the number of half cycles

Transient current = 13.3 + 18.2 = 31.5 amps
Transient impedance = *Eo* / transient current = 12.3 / 31.5 = 0.390 ohms/phase
Resistance = 0.1016 ohms/phase
Transient reactance = $\surd(0.390^2 - 0.1016^2) = 0.377$ ohms/phase

Time constants
From the recording;
$Tds' = 17$ and $Tds'' = 1.0$ where n is the number of half cycles

The current in the damper winding and field winding.

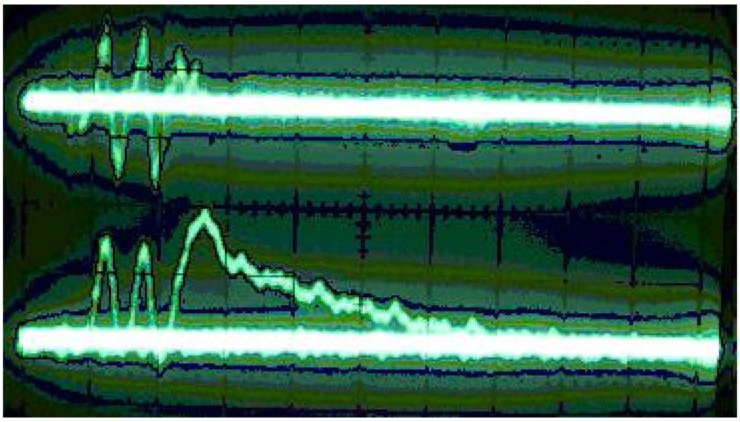


Figure 244; Current in damping winding and main field winding

The current in the damper winding and in the main field coil were recorded. The upper trace is the current in the damper winding and the lower trace is the current in the field winding. The traces initially oscillate with a dc bias and then decay exponentially. The initial oscillations are due to the dc component of the short circuit current.

Calculate the fault current when switched onto an impedance with field current 2.80 amps.
The external impedance consisted of reactance $Xe = 0.5620$ and resistance $Re = 0.1596$ ohms/phase

The subtransient current was calculated.
$E\text{o} = 12.3$ volts
$id = E\text{o} \,/\, [(R + Re)^2/(Xqs'' + Xe)^2 + Xds'' + Xe\,]$
$= 12.3 \,/\, [\,(0.1016 + 0.1596)^2 \,/\, (0.229 + 0.5620) + 0.162 + 0.5620] = 15.2$ amps
$iq = id\,(R + Re) \,/\, (Xq'' + Xe) = 15.2\,(0.1016 + 0.1596) \,/\, (0.229 + 0.5620) = 5.0$ amps
$Isubtr = \sqrt{(id^2 + iq^2)} = 16.0$ amps

The synchronous current was calculated
Try $Isync = 9.8$ amps
From the short circuit test, $N = 0.180$.
Armature reaction $= N\,id = 0.180 \cdot 9.8 = 1.76$ amps reduction in field
Net field $= 2.80 - 1.76 = 1.04$ field amps
$Eq = 7.6 + (0.04/0.5)(10.1 - 7.6) = 7.80$ volts
At 1.04 amps field current, $Xd'' = 0.192 - (.04/.5)(0.192 - 0.185) = 0.191$
and $Xq'' = 0.312 - (.05/.5)(0.312 - 0.283) = 0.310$ ohms/phase
$iq = (R + Re) \,/\, Xq + Xe)\; id = (0.1016 + 0.1596) \,/\, (0.76 + 0.5620) = 0.198$ id
$Eq = iq\,(R + Re) + id\,(Xd'' + Xe) = id\,[\,0.198\,(0.1016 + 0.1596) + 0.191 + 0.5620] = 0.805\;id$
$id = 7.80 \,/\, 0.805 = 9.69$ amps
$iq = 0.198\;id = 1.92$ amps
$Isync = \sqrt{(id^2 + iq^2)} = 9.88$ amps

Therefore *Isync* is between 9.8 and 9.88 amps
Put *Isync* = 9.8 amps
Isubt = 16.0 and *Isync* = 9.8
Theory (1) The decay in the armature current is due to the decay in currents in the damper and field windings;
Transient element = 18.2 / (18.2 + 33.4) (*Isubtr – Isync*) = 2.2 amps
Subtransient element = 33.4 / (18.2 + 33.4) (*Isubtr – Isync*) = 4.0 amps
Fault current = 9.8 + 2.2 exp($-t$ / T') + 4.0 exp ($-t$ / T'') amps
Theory (2) The two time constants are due to the change from saturated to unsaturated leakage reactance;
Armature resistance = 0.1016 ohms/phase
Transient reactance = 0.377 ohms/phase
Circuit resistance = 0.1016 + 0.1596 = 0.261 ohms/phase
Circuit transient reactance = 0.377 + 0.5620 = 0.939 ohms/phase
Circuit transient impedance = 0.975 ohms/phase
Transient current = 12.3 / 0.975 = 12.6 amps
Armature current = 9.8 + (12.6 – 9.8) exp($-t$ / T') + (16.0 – 12.6) exp($-t$ / T'') amps
Armature current = 9.8 + 2.8 exp($-t$ / T') + 3.4 exp($-t$ / T'') amps

Note however that the transient reactance Xd' = 0.377 ohms/phase is significantly different from the unsaturated direct axis reactance Xd'' = 0.191 ohms/phase.

Calculate the time constants by the traditional equations;
$Td'' = Tdo'' (Xd'' + X) / (Xd' + X)$
$Td' = Tdo' (Xd' + X) / (Xd + X)]$

When $X = 0$, The time constants were measured at $Tds' = 17$ and $Tds'' = 1.0$
Td'' is while saturated, $Xd'' = 0.162$ ohms/phase
Td' is while Xd'' is unsaturated, $Xd'' = 0.191$ ohms/phase
$Xd' = 0.377$ and $Xd = 1.60$ ohms/phase
$Tdo'' = Tds'' Xd' / Xd'' = 1.0 \cdot 0.377 / 0.191 = 2.0$
$Tdo' = Tds' Xd / Xd' = 17 \cdot 1.6 / 0.377 = 72$

With $X = 0.5620$;
$Td'' = Tdo'' (Xd'' + X) / (Xd' + X) = 2.0 (0.162 + 0.5620) / (0.377 + 0.5620) = 1.6$
$Td' = Tdo' (Xd' + X) / (Xd + X) = 72 (0.377 + 0.5620) / (1.60 + 0.5620) = 31$

Calculate the time constants by the equations developed in this text;
$Td'' = Tdo'' Tds'' / [Tds'' + (Tdo'' – Tds'') k1]$
where $k1 = (id$ with $X = Xe) / (id$ with $X = 0)$ initial values
$Tdo'' = 2.0$ and $Tds'' = 1.0$
$Td'' = 2.0 \cdot 1.0 / [1.0 + (2.0 – 1.0) (16.0 \text{ amps})/(65.0 \text{ amps})] = 1.6$
$Td' = Tdo' Tds' / [Tds' + (Tdo' – Tds') k1]$
where $k1 = (id$ with $X = Xe) / (id$ with $X = 0)$ final values
$Td' = 72 \cdot 17 / [17 + (72 – 17) \cdot (9.8 / 13.3)] = 21$
The traditional equations give $Td'' = 1.6$ and Td ' = 31 half cycles
The equations developed in this text give $Td'' = 1.6$ and Td ' = 21 half cycles.

Measured result
Isync was measured at 9.7 amps with 2.80 field current

The current through the impedance was recorded and is shown below.
The recording was printed out and the peak to peak value of each half cycle was measured

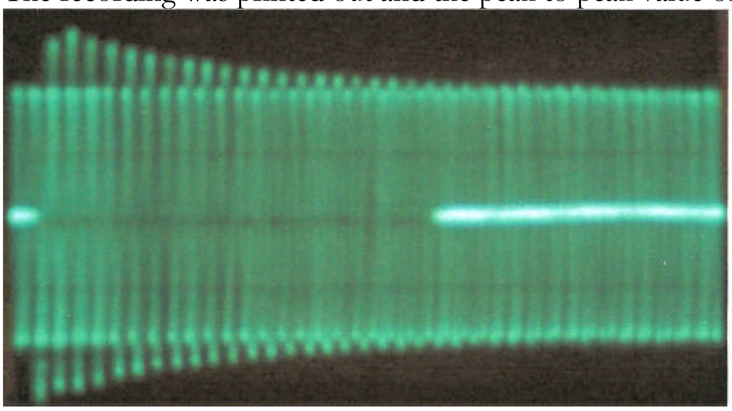


Figure 245; Measured current with impedance in the short circuit

Isync = 9.8 amps. This gives the scale of the recording.

Analysis of the trace from 3rd peak to peak gives;
Short circuit current = $45.8 + 23.0 \exp(–n / 22) + 6.0 \exp(–n / 2.9)$ mm
where n is the number of half cycles.
Isync =45.8 mm = 9.8 amps
Isubtr = 9.8 (45.8 + 23.0 + 6.0) / 45.8 = 16.0 amps

Itr = 45.8 + 23.0 mm = 68.8 mm = 9.8 (68.8 / 45.8) = 14.7 amps
Armature current = 9.8 + 4.9 exp(–*n* / 22) + 1.3 exp(–*n*/ 2.9) amps

Summary
Method (1) Fault current = 9.8 + 2.2 exp(–*n* / 21) + 4.0 exp (–*n* / 1.6) amps
Method (2) Fault current = 9.8 + 2.8 exp(–*n* / 31) + 3.4 exp(–*n* / 1.6) amps
Measured Fault current = 9.8 + 4.9 exp(–*n* / 22) + 1.3 exp(–*n* / 2.9) amps
where *n* is the number of half cycles.

Both methods give similar results within the limits of experimental error but differ from the measured result. For practical purposes, the margin of error is acceptable. There is little if any error in the subtransient and synchronous values. The time constant *Td*' calculated by the method suggested by this text gives a better result.

The tests were repeated at a lower field current and lower short circuit impedance.
The figure 246 shows the armature current with artificial armature reaction when switched onto a zero impedance short circuit with field current 1.56 amps. The final synchronous current was 7.4 amps.

The peak to peak mesurements were taken. The final peak to peak corresponds to the synchronous current 7.4 amps. This gives the scale of the recording.

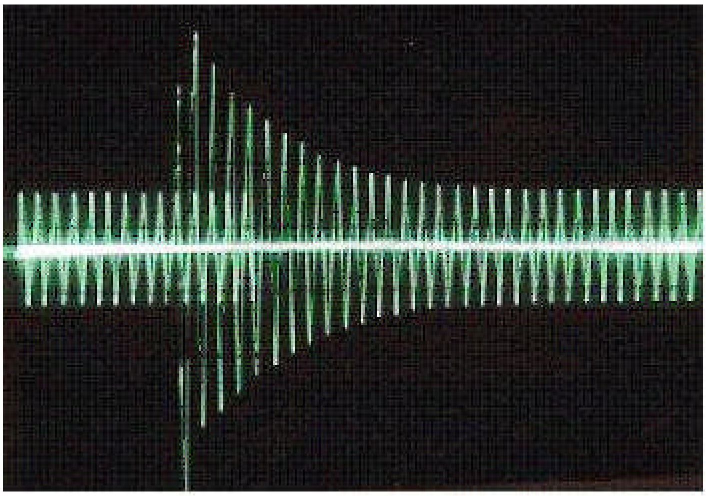


Figure 246; Zero impedance short circuit

The subtransient currents were calculated.

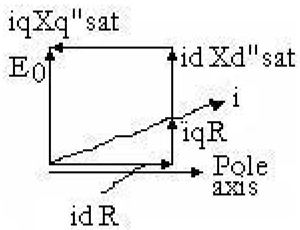


Figure 247: Subtransient values with zero impedance short circuit

At 1.56 field, the emf E_0= 10.3 volts from the open circuit characteristic.
Initially the reactances are partly saturated.
Xd" = 0.182 ohms/phase at 1.56 amps field current.
Xq" = 0.280 ohms/phase at 1.56 amps field current.

R = 0.1016 ohms / phase.

The emf does not change immediately when the short circuit is applied.
$iq\, Xq'' = id\, R$
$iq = 0.1016 / 0.280\; id = 0.363\; id$
$E_0 = iq\, R + id\, Xd''$
$10.3 = (0.363 \cdot 0.1016 + 0.182)\, id = 0.219\, id$
$id = 10.3 / 0.219 = 47.0$ amps
$iq = 0.363 \cdot 47.0 = 17.1$ amps
Subtransient current $= \sqrt{(id^2 + iq^2)} = 50.0$ amps
$Z = 10.3 / 50.0 = 0.206$ ohms per phase

The synchronous current

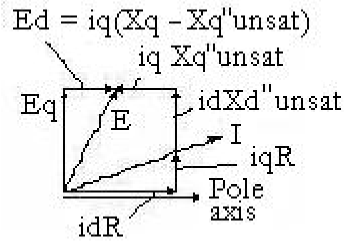


Figure 248; Synchronous values with zero impedance short circuit

The reactances are unsaturated at approx 1.8 volts
$Xd'' = 0.222$ ohms/phase

$Xq'' = 0.313$ ohms/phase
R = 0.1016 ohms / phase.
Field current = 1.56 amps.
$id = S(1.56/2.8)/\sqrt{(1 + R^2 / Xq^2)}$
$= 13.3 \cdot 1.56/2.8 / \sqrt{(1 + 0.1016^2 / 0.76^2)} = 7.34$ amps
$iq = id\, R/Xq = (0.1016 / 0.76)\, id = 0.134\, id = 0.98$ amps
$ia = \sqrt{(id^2 + iq^2)} = 7.41$ amps
$Ed = iq\, R + id\, Xd'' = 0.98 \cdot 0.1016 + 7.34 \cdot 0.222 = 1.73$ volts
$Eq = iq\, (Xq - Xq'') = 0.43$ volts
$E = \sqrt{(Ed^2 + Eq^2)} = 1.8$ volts
Field for 1.8 volts = 0.19 field amps
Armature reaction is equivalent to 1.56 – 0.19 = 1.37 field amps
Let armature reaction = $N\, ia$
$N = 1.37 / 7.41 = 0.185$
This agrees with the previous figure 0.180 within the limits of experimental error

The recorded trace figure 246 was analysed.
Synchronous current = 7.41 amps
Subtransient current = 50.0 amps
The recording of the current was not established till the second half cycle. The trace was analysed from the 2nd to 60th half cycles and can be represented by;

$I = 7.4 + 27.6 \exp(-n / 10) + 15.0 \exp(-n / 0.7)$ amps
Where n is the number of half cycles
$Tds'' = 0.7$ and $Tds' = 10$

Subtransient current = 7.4 + 27.6 +15.0 = 50.0 amps
Transient current = 7.4 + 27.6 = 35.0 amps
Synchronous current = 7.4 amps.
Transient impedance = 10.3/35.0 = 0.294 ohms / phase
Transient reactance = $\sqrt{(0.294^2 - 0.1016^2)} = 0.276$ ohms / phase

Calculate the fault current with field 1.56 amps when switched onto the impedance.
The impedance consisted of reactance $Xe = 0.092$ and resistance $Re = 0.050$ ohms/phase

Subtransient current

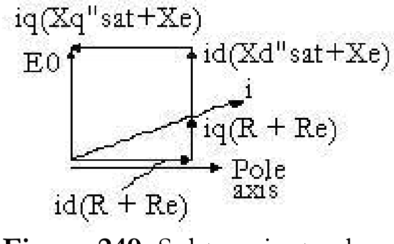


Figure 249; Subtransient values with impedance in the short circuit

At 1.56 amp field, $Xd'' = 0.182$ and $Xq'' = 0.280$ ohms / phase
The *EMF* does not change immediately when the short circuit is applied.
$E_0 = 10.3$ volts
$iq\, (Xq'' + Xe) = id\, (R + Re)$
$iq = id\, (0.1016 + 0.050) / (0.280 + 0.092) = 0.4075\, id$
$E_0 = iq\, R + id\, Xd''$
$10.3 = id\, [0.4075\, (0.1016 + 0.050) + 0.182 + 0.092] = 0.336\, id$

id = 30.7 amps
iq = 12.45amps
Subtransient current $= \sqrt{(id^2 + iq^2)} = 33.1$ amps
Z = 10.3 / 33.1 = 0.311 ohms per phase

The synchronous current was calculated

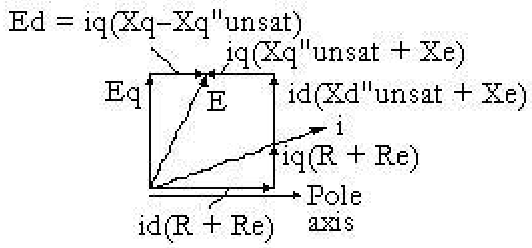


Figure 250; Synchronous values with impedance in the short circuit.

Try *Isync* = 7.1 amps
N = 0.182.
Armature reaction = *N id* = 0.182 · 7.1
= 1.29 amps reduction in field
Net field = 1.56 − 1.29 = 0.27 field amps
Ed = 0.4 + (0.27/0.5)(4.0 – 0.4) = 2.34 volts

At 0.27 amps field current,
Xd" = 0.235 – (0.27/0.5)(0.235 – 0.208) = 0.220 ohms/phase
Xq = 0.313 ohms/phase
iq = (R + *Re*) / *Xq* + *X*e) *id* = (0.1016 + 0.050) / (0.76 + 0.092) id = 0.178 *id*

Ed = *iq* (R + *Re*) + id (*Xd*" + *Xe*) = *id* [0.178 (0.1016 + 0.050) + 0.220 + 0.092] = 0.339 *id*
id = 2.34 / 0.339 = 6.90 amps
iq = 0.178 id = 1.23 amps
Isync $= \sqrt{(id^2 + iq^2)} = 7.01$ amps
Therefore *Isync* is between 7.01 and 7.1 amps
Put *Isync* = 7.1 amps

Isubtr = 33.1 and *Isync* = 7.1
By theory (1)
Transient element = 27.6 / (27.6 + 15.0) (*Isubtr* – *Isync*) = 16.8 amps
Subtransient element = 15.0 / (27.6 + 15.0) (*Isubtr* – *Isync*) = 9.2 amps
Current = 7.1 + 16.8 exp(–t / *T* ') + 9.2 exp (–*t* / *T* ") amps

By theory (2)
Armature resistance = 0.1016 ohms/phase
Transient reactance = 0.276 ohms/phase
Circuit resistance = 0.1016 + 0.050 = 0.1516 ohms/phase
Circuit transient reactance = 0.276 + 0.092 = 0.368 ohms/phase
Circuit transient impedance = 0.398 ohms/phase
Transient current = 10.3 / 0.398 = 25.9 amps
Armature current = 7.1 + (25.9 – 7.1) exp(–*t* / T ') + (33.1 – 25.9) exp(–t / *T* ") amps
Armature current = 7.1 + 18.8 exp(–*t* / T ') + 7.2 exp(–*t* / *T* ") amps

Calculate the time constants.
Using the traditional equations;
$Td'' = \quad Tdo''\ (Xd'' + X) \ / \ (Xd' + X)$
$Td' = \quad Tdo'\ (Xd' + X) \ / \ (Xd + X)]$

When $X = 0$, The time constants were measured at $Tds' = 10$ and $Tds'' = 0.7$
Td'' is while Xd'' is saturated, $Xd'' = 0.182$ ohms/phase
Td' is while Xd'' is unsaturated, $Xd'' = 0.220$ ohms/phase
$Xd' = 0.276$ and $Xd = 1.60$ ohms/phase
$Tdo'' = Tds''\ Xd'/Xd'' = 0.7 \cdot 0.276 \ / \ 0.182 = 1.1$
$Tdo' = Tds'\ Xd \ / \ Xd' = 10 \cdot 1.60 \ / \ 0.276 = 58$
With $X = 0.092$;
$Td'' = Tdo''\ (Xd'' + X) \ / \ (Xd' + X) = 1.1\ (0.182 + 0.092) \ / \ (0.276 + 0.092) = 0.8$
$Td' = Tdo'\ (Xd' + X) \ / \ (Xd + X)] = 58\ (0.276 + 0.092) \ / \ (1.60 + 0.092) = 13$

Using the equations suggested by this text;
$Td'' = Tdo''\ Tds'' \ / \ [Tds'' + (Tdo'' - Tds'')\ k1]$
where $k1 = (id$ with $X = Xe) \ / \ (id$ with $X = 0)$ initial values
$Tdo'' = 1.1$ and $Tds'' = 0.7$
$Td'' = 1.1 \cdot 0.7 \ / \ [0.7 + (1.1 - 0.7)\ (33.1\ \text{amps})/(50.0\ \text{amps})] = 0.8$
$Td' = Tdo'\ Tds' \ / \ [Tds' + (Tdo' - Tds')\ k1]$
where $k1 = (id$ with $X = Xe) \ / \ (id$ with $X = 0)$ final values
$Td' = 58 \cdot 10 / [10 + (58 - 10) \cdot (33.1 / 50.0)] = 14$

Measured result
Isync was measured at 7.1 amps with 1.56 field current

The current through the impedance was recorded and is shown below.
The recording was printed out and the peak to peak value of each half cycle was measured.

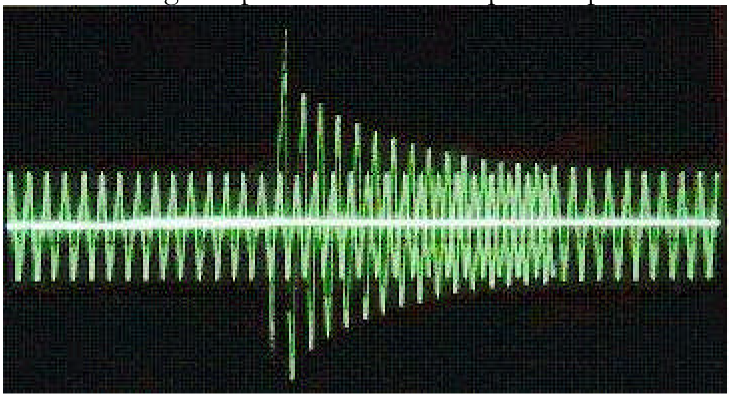


Figure 251; Measured current with impedance in the short circuit.

Isync = 7.1 amps. This gives the scale of the recording.
Analysis of the trace from 2nd peak to peak gives;
Armature current = 7.1 + 15.6 exp(–*n* / 12) + 10.4 exp(–*n* / 0.9) amps
where *n* is the number of half cycles.
Itr = 7.1 + 15.6 = 22.7 amps
Isubtr = 7.1 + 15.6 + 10.4 = 33.1 amps

Summary
Using values for *Td*" and *Td*' calculated above;
Method (1) Current = 7.1 + 16.8 exp(–*n* / 14) + 9.2 exp (–*n* / 0.8) amps
Method (2) Current = 7.1 + 18.8 exp(–*n* / 13) + 7.2 exp(–*n* / 0.8) amps
Measured Current = 7.1 + 15.6 exp(–*n* / 12) + 10.4 exp(–*n* / 0.9) amps
where *n* is the number of half cycles

When calculating the fault current on a full size machine, both methods give a result that is accurate enough for practical purposes. When the response of the automatic voltage regulator is also taken into account, there is little if any difference between the two methods. In the absence of any strong evidence to the contrary, assume that *Xd*" is the saturated value of the leakage reactance and *Xd*' is the unsaturated value of the leakage reactance.

CALCULATION OF FAULT CURRENTS

Generator Reactances and Time Constants
The works short circuit test can be analysed to obtain *Tds* ", *Tds* ', *Xd* " and *Xd* '.
If the machine is also switched onto a low resistance, then values can be obtained for *Xq*" and *Tqs*".

Short circuit and open circuit characteristics
The generator is usually dried out as part of the commissioning procedure. A short circuit of heavy section copper bar is bolted to the generator circuit breaker terminals. The short circuit characteristic is obtained during the dry out and it can be followed by the open circuit characteristic.

Off load tests
With the machine running off load, the effect of a sudden change in excitation can be analysed to obtain values for *Tdo*" and *Tdo*'. It may be possible to do this on load to obtain a value for *Tqo*".

On load records
With the machine on load, the excitation current can be recorded for various loads and power factors to confirm *Xd* and obtain a value for *Xq*.

Note. Any oscilliographs taken of transient conditions should be recorded on an oscillioscope supplied from a different power supply, battery and invertor or a small generator. If the oscilloscope is supplied from the same supply, the trace will disappear off screen due to the system voltage fluctuations that occur in transient conditions.

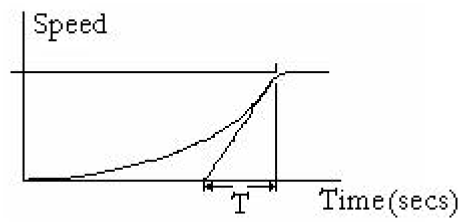


Figure 252; Machine Inertia

Machine Inertia
The inertia, *H*, is measured in kW seconds of kinetic energy at full rated speed per kVA. This can be estimated by plotting the curve of speed against time during a normal start. As the speed approaches full speed, the prime mover power increases till the governor kicks in. Draw the tangent to the curve at the speed corresponding to
full load governor droop

Extend this tangent to the full speed line at the top and the axis at the bottom. The value T seconds is the approximate time to gain full speed kinetic energy if the prime mover supplied rated full load power throughout this period. Thus the kinetic energy is approximately $T \cdot MW$

megawatt seconds where *MW* is the rated full load in megawatts. Hence an approximate value of *H* can be obtained

$$\begin{aligned} H &= T \cdot MW/MVA \\ &= T \cdot pf \end{aligned}$$

Impedance external to the power station
The impedance of the power system between the power station and the fault largely determines the magnitude of the fault. Power lines and cables have resistance and inductive reactance. Transformers have inductive reactance but negligible resistance.

Resistance of Power Lines and cables
Manufacturer's figures for the resistance of line conductors are usually available.

In the absence of manufacturer's figures, the table below gives approximate values for hard drawn copper conductors.

Size mm^2	10	16	35	50	70	95	150
Ohms/km/phase	4.02	1.09	0.596	0.379	0.298	0.181	0.102

Aluminium conductor sizes are usually quoted as equivalent copper size. If the actual size is quoted, multiply the above figures by 1.7

Reactance of Power Lines
Manufacturer's figures are usually available.

The reactance $X = 0.016 \cdot \mu + 0.146 \cdot \text{Log}_{10} (D / r)$ ohms/km
Where μ is permeability, D is conductor spacing and r is conductor radius.
Approximate Reactance of power lines in ohms/phase/km

Size mm^2	10	16	35	50	70	95	150
12 inch spacing	0.34	0.32	0.29	0.28	0.28	0.27	0.25
36 inch spacing	0.41	0.39	0.37	0.35	0.34	0.33	0.32
72 inch spacing	0.45	0.43	0.41	0.40	0.39	0.38	0.36

Reactance of Cables
Manufacturer's figures are usually available.
In the absence of manufacturer's figures, the table below gives the approximate value of the reactance of three core belted cables in ohms/phase/km.

Size mm^2	16	35	50	70	95	150	300	500
1100 Volt	0.081	0.073	0.070	0.069	0.066	0.065	0.063	0.062
3.3 kV	0.089	0.078	0.076	0.073	0.070	0.068	0.066	0.065
11 kV	0.108	0.092	0.088	0.084	0.079	0.076	0.072	0.070

Transformer Impedance
Transformer impedance is nearly all reactance and the resistance can be ignored. The transformer nameplate quotes the "Impedance Volts". This is the volt drop due to the impedance on full load expressed as a percentage of the rated voltage.
If the nameplate value is unavailable, approximate figures are;

Transformer Size kVA	100	250	500	1000	2000	5000	10000
Impedance Volts %	3	3.5	4	5	5.5	7.5	10

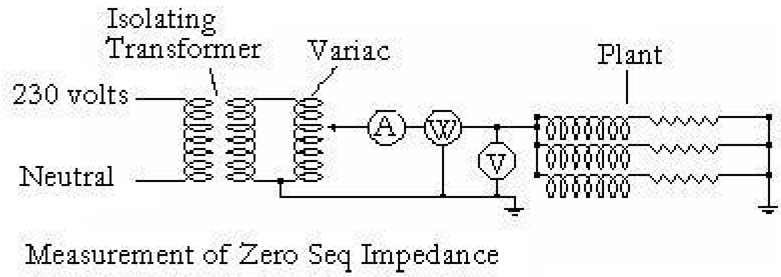
Auto Transformers
If the voltage ratio is near unity, the primary and the secondary are sometimes different connections to the same winding. Transformers of this type are called Auto Transformers. The impedance volts of an auto transformer are about half the above figures.

Positive, Negative and Zero sequence Impedances
The measured or quoted impedances per phase for non rotating plant can be used for both the positive and the negative sequence impedances. The negative phase sequence impedance is usually quoted by the manufacturer for generators. In the absence of manufacturer's figures, the positive sequence impedance could be used but this could lead to significant errors in the calculated fault current.

Zero Sequence Impedances
The zero sequence impedance may be lower or more usually higher by a factor of two or three. It is best to measure the value by a practical test. Power lines with no earth line or a steel earth line have a higher zero sequence impedance. If the impedance of a power line is significant, always measure the zero sequence resistance and reactance on site.

To measure the zero sequence impedance of the generators and other plant, short circuit all three phases each side of the plant. The short circuit at the remote end is then connected to earth. The impedance is then measured between the near end short circuit and earth. This can be done by using a 20 amp Variac transformer, a Voltmeter, an Ammeter and a Wattmeter. The Variac will need to be supplied from a mobile generator or alternatively through an isolating transformer. If it is connected directly to the mains, the neutral connection to earth will negate the readings.

The Zero sequence impedance/phase Z_0 is then 3 · Volts / Current.
The Zero sequence resistance R_0 is 3 · Watts/(Current)^2
The Zero sequence reactance is X_0 is $\sqrt{(Z_0^2 - R_0^2)}$
It may be necessary to correct for the voltage drop across the ammeter and wattmeter current coil or for the current through the voltmeter and wattmeter voltage coil.



$$Z_0 = \frac{3 \cdot \text{Volts}}{\text{Amps}} \quad \text{Ohms/phase}$$

$$R_0 = \frac{3 \cdot \text{Watts}}{\text{Amps}^2}$$

$$X_0 = \sqrt{(Z_0^2 - R_0^2)}$$

Figure 253; Measurement of zero sequence impedance

Calculation of Fault Currents
Where a power system contains transformers, impedances in series at different voltages cannot be simply added. They must all be converted to the equivalent at the same voltage.

A convenient way to do this is to evaluate each impedance as the per cent volt drop when carrying a "base load".
The base load can be any value but it is common to use 10 MVA.

Impedances to a base 10 MVA
The impedance Z to base 10 MVA is given by;

Z = (% Plant Impedance) · 10 / (Plant MVA)
Z = 1000 · (Impedance in ohms per phase) / (line kV)^2

Impedances in series and at different voltages each expressed to a common base 10 MVA can be a added numerically, ie $R = R1 + R2$ and $X = X1 + X2$.

Fault MVA
Fault MVA = [1000 /(total % impedance including the generator subtransient reactance to base 10 MVA)] MVA

Short Circuit heating
Temp rise in deg C = [Amps/mm^2]^2 · (time in seconds) / 202
Thus the minimum size of 11 kV cable to carry 250 MVA for 2 seconds is about 200mm^2.

Force between conductors
Mean Force = $2 I^2 / (10{,}000\ d)$ Newtons/metre run
where d is the conductor spacing in mm

Peak Force = $(1.8 \sqrt{2}\, I)^2\ \ 2/(10{,}000\ d)$
$= 1.30\ I^2 / (1{,}000\ d)$ Newtons/metre run

Motor Contribution
Add induction motor starting currents to the fault current. This is normal practice, and some standards require it. However, in a fault, the voltage falls and the frequency usually rises. Both cause the motor to take more current rather than generate. In a heavy fault causing near zero voltage, induction motors are likely to generate at slip frequency due to residual magnetism in the rotor. This will "beat" with the fault current and there will be times when the currents are in anti phase.

SYMMETRICAL COMPONENTS

Symmetrical Components
The method of expressing unbalanced currents by symmetrical components is well documented and covered in many textbooks. The following is a summary of what is published elsewhere.

Evaluating Symmetrical Components
The currents in a three phase system can be represented by the sum of three symmetrical three phase currents. The Figure shows the symmetrical components of three very unbalanced currents *IA*, *IB* and *IC*. I_1, I_2 and I_0 are the positive, negative and zero sequence components. h is the operator that rotates the vector by 120 degrees.

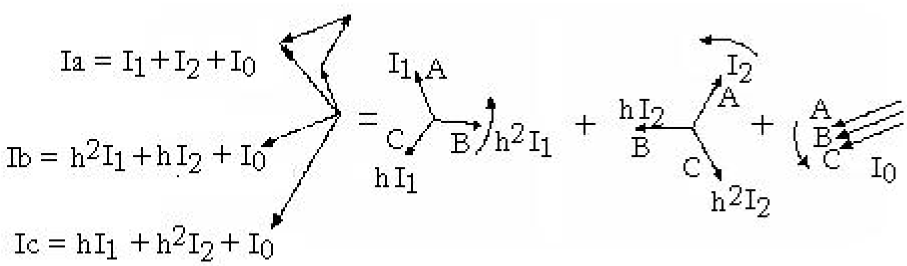


Figure 254; Symmetrical Components

The vector equations are true;

$$Ia = I_0 + I_1 + I_2$$
$$Ib = I_0 + h^2 I_1 + h I_2$$
$$Ic = I_0 + h I_1 + h^2 I_2$$

h is the operator that rotates the vector by 120 degrees.

$$h = -1/2 + j\sqrt{3}/2$$
$$h^2 = -1/2 - j\sqrt{3}/2$$
$$h^3 = 1$$

Also $1 + h + h^2 = 0$

Adding $Ia + Ib + Ic = 3 \cdot I_0$

$$Ia + h \cdot Ib + h^2 \cdot Ic = 3 \cdot I_1$$
$$Ia + h^2 \cdot Ib + h \cdot Ic = 3 \cdot I_2$$

These equations show that there is always one but only one set of symmetrical components for any given three currents

Impedance drop
Let Z_1, Z_2 and Z_0 be the impedances per phase of the circuit to the positive, negative and zero sequence components of the current.

Generator emfs are;

$$Ea = E$$
$$Eb = h^2 \cdot E$$
$$Ec = h \cdot E$$

$$Va = E - Ia \cdot Z$$
$$= E - (I_1 \cdot Z_1 + I_2 \cdot Z_2 + I_0 \cdot Z_0)$$

$Vb = h^2 \cdot E - (h^2 \cdot I_1 \cdot Z_1 + h \cdot I_2 \cdot Z_2 + I_0 \cdot Z_0)$
$Vc = h \cdot E - (h \cdot I_1 \cdot Z_1 + h^2 \cdot I_2 \cdot Z_2 + I_0 \cdot Z_0)$

Three phase fault (LLL)
$Ia = Ifault$
$Ia + Ib + Ic = 0$
$Va = Vb = Vc = Vfault$

Substituting in the above equations and simplifying;
$Vfault = 0$, $I_1 = Ifault$, $I_2 = 0$ and $I_0 = 0$
$Ifault = E / Z_1$

For fault calculations put all impedances in terms of base 10MVA. Put Z_1 equal to the sum of all positive sequence impedances between the generators and the fault.
Fault current = 1000 /[(√3) (*line kV at the fault*) (Z_1)] kAmps

Single phase to ground (LG)
$Ia = Ifault$
$Va = Ib = Ic = 0$
Substituting and simplifying;
$I_1 = I_2 = I_0 = Ifault / 3$
$Ifault = 3 \cdot E / (Z_1 + Z_2 + Z_0)$

For fault calculations put all impedances in terms of base 10MVA. Put Z_1 and Z_2 equal to the sum of all positive and negative sequence impedances between the generator neutral and the fault. If there is a delta/star transformer between the generator and the fault, put Z_0 equal to the zero sequence impedance between the transformer neutral and the fault. If there is no transformer Z_0 is the zero sequence impedance between the generator neutral and the fault.

Fault current = 3000 /[(√3) (*line kV at the fault*) ($Z_1 + Z_2 + Z_0$)] kAmps

Line to line (LL)
$Ia = 0$
$Ib = -Ic = Ifault$
$Vb = Vc$
Substituting and simplifying;
$I_0 = 0$
$I_1 = -I_2 = (h - h^2) \cdot Ifault/3$
$Ifault = E \cdot \sqrt{3} / (Z_1 + Z_2)$

For fault calculations put all impedances in terms of base 10MVA. Put Z_1 and Z_2 equal to the sum of all positive and negative sequence impedances between the generator neutral and the fault.

Fault current = 1000 /[(*line kV at the fault*) ($Z_1 + Z_2$)] kAmps

Line to line and ground fault (LLG)

$Ia = 0, \quad Vb = Vc = 0$

Substituting and simplifying;

$I_1 = E / (Z_1 + Z_0 \cdot Z_2 / (Z_0 + Z_2))$

$I_2 = - I_1 \cdot Z_0 / (Z_0 + Z_2)$

$I_0 = - I_1 \cdot Z_2 / (Z_0 + Z_2)$

$Ib = I_0 + h^2 I_1 + h I_2$

$Ic = I_0 + h I_1 + h^2 I_2$

Current to ground = $3 \cdot I_0$

For fault calculations, it is best to calculate the symmetrical components and then add them vectorally to obtain *Ib* and *Ic*. The current to ground, which will be needed if there is earth fault protection, is three times the zero sequence component.

Using these values for the fault calculations, the positive sequence current and the positive sequence busbar voltage are obtained.

Power in terms of symmetrical components

Power due to I_1 is the sum of the vector dot products;

$W = E \bullet I_1 + h^2 E \bullet h^2 I_1 + h E \bullet h I_1$

$\quad = E \cdot I_1 \cdot \mathrm{Cos}(\phi_1) + E \cdot I_1 \cdot \mathrm{Cos}(\phi_1) + E \cdot I_1 \cdot \mathrm{Cos}(\phi_1)$

$\quad = \sqrt{3} \cdot (line\ kV) \cdot I_1 \cdot \mathrm{Cos}(\phi_1)$

Power due to I_2 is the sum of the vector dot products;

$W = E \bullet I_2 + h^2 E \bullet h I_2 + h E \bullet h^2 I_2$

$= E \cdot I_2 \cdot \mathrm{Cos}(\phi_2) + E \cdot I_2 \cdot \mathrm{Cos}\,(2\pi/3 + \phi_2) + E \cdot I_2 \cdot \mathrm{Cos}\,(2\pi/3 - \phi_2)$

$= E \cdot I_2 \cdot [\,\mathrm{Cos}(\phi_2) - (1/2) \cdot \mathrm{Cos}(\phi_2) - (\sqrt{3}/2) \cdot \mathrm{Sin}(\phi_2) - (1/2) \cdot \mathrm{Cos}(\phi_2) + (\sqrt{3}/2) \cdot \mathrm{Sin}(\phi_2)\,]$

$= 0$

Power due to I_0 is the sum of the vector dot products;

$W = E \bullet I_0 + h^2 E \bullet I_0 + h E \bullet I_0 = 0$

Thus the total power due to the symmetrical components is;

$W = \sqrt{3} \cdot (line\ kV) \cdot I_1 \cdot \mathrm{Cos}(\phi_1)$

where W is in MW, I_1 is in kiloamps and ϕ_1 is the phase angle of I_1

Currents in the primaries of Delta Star Transformers with a Line to Ground fault

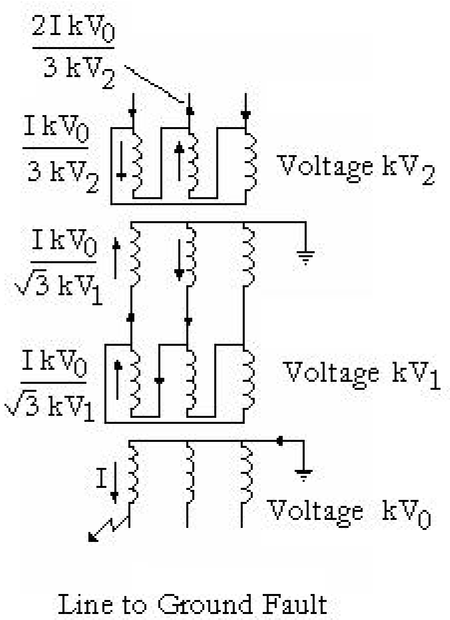


Figure 255; Delta star transformer Line to Ground fault

Use the turns ratio, not the line voltage ratio to calculate the currents on the primaries of Delta / Star Transformers. A single line to ground fault of I in one phase of the secondary of the transformer gives rise to a current of $I\,kv0/(\sqrt{3}\,kv1)$ in two phases of the primary. A further delta star transformer has a current of $I\,kv0/(3\,kv2)$ in two phases of its primary and a current of $2I\,kv0/(3\,kv2)$ in the third phase.

A Star/Star transformer with a third Delta winding, ie Star/Delta/Star has a current of $(I/3)\,kv0/kv1$ in two phases of the Star primary and a current of $(2\,I/3)\,kv0/kv1$ in the third phase

Currents in the primaries of Delta Star Transformers with a Line to Line fault

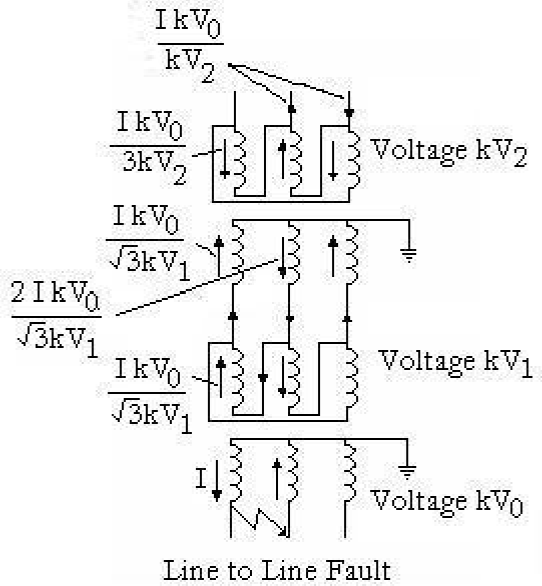


Figure 256; Delta star transformer Line to Line fault

.

A Line to Line Fault of *I* at *kv0* on the secondary of a delta/star transformer appears as a current of *I kv0*/(√3 *kv1*) on two phases of the primary.
The third phase has a current of 2*I kv0*/(√3 *kv1*).
A further delta/star transformer has a current of *I kv0*/ *kv2* in two phases.

A Star/Delta/Star transformation gives a current of *I kv0*/*kv1* in two phases of the primary.

Currents in the primaries of Delta Star Transformers with a Double Line to Ground fault

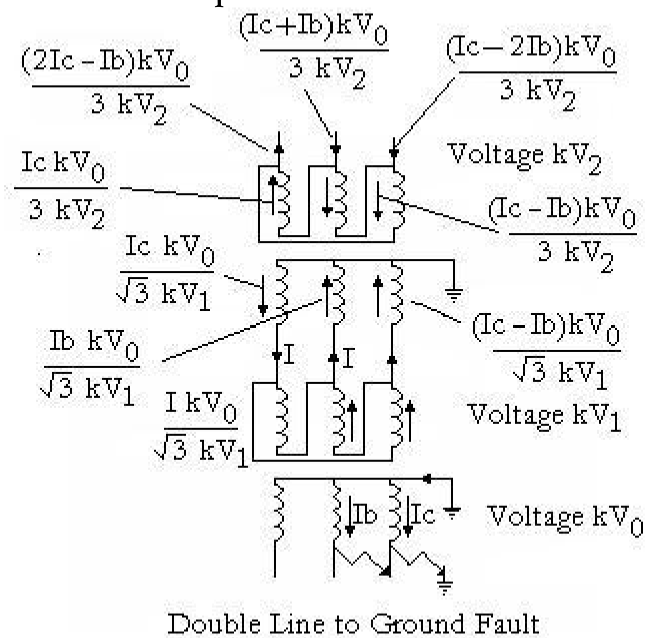


Figure 257; Delta star transformer Double Line to Ground fault

Double Line to ground fault involves the vector sum and difference between the currents on each phase. Thus it is best to keep the fault currents in terms of the symmetrical components until final evaluation.

Combination of Fault current and Load current

LLL and LLLG Faults

The Fault current and Load current are both positive phase sequence only

Hence Total Current = vector sum of Fault current $Ifault$ and Load current IL

$I = \sqrt{[\{Ifault \cdot \cos(\phi F) + IL \cdot \cos(\phi L)\}^2 + \{Ifault \cdot \sin(\phi F) + IL \cdot \sin(\phi L)\}^2]}$

where $\phi F = \arctan(XF/RF)$ and $\phi L = \arctan(XL/RL)$

LG Fault

The Positive, Negative and Zero phase sequence currents are all in phase on the line with the fault. The fault current Ifault is the sum of these currents.

Hence on the phase with the fault,

Total current = vector sum of $Ifault$ and IL

Total Current $= \sqrt{[\{Ifault \cdot \cos(\phi F) + IL \cdot \cos(\phi L)\}^2 + \{Ifault \cdot \sin(\phi F) + IL \cdot \sin(\phi L)\}^2]}$

where $\phi F = \arctan(XF/RF)$ and $\phi L = \arctan(XL/RL)$

On the other two phases, the fault current is zero.

LL Fault

Zero sequence component = 0

Assume the fault is between phases B and C

On phase A, the Negative sequence component is equal and opposite to the Positive sequence component. The Fault current is zero.

On phase B,
Fault current = $h^2 \cdot$ Positive seq component + $h \cdot$ Negative seq component
= $(h^2 - h) \cdot$ Positive seq component
Fault current = √3 times Positive sequence component of the Fault current
This current lags the voltage Eb by $(\phi F - \pi/6)$
On phase C,
Fault current is equal and opposite to the Fault current on phase B
It lags the voltage Ec by $(\phi F + \pi/6)$

Total current = $\sqrt{[\ \{Ifault \cdot \cos(\phi T) + IL \cdot \cos(\phi L)\}^2 + \{Ifault \cdot \sin(\phi T) + IL \cdot \sin(\phi L)\}^2\ ]}$
where $\phi F = \arctan(XF/RF)$ and $\phi L = \arctan(XL/RL)$
On phase B $(\phi T) = (\phi F - \pi/6)$
On phase C $(\phi T) = (\phi F + \pi/6)$

LLG Faults
The positive sequence component of the Fault Current can be obtained directly by putting;
$XF = X_1 + (R_0^{\ 2} \cdot X_2 + R_2^{\ 2} \cdot X_0 + X_0 \cdot X_2 \cdot (X_0 + X_2)) \ / \ ((R_0 + R_2)^2 + (X_0 + X_2)^2)$
$RF = R_1 + (R_0 \cdot R_2 \cdot (R_0 + R_2) + R_2 \cdot X_0^{\ 2} + R_0 \cdot X_2^{\ 2}) \ / \ ((R_0 + R_2)^2 + (X_0 + X_2)^2)$

This impedance can be combined with the Load Impedance to obtain the equivalent impedance $R + jX$. This enables the Total Positive sequence component to be obtained which can then be separated into the Load Current and Positive Sequence Component of the Fault Current.

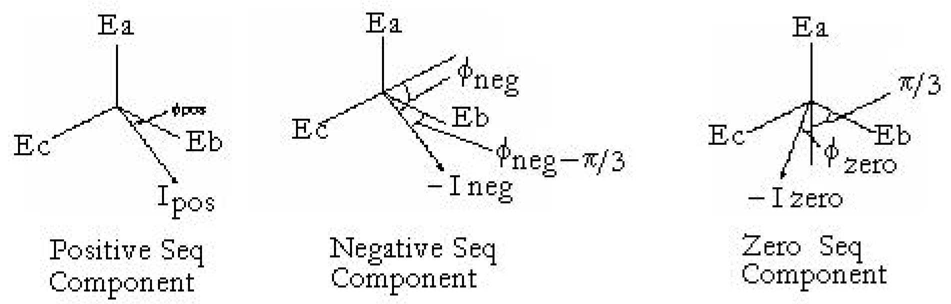


Figure 258; Symmetrical components

The Negative and Zero components of the Fault Current can then be obtained.

Let Positive Seq Component be *Ipos* at phase angle ϕPos
$\phi Pos = \arctan(XF/RF)$

Let Negative Seq Component be *Ineg* at phase angle ϕNeg on phase A
Put $R_3 = R_0 + R_2$ and $X_3 = X_0 + X_2$
$Ineg = -Ipos \cdot (R_0 + j\,X_0) \ / \ (R_3 + j\,X_3)$
$= -Ipos \cdot [\ (R_0 \cdot R_3 + X_0 \cdot X_3) + j(R_3 \cdot X_0 - R_0 \cdot X_3)] \ / \ (R_3^{\ 2} + X_3^{\ 2})$
Hence magnitude of *Ineg* is given by
$Ineg = -\ Ipos \cdot /[(R_0 \cdot R_3 + X_0 \cdot X_3)^2 + (R_3 \cdot X_0 - R_0 \cdot X_3)^2\ ] \ / \ (R_3^{\ 2} + X_3^{\ 2})$
$\phi Neg = \phi F + \arctan[\ (R_3 \cdot X_0 - R_0 \cdot X_3) \ / \ (R_0 \cdot R_3 + X_0 \cdot X_3)\ ]$

Interchange suffix 2 and 0 to get;
Magnitude of *Izero* is given by
$Izero = -Ipos \cdot /[(R_2 \cdot R_3 + X_2 \cdot X_3)^2 + (R_3 \cdot X_2 - R_2 \cdot X_3)^2\ ] \ / \ (R_3^{\ 2} + X_3^{\ 2})$

$$\phi Zero = \phi F + \arctan[\,(R_3 \cdot X_2 - R_2 \cdot X_3) \,/\, (R_2 \cdot R_3 + X_2 \cdot X_3)\,]$$

Thus the current in all phases with and without Load Current can be obtained by the vector addition of the relevant currents.
Currents in phase B

Fault Current in phase B =
$$\sqrt{[\{Ipos \cdot \mathrm{Cos}(\phi Pos) + Ineg \cdot \mathrm{Cos}(\phi Neg - \pi/3) + Izero \cdot \mathrm{Cos}(\phi Zero + \pi/3)\}^2 + \{Ipos \cdot \mathrm{Sin}(\phi Pos) + Ineg \cdot \mathrm{Sin}(\phi Neg - \pi/3) + Izero \cdot \mathrm{Sin}(\phi Zero + \pi/3)\}^2\,]}$$

Similarly, Fault Current in phase C =
$$\sqrt{[\{Ipos \cdot \mathrm{Cos}(\phi Pos) + Ineg \cdot \mathrm{Cos}(\phi Neg + \pi/3) + Izero \cdot \mathrm{Cos}(\phi Zero - \pi/3)\}^2 + \{Ipos \cdot \mathrm{Sin}(\phi Pos) + Ineg \cdot \mathrm{Sin}(\phi Neg + \pi/3) + Izero \cdot \mathrm{Sin}(\phi Zero - \pi/3)\}^2\,]}$$

Current to ground = $3 \cdot Izero$

For parts of the network carrying both Load Current and Fault Current,
put $Ipos = Ifault + Iload$ but keeping *Ineg* and *Izero* unchanged as these currents do not flow in the load circuit.

COMMISSIONING AND FAULT FINDING

To Commission new Electrical Plant, the following tests are usually carried out.

1.) Dry out
2.) Operation tests
3.) Primary Injection
4.) Secondary Injection
5.) Pressure tests
6.) Phase polarity test

Dry Out
All new plant with insulation that can absorb moisture must be dried out before any voltage tests are carried out. Motors can be dried out by applying a low voltage ac supply to the machine. Use a voltage that is too low to rotate the machine and raise the current to a figure below full load current. The cooling fan is not supplying cooling air, so the current must not reach the full load value.

At regular intervals of time, the current is switched off and the insulation resistance is measured. As the temperature rises, the insulation resistance falls until a steady temperature is reached. The insulation resistance then rises as the machine dries out. When the machine is dry, the insulation resistance again levels off.

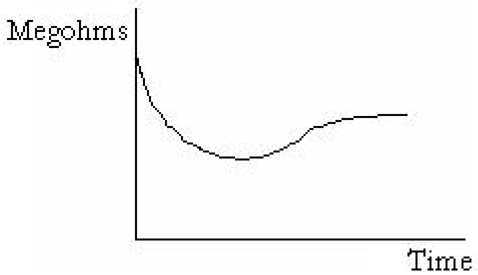


Figure 259; Machine Dry Out

Generators can be dried out by applying a short circuit to earth on all phases and the machine run on hand excitation control. The short circuit connection must be rated for continuous full load current. The normal switchgear earthing gear must on no account be used as it is usually rated for only 30 seconds. A failure of this connection will lead to a catastrophe, the machine voltage will rise rapidly. As the connection melts, a high power fault close to the power station busbars will develop leading to a power station busbar fault.

Some operators remove the generator trip fuses during the dry out. If the machine circuit breaker trips inadvertently, the voltage could rise above the machine rating before the insulation is fully dry and permanently damage the machine.

Dry out may not be possible if the machine is of the brushless excitation design. These machines have an ac exciter on the same shaft as the generator. The exciter field is the stator. The exciter rotor generates ac which is converted to dc by silicon rectifiers at the generator field on the generator rotor. The generator stator is the generator armature. The voltage regulator controls the exciter field and hand control of the exciter may not be possible.

Operation tests
With the switchgear in the racked out position, attach the secondary jump leads. Operate all the closing and tripping controls and trip the switch by all the protection relays.

Primary Injection
Apply a short circuit on the plant and connect a high current low voltage ac supply from a Primary Injection set. Apply the full rated ac current and measure the current in all the CT secondary circuits. Use a milliammeter to display the residual current in earth fault circuits. With all phases earthed on one side of the plant, apply the primary injection on the other side. Apply the current between the red and yellow phases, repeat between the yellow and blue phases and repeat again between the blue and red phases. Finally apply the injection current between one phase and earth. Record the current in all CT secondary circuits on each test and check the results are as expected.

Secondary Injection
Apply a low power ac current to each CT secondary circuit and check the operation of all protection relays and the instrumentation. Time the operation of time lagged relays.

Voltage transformer secondary circuits are also tested at this time. Remove the voltage transformer fuses to prevent any feedback that might make the voltage transformer primary live at full voltage. If a new power station is being commissioned, the synchronising panel will also need to be tested.

Pressure Tests.
The main electrical circuit is tested with a high voltage dc test set. If the neutral is earthed through an impedance, test the red phase with the yellow phase against the blue phase and earth. Repeat the test with the yellow phase and blue phase against the red phase and earth. The test voltages are at prescribed levels, eg use a 30 kV dc test voltage on an 11kV rms line voltage system. Beware, the dc voltage charges up the capacitance in cables to a lethal amount. Discharge the cables after the test, wait a few minutes and discharge them again. Repeat again and again till fully discharged. Keep the cables earthed while making the connections for the next test.

If the neutral is solidly earthed, the operating voltage to earth is $1/\sqrt{3}$ times the voltage between lines and the pressure tests should take account of this lower voltage.

Phase polarity test
When all the above tests have been successfully done, the plant may be energised for the first time. However if the plant is part of a duplicate supply, the phase connection must be checked before switching the plant into parallel operation. Energise the plant from one end and check the polarity of the voltage transformer secondary with another voltage transformer secondary. Switch off the new plant and energise from the other end and check the voltage transformer polarities again. If there is no suitable voltage transformer, then a high voltage voltmeter is required.

Automatic Voltage Regulator compounding
When commissioning a generator, the AVR compounding will need to be checked. Incorrect compounding is a common fault on new machines. There is always an earth connection in CT secondary circuits and an earth connection on voltage transformer secondary circuits. The AVR compounding connects a current transformer circuit to a voltage transformer circuit and if these both have an earth connection they may short out the compounding. When the generator supplies a lagging current, the machine reactance lowers the voltage. The AVR compounding must not eliminate this voltage drop or the machines will not run in parallel.

Fault finding
High impedance faults
A high impedance fault on one phase of a long underground cable or overhead power line can be located by a high voltage bridge.
Connect the remote end of the faulty core to a healthy core. Connect the high voltage bridge to this healthy core and the faulty core.

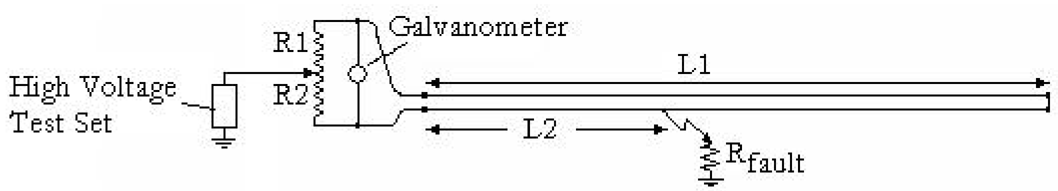


Figure 260; Locating a cable fault

Supply the bridge from a high voltage test set and balance the bridge on the slider. Note the readings *R1* and *R2*.

Although the fault resistance may be several thousand ohms and the cable core only a few ohms, the ratio $[L1 + (L1 - L2)]/L2$ is equal to $R1/R2$
Thus $L2/L1 = 2\ R2/(R1 + R2)$

It is prudent to keep a record of the exact location of all cable joints on a long underground cable. The fault is usually at a joint unless there is obvious damage above ground.

PROGRAM MANUAL

Getting Started

When the program is running, data is written to the source folder. Therefore the program will not run from a CD. Copy and Paste the entire contents of the CD to your own Folder on your hard drive or onto a floppy disc. In Windows, press the [Start] + [e] keys, click on the drive for the CD, for example "My Computer / Disc Drive D", click on "Edit", click on "Copy", click on your chosen destination Folder and click on "Edit" and click on "Paste".

Double click on the application program "Fault Calculations Rev 1.exe" and page 1 appears.

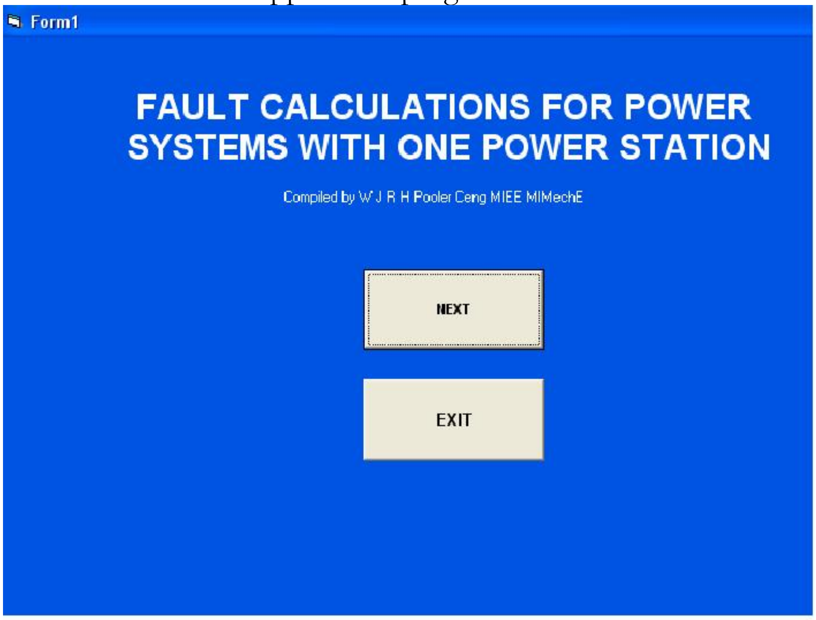


Figure 261; Page 1

Click on Next and page 2 appears

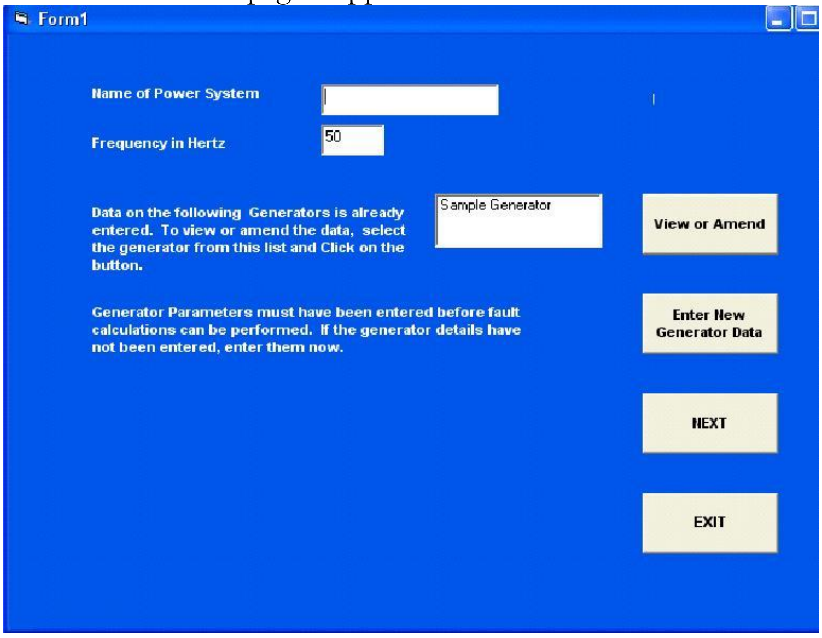


Figure 262; Page 2

Enter the name for the Power System although the program will run if this box is left blank.
For a trial run, click on "Sample Generator". It turns blue when selected.
With a generator selected, click on "View or Amend" and page 3 appears.

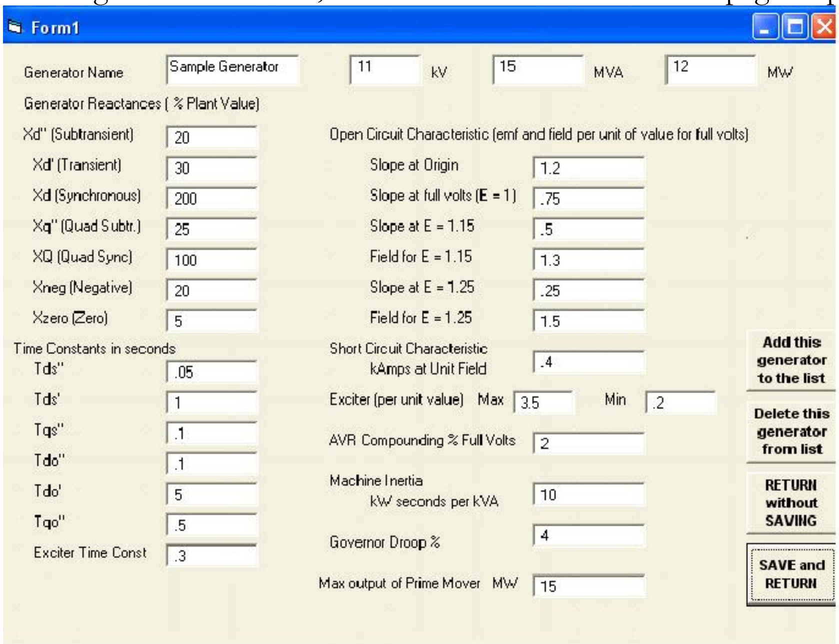


Figure 263; Page 3

This shows the data entered for the selected generator. Click on "Return without saving"

Alternatively, you can add your own generator data at this stage.
In this case enter the name for your generator in the top left hand box and your data in every box.
Enter the generator kV, MVA and MW in the appropiate boxes.
Enter the reactances as the plant values.
Enter the time constants in seconds.
Enter the Open Circuit characteristic with EMF as per unit of the rated value and Field as per unit of the field for rated EMF. This is entered as Slope at the origin, Slope at E = 1, Slope at E = 1.15 and Slope at E = 1.25.
Enter the per unit value of the Field at E = 1.15 and at E = 1.25.
Enter the Short Circuit Characteristic as kiloamps at unit Field.
Enter the maximum and minimum available exciter output as per unit value of the value for open circuit rated volts.
Enter the AVR compounding voltage on full load current as per cent of the AVR rated voltage. It is assumed that the compounding voltage is 90 degrees out of phase with the rated voltage at the machine design power factor.
Enter the machine inertia as kW seconds per kVA. If T is the estimated time in seconds for the machine to run up from stationary to full speed with the prime mover developing full power throughout the run up, then H = T · (power factor).
Enter the governor droop, the change in speed from no load to full load with no manual adjustment expressed as as per cent of the full speed.
Finally enter the maximum available power output of the prime mover.
Click on "Add this generator to the list". This saves the data for the new generator.
Click on "Return without Saving" or "Save and Return" to return to page 2.

Whenever the program is run again, this generator will appear on page 2.

After returning to page 2, click on Next to move to page 4.

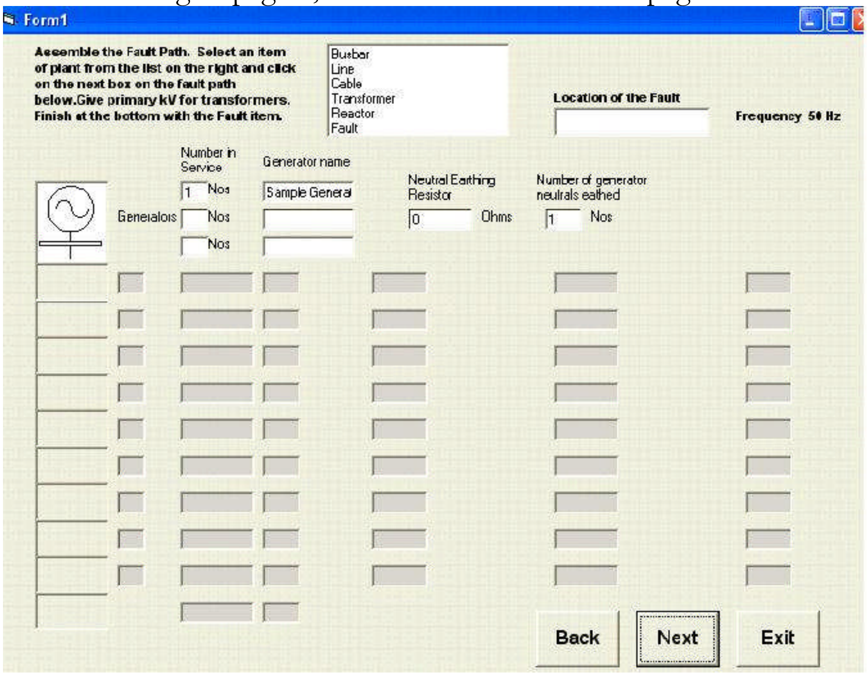


Figure 264; Page 4

Enter the name of each type of generator and the number in service for this fault calculation. Up to three different types of generator all running in parallel can be entered.

Build up the fault path. Click on the item of plant in the box at the top of the page. It turns blue, then click on the box below the generator symbol. Enter the plant size in the appropriate boxes to the right of each item.

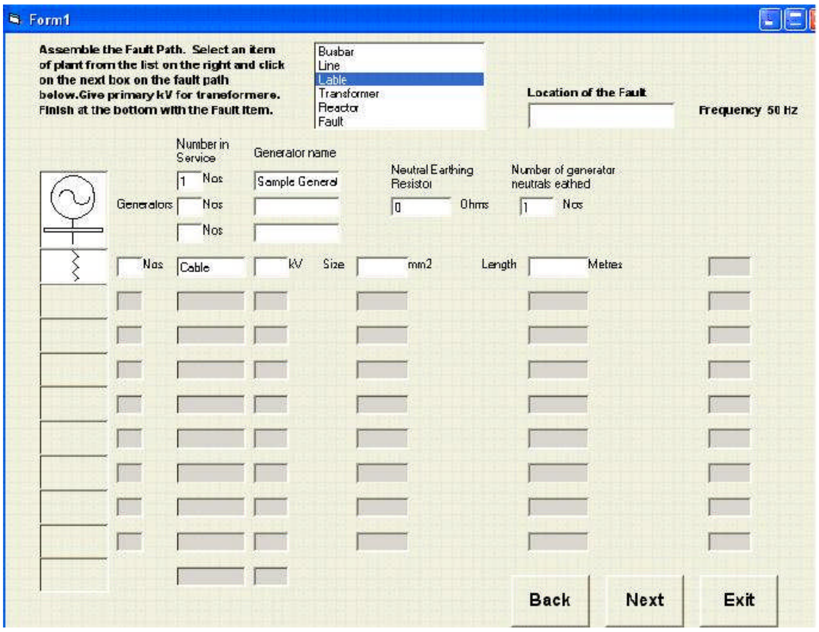


Figure 265; Page 4 continued

Build up the fault path by clicking on an item in the top box then clicking on the fault path box on the left of the page. Enter the details for the item on its right.

Finally click on Fault in the top box and click on the box at the end of the fault path
When the Fault Path is complete, it will look something like
Check that data has been entered in all the white boxes. The page should appear like this.

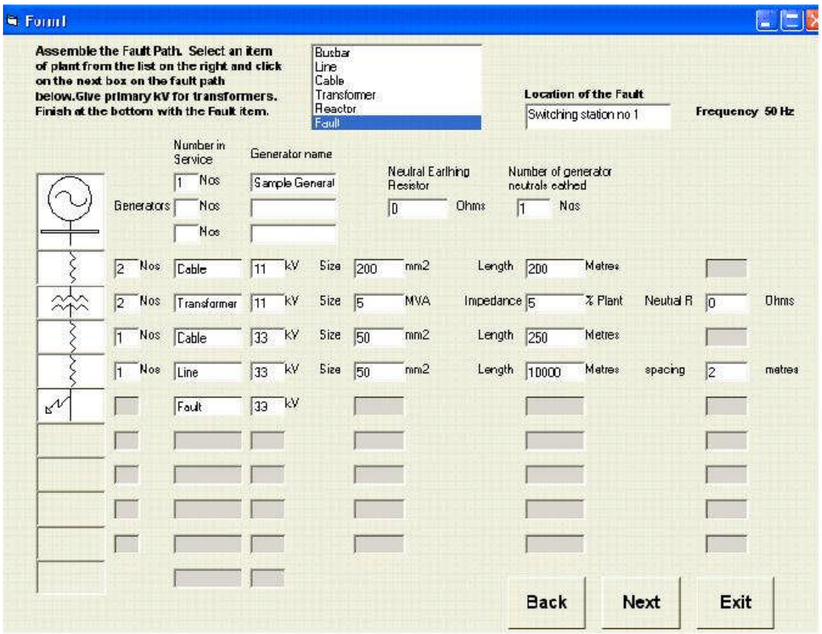


Figure 266; Page 4 continued.

Click on Next and page 5 appears.

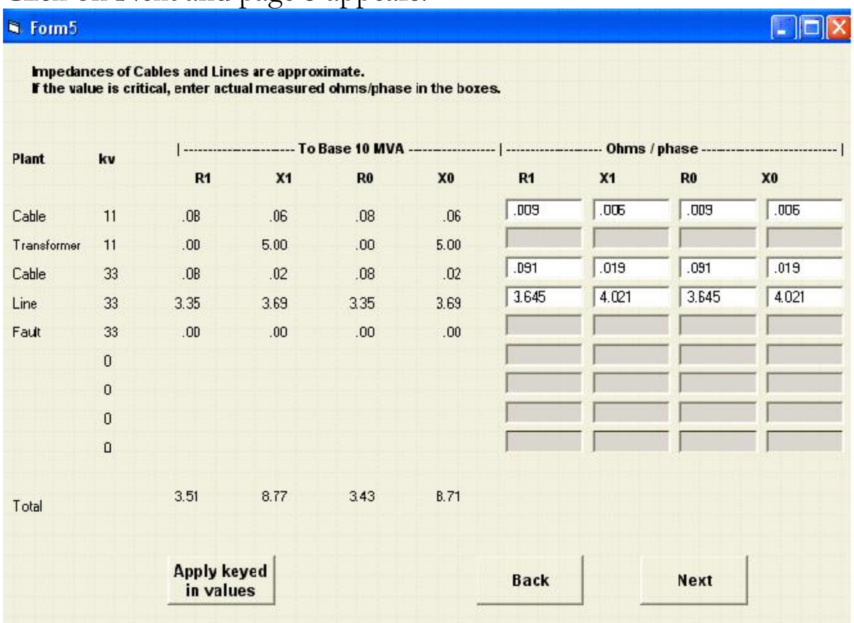


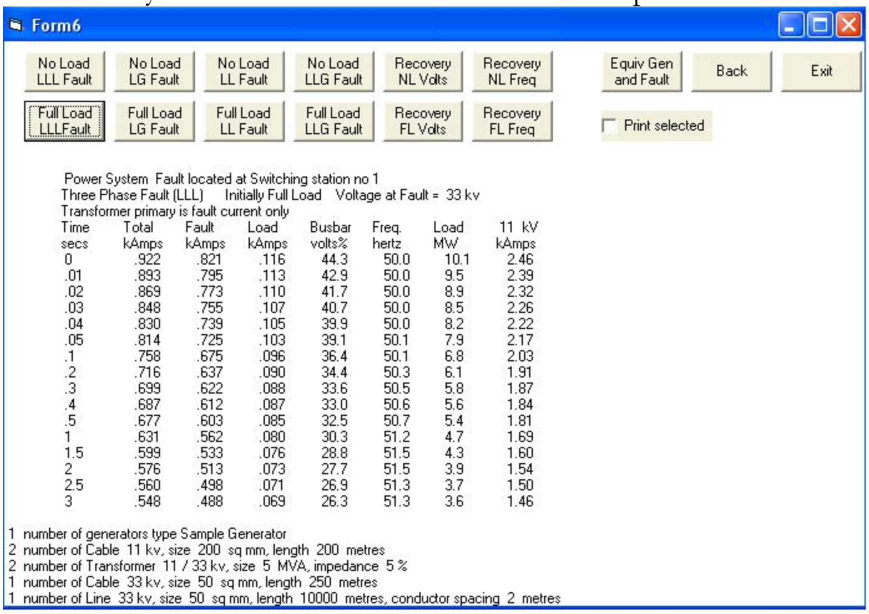
Figure 267; Page 5

This page displays the impedances of each item of plant based on typical values. If more accurate data is available, the values expressed as ohms/phase or as % to base 10 MVA can be entered. Click on “Apply keyed in values” and the keyed in values will be used in the calculations.
If the R0 and X0 values for a power line are significant, they should be measured on site and the actual value entered. They may be as much as five times higher.

Click on Next and the last page appears.

Figure 268; Page 6

Click on any button and the Fault Calculations are performed and Displayed



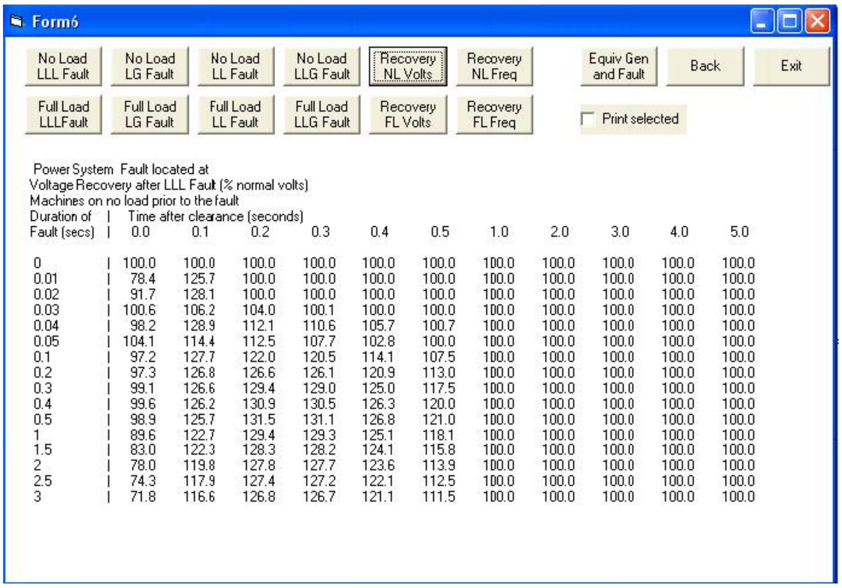
Power System Fault located at Switching station no 1
Three Phase Fault (LLL) Initially Full Load Voltage at Fault = 33 kv
Transformer primary is fault current only

Time secs	Total kAmps	Fault kAmps	Load kAmps	Busbar volts%	Freq. hertz	Load MW	11 kV kAmps
0	.922	.821	.116	44.3	50.0	10.1	2.46
.01	.893	.795	.113	42.9	50.0	9.5	2.39
.02	.869	.773	.110	41.7	50.0	8.9	2.32
.03	.848	.755	.107	40.7	50.0	8.5	2.26
.04	.830	.739	.105	39.9	50.0	8.2	2.22
.05	.814	.725	.103	39.1	50.1	7.9	2.17
.1	.758	.675	.096	36.4	50.1	6.8	2.03
.2	.716	.637	.090	34.4	50.3	6.1	1.91
.3	.699	.622	.088	33.6	50.5	5.8	1.87
.4	.687	.612	.087	33.0	50.6	5.6	1.84
.5	.677	.603	.085	32.5	50.7	5.4	1.81
1	.631	.562	.080	30.3	51.2	4.7	1.69
1.5	.599	.533	.076	28.8	51.5	4.3	1.60
2	.576	.513	.073	27.7	51.5	3.9	1.54
2.5	.560	.498	.071	26.9	51.3	3.7	1.50
3	.548	.488	.069	26.3	51.3	3.6	1.46

1 number of generators type Sample Generator
2 number of Cable 11 kv, size 200 sq mm, length 200 metres
2 number of Transformer 11 / 33 kv, size 5 MVA, impedance 5 %
1 number of Cable 33 kv, size 50 sq mm, length 250 metres
1 number of Line 33 kv, size 50 sq mm, length 10000 metres, conductor spacing 2 metres

Figure 269; Page 6 continued.

Click on "Recovery NL Volts" to see the over voltage after the fault is cleared if the machine was on no load before the fault.



Power System Fault located at
Voltage Recovery after LLL Fault (% normal volts)
Machines on no load prior to the fault

Duration of Fault (secs)	Time after clearance (seconds) 0.0	0.1	0.2	0.3	0.4	0.5	1.0	2.0	3.0	4.0	5.0
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.01	78.4	125.7	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.02	91.7	128.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.03	100.6	106.2	104.0	100.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.04	98.2	128.9	112.1	110.6	105.7	100.7	100.0	100.0	100.0	100.0	100.0
0.05	104.1	114.4	112.5	107.7	102.8	100.0	100.0	100.0	100.0	100.0	100.0
0.1	97.2	127.7	122.0	120.5	114.1	107.5	100.0	100.0	100.0	100.0	100.0
0.2	97.3	126.8	126.6	126.1	120.9	113.0	100.0	100.0	100.0	100.0	100.0
0.3	99.1	126.6	129.4	129.0	125.0	117.5	100.0	100.0	100.0	100.0	100.0
0.4	99.6	126.2	130.9	130.5	126.3	120.0	100.0	100.0	100.0	100.0	100.0
0.5	98.9	125.7	131.5	131.1	126.8	121.0	100.0	100.0	100.0	100.0	100.0
1	89.6	122.7	129.4	129.3	125.1	118.1	100.0	100.0	100.0	100.0	100.0
1.5	83.0	122.3	128.3	128.2	124.1	115.8	100.0	100.0	100.0	100.0	100.0
2	78.0	119.8	127.8	127.7	123.6	113.9	100.0	100.0	100.0	100.0	100.0
2.5	74.3	117.9	127.4	127.2	122.1	112.5	100.0	100.0	100.0	100.0	100.0
3	71.8	116.6	126.8	126.7	121.1	111.5	100.0	100.0	100.0	100.0	100.0

Figure 270; Voltage after fault clearance.

Click on "Recovery FL Volts" to see the over voltage that occurs after a fault if the machine was on Full Load before the fault.

Click on "Back" to return to page 4 for more calculations with a new Fault Path.

Finally Click on Exit on any page to leave the program.

INDEX

AC Bridge Circuits 64
AC Commutator motors 137
AC machine emf 101
AC, Alternating Current 45
Addition of AC quantities 48
Ampere, Amp 17
Armature Reaction, AC machine 120-123
Armature Reaction, DC machine 80
AVR Auto Voltage Regulator 132
Average Value of AC 45
B, Magnetic Field 15
Bearings 84
Brush drop 97
Brush friction 95
Brush loss
Brushless generator 101
Buchholtz protection 182
C, Capacitance 35
Cables 158
Capacitance of concentric tubes 37
Capacitance of parallel plates 36
Capacitance of sphere 35
Capacitance on AC 54
Capacitive load on AC generator 119,122
Circulating current protection 179
Coil span, kp 106
Commutator 75
Compensating winding, DC machine 80
Contactors 165
Corkscrew Rule 16
Coulomb, q 17
Current Transformers 165
Damping winding 119
DC circuits 40
DC compound motor 80
DC machine emf 77
DC machine with split field 89
DC motor losses 93
DC shunt and series generators 78, 90, 91
DC shunt and series motors 78
Decelleration test 96
Delta/Star Transformation 63, 148, 227
Direct and quadrature axes 120
Distance Protection 180
Distribution factor, *kd* 107, 109
Distribution lines 157
E, *EMF*, Electro Motive Force 25
Earth fault protection 179
Earthing Electrode 160
Earthing Resistor 161
Eddy currents 72
Electrical degrees 103
Electromagnet 73
Energy in a magnetic field 72
Energy stored in capacitance 39
Energy stored in inductance 29
F, *MMF*, Magnetising Force 19
f, frequency 39, 85
Farad, microfarad 35
Faraday's Law 25
Fault MVA 222
Fault program manual 236
Flux in three phase machine 106
Flux in transformers 147
Flux temperature relation 69
Flux 15
Force between conductors 18, 222
Form Factor 46
Fractional slots 110
Frequency, f 47, 101
G, Gauss, Magnetic Field 15
Generator impedances 192
Generator protection 180
Gilbert 22
H, Magnetising Force 16
Harmonics 111
Henry, Self Inductance 27
High Rupturing (HRC) Fuses 179
Hopkinson Knapp test 94
Hydrogen cooling 103
Hysteresis 69, 70
I, *i*, Current 16, 17
Impedance, Z 58
Inductance on AC 53
Induction motor, single phase 142
Induction motor, three phase 139
Instruments, dynamometer 172
Instruments, electrostatic 174
Instruments, induction meter 175
Instruments, moving coil 171
Instruments, moving iron 171, 172
Insulation Class 144
Insulation temperature 144
Interpoles 80
Iron cored Reactor 56
k, Dialectric Constant 33

kd, Distribution factor 108
kp, Pitch factor 106
Kirchoff's Laws 41
L, Inductance 27
Leakage Reactance 120
Left Hand Rule 18
Lenz's Law 25
Line impedances 220, 221
Line insulators 157
Line to ground fault 225
Line to line fault 225
Machine rating 103
Magnetic Circuit 22
Magnetic Field, B 16
Magnetic Flux, Φ 15
Magnetic Properties of materials 68
Magnetizing Force 16, 17
Magneto Motive Force, MMF 22
Mercury arc rectifier 151
Motor protection 182
Neutral Earth 160
Oe, Oersted 16
Ohm, Ω 26
Open Circuit test 123
Overcurrent and e/f protection 179
Parallel operation of generators 130
Peak Factor 46
Permeability 16
Pitch factor, *kp* 106
Potentiometer 44
Power measurement 51
Power systems 184
Power, AC induction motor 141
Power, DC machine 77
Power, single phase system 49, 50
Power, symmetrical components 226
Power, three phase system 51
Q, Coil at resonance 59
R, Resistance 22, 45
Rating of machines 103
Reactance XC 54
Reactance XL 53
Reactance X 53
Rectifier multi phase 152
Rectifier single phase 153, 154
Rectifier three phase 153
Rectifier with capacitor 154
Resistance on AC 53
Resistance, R 26
Resonance 58, 59
Restricted earth fault protection 179
Right Hand Rule 26
Ring Main 188
RMS Value of AC 45
Self Inductance, L 27
Short Circuit test 123
Slot ripple 116
Speed control DC motor 86, 87
Stability of AC machines 135
Starter for DC motor 84
Surge Diverters 157
Swinburne test for DC motor 93
Switchgear close and trip circuits 166
Switchgear types 163
Symmetrical components 224
Synchronising AC machines 132
Synchronising circuits 169
Synchronous condenser 136
Synchronous motor 136
Synchronous Reactance, Xd 126
Tesla, magnetic field B 15
Thevenin's Theorem 42
Three phase fault 225
Three phase generation 102
Three phase system 50
Torque, AC induction motor 141
Torque, DC machine 77
Transformer cores 147
Transformer flux and emf 147
Transformer losses 148
Transformer magnetising current 149
Transformer protection 181
Transformers, delta/star 148
Transformers, unbalanced load 150
Transmission lines 157
Unit pole 15
V, Volt 25
Vector Diagram AC generator 127
Vector Representation of AC 47
Voltage transformers 168
Ward Leonard set 87
Weber 15
Wheatstone Bridge 43
Winding types 83
X, Reactance 53
Z, Impedance 58