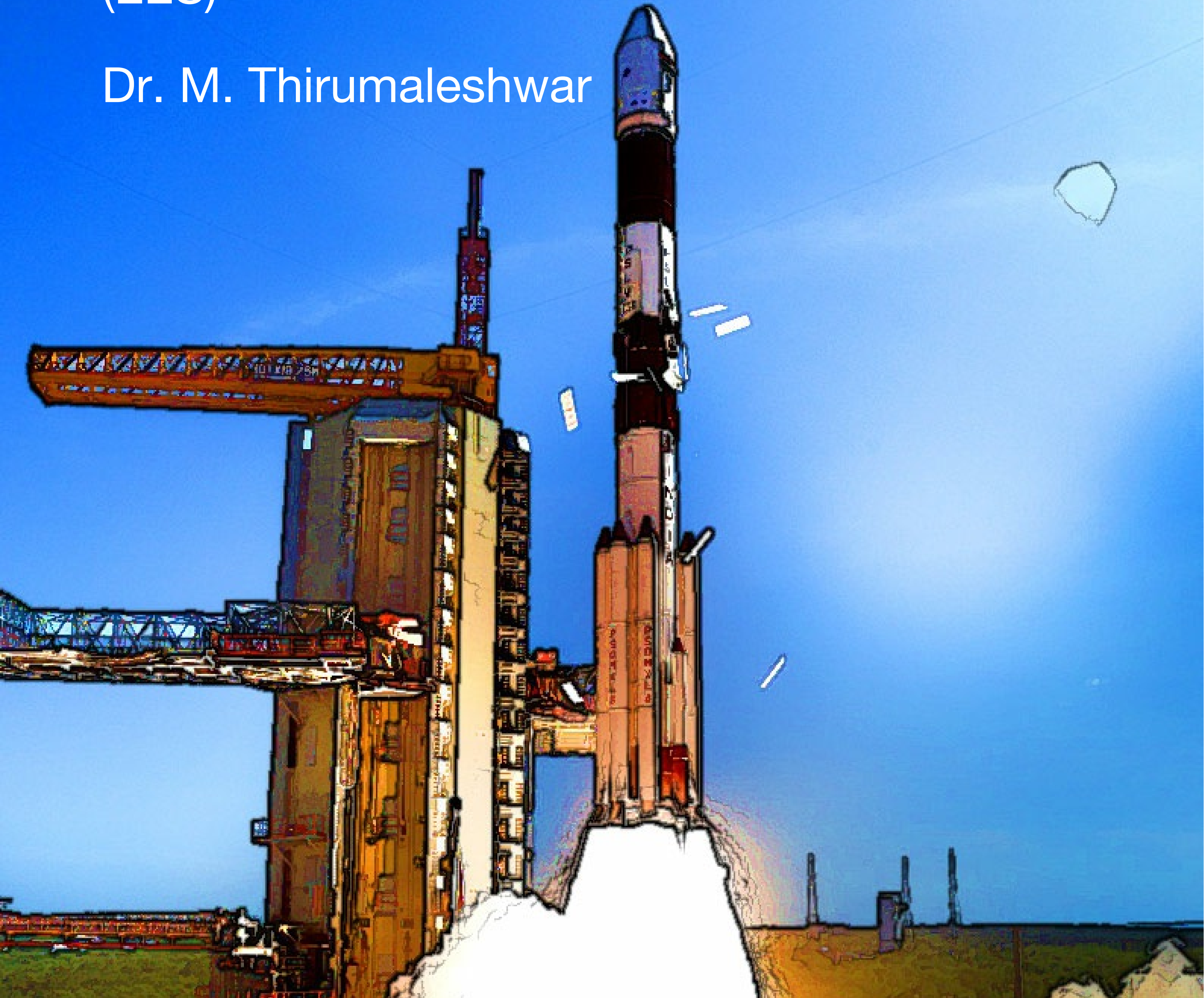
# Cryog. Engineering: Software Solutions Part-III-B

Cryogenic Refrigeration systems: Problems (EES)

Dr. M. Thirumaleshwar



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DR. M. THIRUMALESHWAR

# CRYOGENIC ENGINEERING: SOFTWARE SOLUTIONS PART-III-B

## CRYOGENIC REFRIGERATION SYSTEMS – PROBLEMS (EES)

Cryogenic Engineering: Software Solutions Part-III-B: Cryogenic Refrigeration systems – Problems (EES)
1st edition

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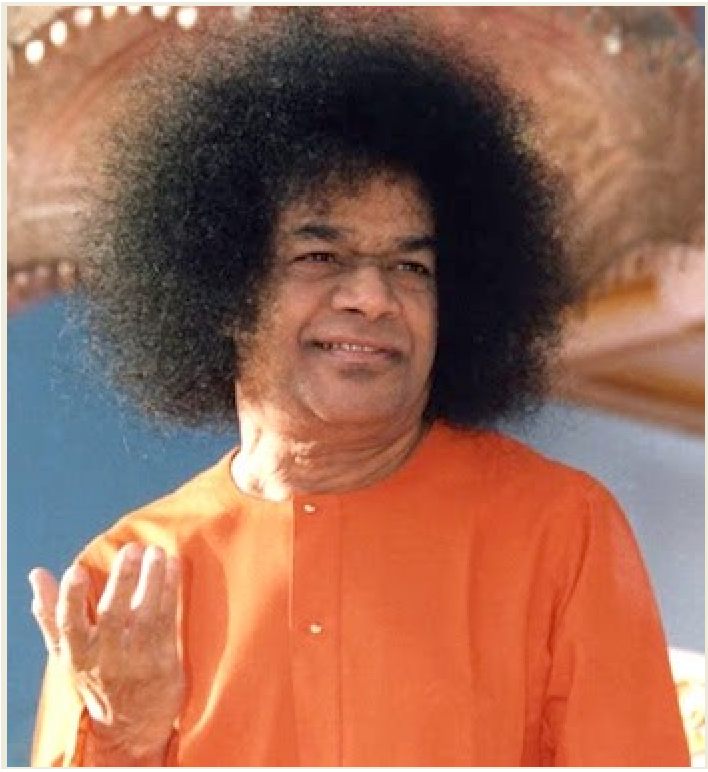
# CONTENTS

**Cryogenic Engineering: Software Solutions Part-III-B:
Cryogenic Refrigeration systems – Problems (EES)**

# DEDICATION

**This work is lovingly dedicated at the lotus feet of**

**Bhagavan Sri Sathya Sai Baba**



"There is only one religion, the religion of Love.
There is only one caste, the caste of Humanity.
There is only one language, the language of the Heart.
There is only one God, He is Omnipresent."

**"Help Ever, Hurt Never!"**

***…Bhagavan Sri Sathya Sai Baba***

# PREFACE

This book, viz. **Cryogenic Engineering: Software solutions – Part-III-B**, is the third volume in the series on **Cryogenic Engineering: Software Solutions**.

**This part is being published in two volumes, viz. Cryogenic Engineering: Software solutions – Part-III A and Part-III B.** Part-III A contains a brief summary of background theory, definitions and formulas and the problems solved with Mathcad. Part-III B contains problems solved with Engineering Equation Solver (EES). ***So, it is advisable that one refers to both the parts.***

As with the Part-I and II of the series, focus here is on the solutions of problems in cryogenic engineering using software such as Mathcad and Engineering Equation Solver (EES). *Only the essential theory and summary of equations required for calculations are given at the beginning of the chapter.*

*Advantages* of using computer software to solve problems are reiterated:

i) It helps in solving the problems fast and accurately
ii) Parametric analysis (what-if analysis) and graphical visualization is done very easily. This helps in an in-depth analysis of the problem.
iii) Once a particular type of problem is solved, it can be used as a *template* and solving similar problems later becomes extremely easy.
iv) In addition, one can plot the data, curve fit, write functions for various properties or calculations and re-use them.
v) These possibilities create interest, curiosity and wonder in the minds of students and enthuse them to know more and work more.

This book, viz. **Cryogenic Engineering: Software solutions – Part-III-B** deals with the **Cryogenic refrigeration systems. Here, based on the theory and equations summarized in Part-III-A, many Functions and Procedures are written using Engineering Equation Solver (EES) and problems are solved to illustrate the use of these Functions/Procedures.**

**EES has a great advantage that it has in-built Functions for properties of most of the cryogenic fluids and materials required for our calculations. Thus, it is very convenient to use EES.**

In Part-III A of this book, theoretical background and summary of calculation equations are given on various topics under **Cryogenic refrigeration systems. Topics covered are:**

Thermodynamically ideal isothermal source system, concept of Coeff. of Performance (COP) and Figure of Merit (FOM), thermodynamically ideal isobaric source system, simple Linde-Hampson (L-H) refrigerator, pre-cooled L-H system, Claude refrigeration system, cold gas refrigeration system, Philips (or Stirling cycle) refrigerator, Vuilleumier cycle refrigerator, Solvay cycle and Gifford-McMahon (GM) cycle refrigerators etc., Magnetic cooling, i.e. Adiabatic demagnetization of paramagnetic materials, calculation of magnetic moments and entropy of paramagnetic materials, $He^3$-$He^4$ Dilution refrigerator, and Nuclear adiabatic demagnetization. Importance of regenerators used in Philips and GM cycle refrigerators and the design equations are summarized. Several Functions are written in Mathcad to simplify the standard and most required calculations. Students, teachers, researchers and professionals may find them very useful.

S.I. Units are used throughout this book. Wide variety of worked examples presented in the book should be useful for those appearing for University, AMIE and Engineering Services examinations.

**Acknowledgements**: Firstly, I would like to **thank all my students**, who have been an inspiration to me in all my academic efforts.

Sincere thanks are due to **Rev. Fr. Joseph Lobo**, Director, St. Joseph Engineering College, Mangalore, for his kindness, regard and words of encouragement.

I am also thankful to **Dr. Thirumaleshwara Bhat**, Principal, Sri Madhwa Vadiraja Institute of Technology and Management, Bantakal, Udupi, for giving me support in my academic activities.

I gratefully remember my former colleagues at the Cryogenics section of Technical Physics Division, Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology, Indore for their sincere cooperation in a true spirit of team-work in all the projects that we undertook.

I particularly salute and admire the vision and foresight of former Heads of Technical Physics Division, BARC viz. late Mr. C. Ambasankaran, Mr. R.Y. Deshpande, Dr. S.R. Gowariker and late Mr. S.S. Ramamurthy in initiating and guiding many of the 'first of its kind' projects for Indian Space Research Organization (ISRO), wherein the Cryogenics section was deeply involved in the design and execution of the projects.

**I am especially grateful to Prof. R.G. Scurlock, former Director of Institute of Cryogenics, University of Southampton, (U.K.) for writing a message for the Part-I of this series.**

It was indeed gracious of my former Professor, under whom I studied for M.Sc. in Cryogenics at the University of Southampton, U.K. during 1970–72, and worked as a Visiting Research Fellow during 1993–94, to honor me by writing this message.

My special thanks to **Bookboon.com** for publishing this *free ebook*. **Ms Karin Jakobsen** and the editorial staff have been most patient and helpful.

Finally, I would like to express my sincere thanks and appreciation to my **wife, Kala,** who, as usual, has given me continuous support, help and encouragement in all my academic activities, making many silent sacrifices.

**M. Thirumaleshwar**
**January, 2017**
**Email:** tmuliya@rediffmail.com

**Note:** Along with this book, two zip files are also available for free download. They contain several 'stand alone' .exe files to calculate performance parameters of many Cryogenics liquefiers and refrigerators. They should be of great use for students, teachers and researches for quick verification of their calculations.

# ABOUT THE AUTHOR

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D (cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

**He was Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.**

A book entitled **"Fundamentals of Heat and Mass Transfer"** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has authored a *free e-book* entitled **"Software Solutions to Problems on Heat Transfer"** wherein problems are solved using 4 software viz. Mathcad, EES, FEHT and EXCEL. This book, containing about 2750 pages, is presented in 9 parts and all the 9 parts can be downloaded *for free* from www.bookboon.com

He has also authored ***free e-books*** **on Thermodynamics** entitled **"Basic Thermodynamics: Software Solutions" and "Applied Thermodynamics: Software Solutions"** wherein problems are solved using 3 software viz. Mathcad, EES, and TEST. Each of these titles is presented in 5 parts and all the books can be downloaded *for free* from www.bookboon.com

In addition, he has authored following two useful *free ebooks*:

i) Applied Thermodynamics: Software Solutions: Vapor compression Refrigeration cycle + Problems (Mathcad)

http://bookboon.com/en/applied-thermodynamics-software-solutions-vapor-ebook

ii) Applied Thermodynamics: Software Solutions: Vapor Power cycles (Rankine cycle) + Problems (Mathcad)

http://bookboon.com/en/applied-thermodynamics-software-solutions-vapor-po-ebook

His earlier *free ebooks in this* **Cryogenic Engineering series**, viz. **Cryogenic Engineering: Software Solutions – Parts-I, II-A and II-B** were published by Bookboon about an year ago.

He has also authored **three motivational, free ebooks**, published by www.bookboon.com, entitled as follows:

1. Towards Excellence… How to Study (A Guide book to Students)
2. Towards Excellence… How to teach (A guide book to Teachers)
3. Towards Excellence… Seminars, GD's and Personal Interviews

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

# ABOUT THE SOFTWARE USED

Following three software are used while solving problems in this book series:

1. Mathcad 7 and Mathcad 15 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and

For a brief introduction to Mathcad, EES and EXCEL see the chapter 1 of the following *free ebook* by the author:

**"Software Solutions to Problems on Heat Transfer – CONDUCTION-Part-I":**
http://bookboon.com/en/software-solutions-to-problems-on-heat-transfer-ebook

# 4 CRYOGENIC REFRIGERATION SYSTEMS: PROBLEMS (EES)

**Learning objectives:**

1. In this chapter, topics on 'cryogenic refrigeration systems' are dealt with.
2. Part-III-A gave theoretical background and summary of calculation equations on various topics: thermodynamically ideal isothermal source system, Coeff. of Performance (COP), Figure of Merit (FOM), thermodynamically ideal isobaric source system, simple Linde-Hampson (L-H) refrigerator, pre-cooled L-H system, Claude refrigeration system, cold gas refrigeration system, Philips (or Stirling cycle) refrigerator, Vuilleumier cycle refrigerator, Solvay cycle and Gifford-McMahon (GM) cycle refrigerators, Magnetic cooling, i.e. Adiabatic demagnetization of paramagnetic materials, calculation of magnetic moments and entropy of paramagnetic materials, $He^3$-$He^4$ Dilution refrigerator, and Nuclear adiabatic demagnetization. Many numerical problems are solved to illustrate the ease of computer calculations using Mathcad software.
3. Here, in Part-III-B, we write many Functions and Procedures in Engineering Equation Solver (EES) to simplify the calculations.
4. Several problems are worked out to illustrate the use of these EES Functions/ Procedures.

## 4.1 DEFINITIONS, STATEMENTS AND FORMULAS USED[1–9]: SEE PART-III-A

## 4.2 PROBLEMS SOLVED WITH MATHCAD: SEE PART-III-A

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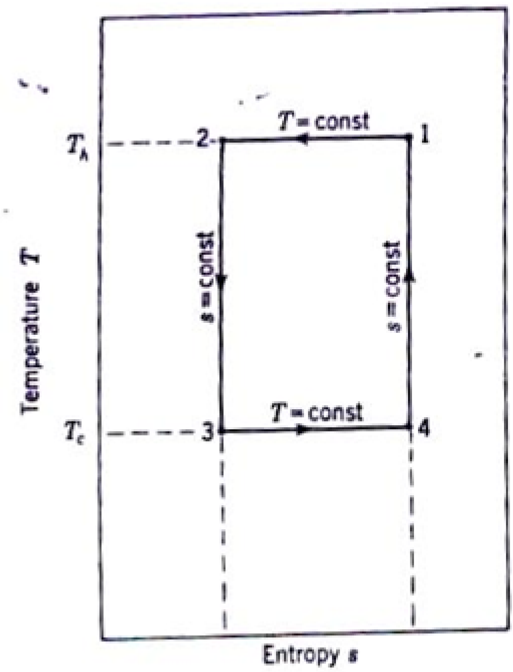
## 4.3 PROBLEMS SOLVED WITH EES

"**Prob. 4.3.1** Write an EES Procedure to calculate the COP and Work requirement for a Thermodynamically Ideal, Isothermal source system.[1]. And, plot the COP against various source temps. Tc for a constant sink temp. Th = 300 K."




**First, write the EES Procedure:**

PROCEDURE Ideal_isoth(Tc, Th:COP_i, WbyQ)

"Inputs: Tc, Th … cold and hot temps, Temp in K"

"Outputs: COP_i, WbyQ (kJ/kJ)"

COP_i:=Tc / (Th – Tc)

WbyQ:= 1 / COP_i "[kJ/kg]"

END

"==============================================================="

Now, use this Procedure to do calculations required:

"Ex:"

Tc = 77 "K"

Th = 300 "K"

CALL Ideal_isoth(Tc, Th:COP_i, WbyQ)

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

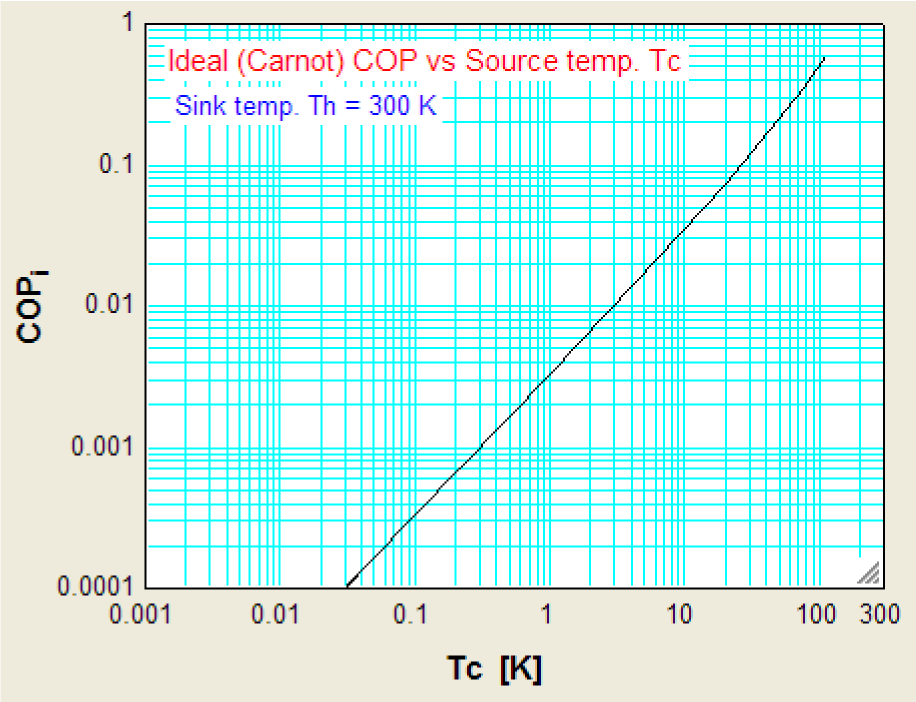
$COP_i$ = 0.3453    Tc = 77 [K]    Th = 300 [K]    WbyQ = 2.896 [kJ/kJ]

**To plot COP vs Tc:**

**First, produce the Parametric Table:**

| 1..8 | Tc [K] | Th [K] | $COP_i$ | WbyQ [kJ/kJ] |
|---|---|---|---|---|
| Run 1 | 0.001 | 300 | 0.000003333 | 299999 |
| Run 2 | 0.056 | 300 | 0.0001867 | 5356 |
| Run 3 | 0.01 | 300 | 0.00003333 | 29999 |
| Run 4 | 1 | 300 | 0.003344 | 299 |
| Run 5 | 4.2 | 300 | 0.0142 | 70.43 |
| Run 6 | 20.4 | 300 | 0.07296 | 13.71 |
| Run 7 | 77.4 | 300 | 0.3477 | 2.876 |
| Run 8 | 112 | 300 | 0.5957 | 1.679 |

**Now, plot the results:**

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**Prob. 4.3.2** Determine the minimum work required to remove 200 W from a region at 20.4 K if the sink temp is: (a) 300 K (b) 77 K. [1]

**EES Solution:**

Q = 200 "W… at 20.4 K"

Tc = 20.4 [K]

Th1 = 300 [K]

Th2 = 77 [K]

"W1 = Min. work when Th = 300 K:"

CALL Ideal_issoth(Tc, Th1:COP_i_1, W1byQ)

W1 = W1byQ * Q "W … Min. work reqd when Th = 300 K"

"W2 = Work when Th = 77 K:"

CALL Ideal_issoth(Tc, Th2:COP_i_2, W2byQ)

W2 = W2byQ * Q "W … Min. work reqd when Th = 77 K"

"=============================================================="

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | | |
|---|---|---|---|
| $COP_{i,1}$ = 0.07296 | $COP_{i,2}$ = 0.3604 | Q = 200 [W] | Tc = 20.4 [K] |
| Th1 = 300 [K] | Th2 = 77 [K] | W1 = 2741 [W] | W1byQ = 13.71 |
| W2 = 554.9 [W] | W2byQ = 2.775 | | |

**Thus:**

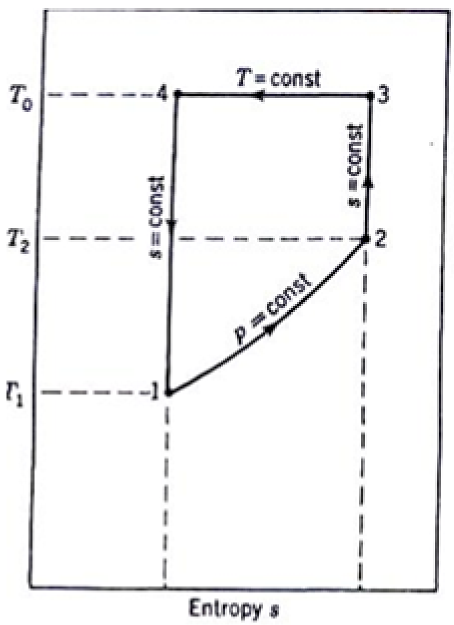
**Minimum work required to remove 200 W from a region at 20.4 K:**

**When Th = 300 K: W1 = 2741 W … Ans.**

**When Th = 77 K: W2 = 554.9 W … Ans.**

---

**Prob. 4.3.3** Write an EES Procedure to calculate COP and WbyQ for a Thermodynamically Ideal, Isobaric source system, for an ideal gas. Then, find COP and WbyQ when source temp is from T1 = 112 K to T2 = 190 K, and sink temp T0 = 300 K. Plot COP vs T1by T0.



**EES Solution:**

**First, write the EES Procedure:**

PROCEDURE Ideal_isobaric_IdealGas(T2byT1, T1byT0:COP_i, WbyQ)

"Gives COP and Wby Q for Ideal, isobaric. source refrig … Source temp: from T1 to T2, Sink at Th"

"Inputs: T2byT1 = T2/T1, T1byT0 = T1/T0 … where T1 = refrign supplied from lower temp T1 to higher temp. T2,
heat rejected to sink temp at T0, all temps. in K"

"Outputs: COP_i, WbyQ (kJ/kJ)"

COP_i:= (T2byT1 – 1) / ((1 / T1byT0) * ln(T2byT1) – T2byT1 + 1)

WbyQ:= 1 / COP_i "[kJ/kg]"

END

"==========================================================="

**Now, solve the problem:**

T1 = 112 [K]

T2 = 190 [K]

T0 = 300 [K]

"Therefore:"

T2byT1 = T2 / T1

T1byT0 = T1 / T0

"Now, call the Procedure:"

CALL Ideal_isobaric_IdealGas(T2byT1, T1byT0:COP_i, WbyQ)

Results:

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | | |
|---|---|---|---|
| $COP_i$ = 0.9683 | T0 = 300 [K] | T1 = 112 [K] | T1byT0 = 0.3733 |
| T2 = 190 [K] | T2byT1 = 1.696 | WbyQ = 1.033 [kJ/kJ] | |

**Note the results:**

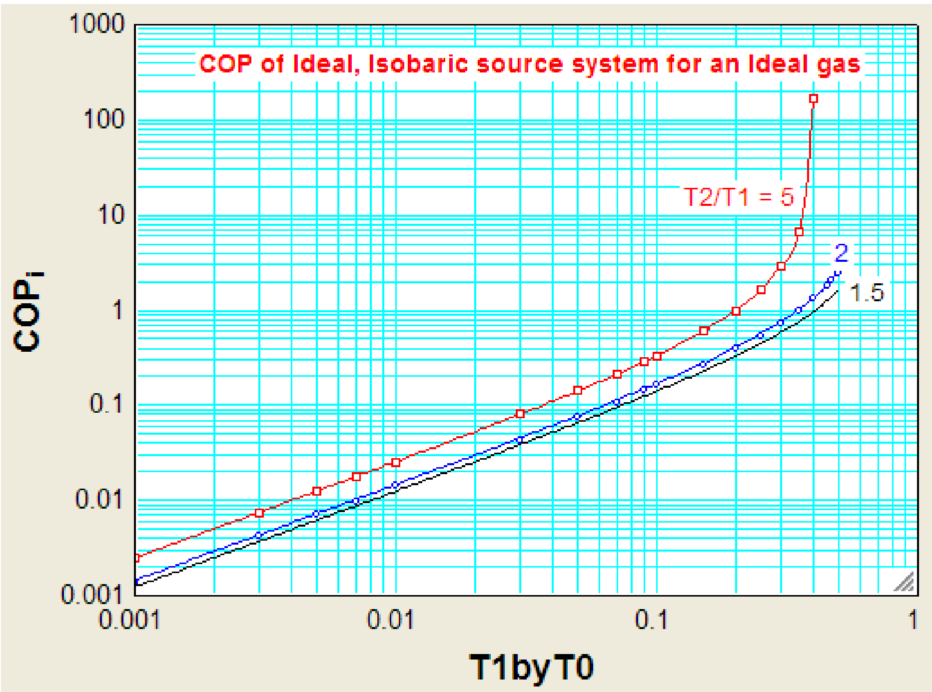
**COP = 0.9683, WbyQ = 1.033 kJ/kJ … Ans.**

**Now, to plot COP vs T1byT0 for different values of T2byT1:**

First, compute the Parametric Table:

| | | T2/T1 = 1.5 | T2/T1 = 2 | T2/T1 = 5 |
|---|---|---|---|---|
| 1..19 | T1byT0 | $COP_i$ | $COP_i$ | $COP_i$ |
| Run 1 | 0.001 | 0.001235 | 0.001445 | 0.002492 |
| Run 2 | 0.003 | 0.003713 | 0.004347 | 0.007512 |
| Run 3 | 0.005 | 0.006204 | 0.007266 | 0.01258 |
| Run 4 | 0.007 | 0.008707 | 0.0102 | 0.01771 |
| Run 5 | 0.01 | 0.01249 | 0.01464 | 0.02549 |
| Run 6 | 0.03 | 0.03842 | 0.04524 | 0.08057 |
| Run 7 | 0.05 | 0.06571 | 0.07774 | 0.1419 |
| Run 8 | 0.07 | 0.09448 | 0.1123 | 0.2106 |
| Run 9 | 0.09 | 0.1248 | 0.1492 | 0.2881 |
| Run 10 | 0.1 | 0.1407 | 0.1686 | 0.3307 |
| Run 11 | 0.15 | 0.227 | 0.2762 | 0.5944 |
| Run 12 | 0.2 | 0.3274 | 0.4056 | 0.9883 |
| Run 13 | 0.25 | 0.4457 | 0.5641 | 1.641 |
| Run 14 | 0.3 | 0.5872 | 0.7631 | 2.931 |
| Run 15 | 0.35 | 0.7593 | 1.02 | 6.685 |
| Run 16 | 0.4 | 0.9734 | 1.365 | 169.5 |
| Run 17 | 0.45 | 1.247 | 1.851 | |
| Run 18 | 0.47 | 1.379 | 2.106 | |
| Run 19 | 0.5 | 1.608 | 2.589 | |

**And, now, plot the results:**



**Prob. 4.3.4** An ideal, isobaric source refrigerator operates with 300 K as the sink temp, while the source temp varies between 70 K and 90 K. For an ideal gas, determine the COP. Compare this with the COP of a Carnot refrigerator operating between: (a) 300 K and 70 K (b) 300 K and 90 K. [1]

**EES Solution:**

"For Ideal, isobaric source refrigerator:"

T1 = 70 [K]

T2 = 90 [K]

T0 = 300 [K]

"Therefore:"

T2byT1 = T2 / T1

T1byT0 = T1 / T0

"Now, call the Procedures:"

"For Isobaric source:"

CALL Ideal_isobaric_IdealGas(T2byT1, T1byT0:COP_i, WbyQ)

"For Carnot Refrigerator: between 300 K and 70 K"

Tc1 = 70 [K]

Th = 300 [K]

CALL Ideal_isoth(Tc1, Th:COP_i_1, W1byQ)

"For Carnot Refrigerator: between 300 K and 90 K"

Tc2 = 90 [K]

CALL Ideal_isoth(Tc2, Th:COP_i_2, W2byQ)

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | | |
|---|---|---|---|
| $COP_i$ = 0.361 | $COP_{i,1}$ = 0.3043 | $COP_{i,2}$ = 0.4286 | T0 = 300 [K] |
| T1 = 70 [K] | T1byT0 = 0.2333 | T2 = 90 [K] | T2byT1 = 1.286 |
| Tc1 = 70 [K] | Tc2 = 90 [K] | Th = 300 [K] | W1byQ = 3.286 |
| W2byQ = 2.333 | WbyQ = 2.77 [kJ/kJ] | | |

**Thus:**

**For Isobaric source refrig.:**

**COP = 0.361 … Ans.**

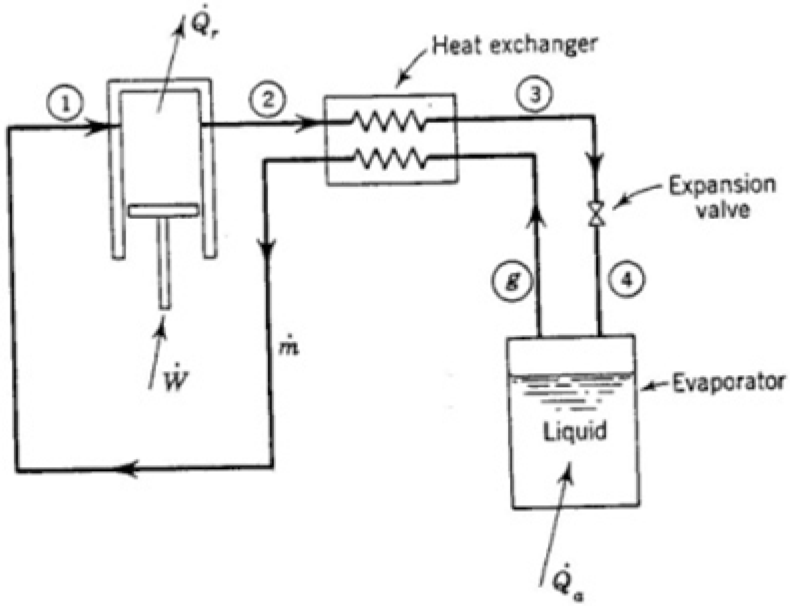
**For Carnot refrig operating between 70 K and 300 K.:**

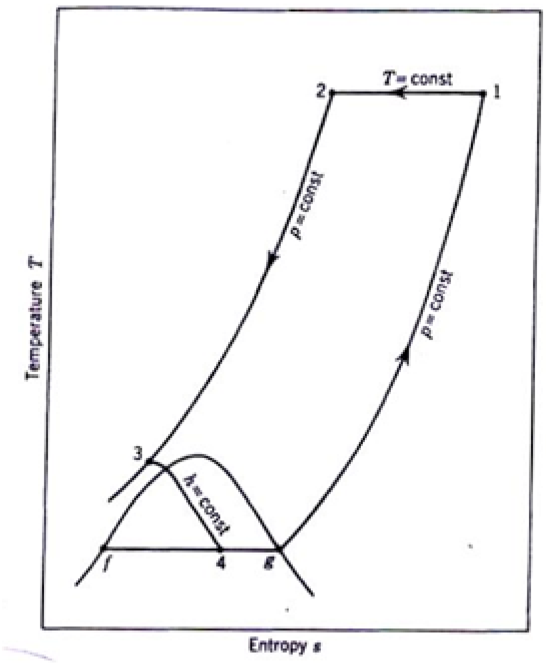
**COP_i_1 = 0.3043 … Ans.**

**For Carnot refrig operating between 90 K and 300 K.:**

**COP_i_2 = 0.4286 … Ans.**

**Prob. 4.3.5** Write an EES Procedure to find the parameters of a J-T Refrigeration system with various working fluids.




**EES Procedure:**

PROCEDURE JT_Refrign_system(Fluid$, P1, T1, P2, eta_co, eta_hx: Tsat_fluid, Qa_by_m, W_by_m, COP, COP_i, FOM)

"Gives various parameters for a JT refrign. system:"

```
"Inputs: Fluid$: desired fluid (real)--- Nitrogen, Oxygen, Argon.
P1, P2 (kPa), T1 (K),

eta_co = compressor overall effcy., eta_hx = heat exchanger effcy."

"Outputs: Tsat_fluid (K), Qa_by_m (kJ/kg), W_by_m (kJ/kg), COP = Qa_by_W, COP_i,
FOM = COP/COP_i"

h1:= ENTHALPY(Fluid$, T=T1, P=P1)
s1:= ENTROPY(Fluid$, T=T1, P=P1)

h2 := ENTHALPY(Fluid$, T=T1, P=P2)
s2 := ENTROPY(Fluid$, T=T1, P=P2)

hg := ENTHALPY(Fluid$, x=1, P=P1)

Tsat_fluid := T_sat(Fluid$, P = P1)

W_by_m := (T1 * (s1 – s2) – (h1 – h2)) / eta_co
```

Qa_by_m := (h1 – h2) – (1 – eta_hx) * (h1 – hg)

COP := Qa_by_m / W_by_m

COP_i:= Tsat_fluid / (T1 – Tsat_fluid)

FOM:= COP / COP_i

END

"==============================================================="

"**Prob. 4.3.6** In a J-T Refrigeration system with Nitrogen as working fluid, the system works between 0.5 bar, 300 K and 152 bar, with isothermal compression. Overall efficiency of compressor is 70%, and heat exchangers effcy is 96.5%. Find out the refrig. effect, work required, COP and FOM."

**EES Solution:**

Fluid$ = 'Nitrogen';

P1 = 0.5 * 100 [kPa]

T1 = 300 [K]

P2 = 152 * 100 [kPa]

eta_co = 0.7

eta_hx = 0.965

CALL JT_Refrign_system(Fluid$, P1, T1, P2, eta_co, eta_hx: Tsat_fluid, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

**Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]**

| | | |
|---|---|---|
| COP = 0.02555 | $COP_i$ = 0.3148 | $\eta_{co}$ = 0.7 |
| $\eta_{hx}$ = 0.965 | Fluid$ = 'Nitrogen' | FOM = 0.08116 |
| P1 = 50 [kPa] | P2 = 15200 [kPa] | $Qa_{by,m}$ = 18.58 [kJ/kg] |
| T1 = 300 [K] | $Tsat_{fluid}$ = 71.83 [K] | $W_{by,m}$ = 727.1 [kJ/kg] |

**Thus:**

**Refrig. effect = Qa_by_m = 18.58 kJ/kg … Ans.**

**Work required = W_by_m = 727.1 kJ/kg … Ans.**

**COP = 0.02555 … Ans.**

**COP_i = 0.3148 … Ans.**

**FOM = 0.8116 … Ans.**

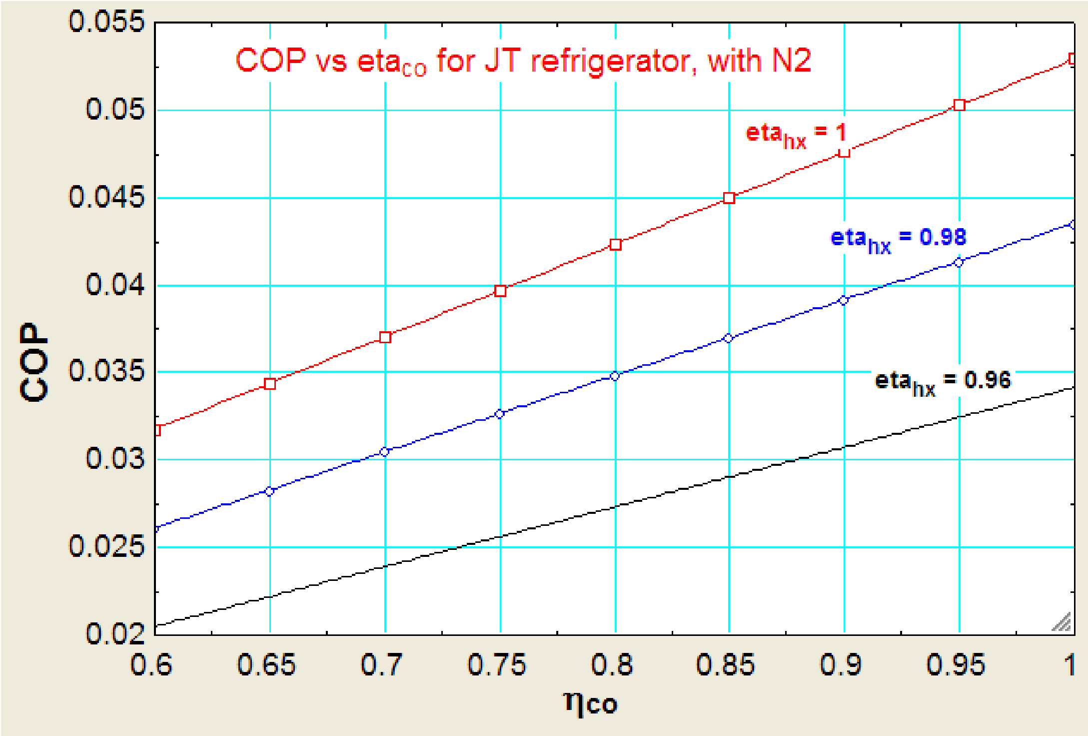
---

**Plot COP against compressor effcy, as the compressor effcy varies from 0.6 to 1, for eta_HX values of 0.96, 0.98 and 1:**

**First, compute the Parametric Table:**

| | | eta_hx = 0.96 | eta_hx = 0.98 | eta_hx = 1 |
|---|---|---|---|---|
| 1..9 | $\eta_{co}$ | COP | COP | COP |
| Run 1 | 0.6 | 0.02049 | 0.02612 | 0.03174 |
| Run 2 | 0.65 | 0.0222 | 0.0283 | 0.03439 |
| Run 3 | 0.7 | 0.02391 | 0.03047 | 0.03704 |
| Run 4 | 0.75 | 0.02562 | 0.03265 | 0.03968 |
| Run 5 | 0.8 | 0.02732 | 0.03483 | 0.04233 |
| Run 6 | 0.85 | 0.02903 | 0.037 | 0.04497 |
| Run 7 | 0.9 | 0.03074 | 0.03918 | 0.04762 |
| Run 8 | 0.95 | 0.03245 | 0.04135 | 0.05026 |
| Run 9 | 1 | 0.03416 | 0.04353 | 0.05291 |

**Now, plot the results:**

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**Prob. 4.3.7** In a J-T Refrigeration system with Argon as working fluid, the system works between 1.013 bar, 300 K and 202.6 bar, with isothermal compression. Efficiencies of compressor and heat exchangers are 72% and 98% respectively. Find out the refrig. effect, work required, COP and FOM.

**EES Solution:**

Fluid$ = 'Argon';

P1 = 1.013 * 100 [kPa]

T1 = 300 [K]

P2 = 202.6 * 100 [kPa]

eta_co = 0.72

eta_hx = 0.98

CALL JT_Refrign_system(Fluid$, P1, T1, P2, eta_co, eta_hx: Tsat_fluid, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | |
|---|---|---|
| COP = 0.06629 | $COP_i$ = 0.4104 | $\eta_{co}$ = 0.72 |
| $\eta_{hx}$ = 0.98 | Fluid$ = 'Argon' | FOM = 0.1615 |
| P1 = 101.3 [kPa] | P2 = 20260 [kPa] | $Qa_{by,m}$ = 29.96 [kJ/kg] |
| T1 = 300 [K] | $Tsat_{fluid}$ = 87.3 [K] | $W_{by,m}$ = 451.9 [kJ/kg] |

**Thus:**

**Refrig. effect = Qa_by_m = 29.96 kJ/kg … Ans.**

**Work required = W_by_m = 451.9 kJ/kg … Ans.**

**COP = 0.06629 … Ans.**

**COP_i = 0.4104 … Ans.**

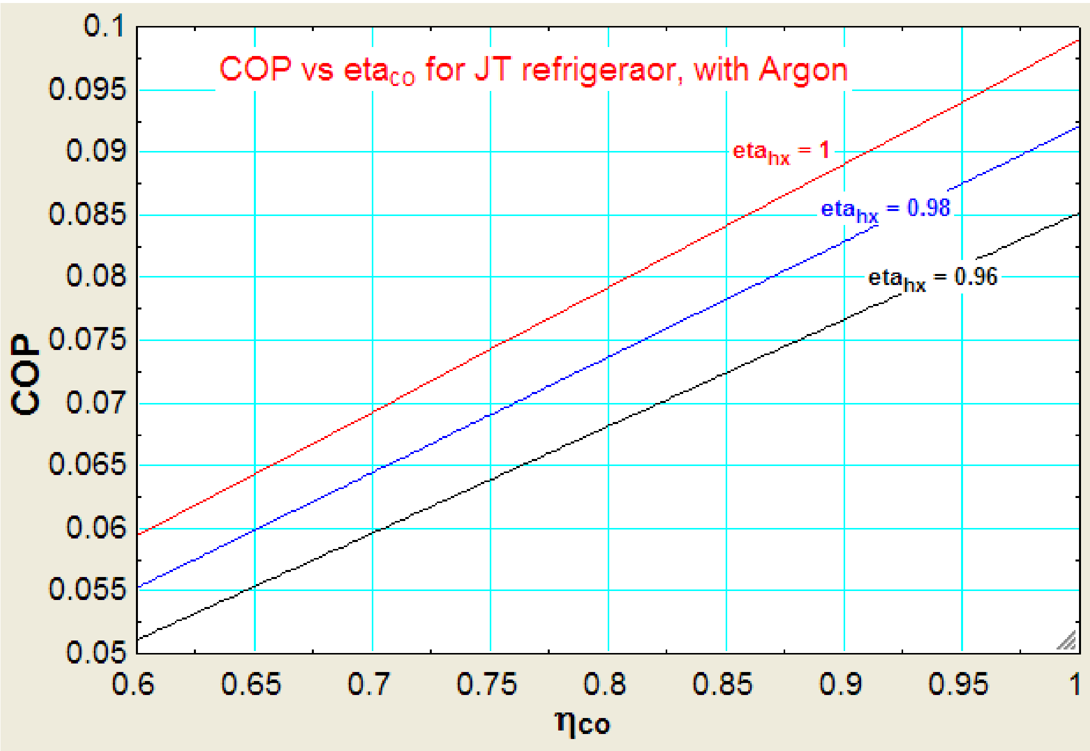
**FOM = 0.1615 … Ans.**

**Plot COP against compressor effcy, as the compressor effcy varies from 0.6 to 1, for eta_HX values of 0.96, 0.98 and 1:**

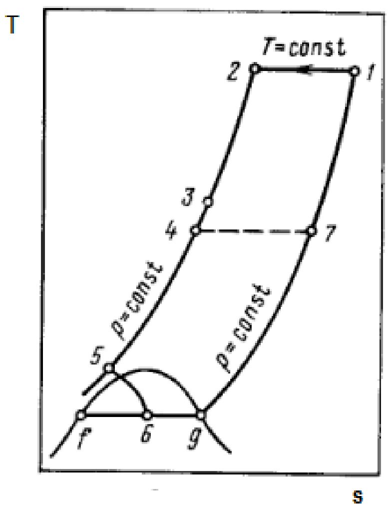
**First, compute the Parametric Table:**

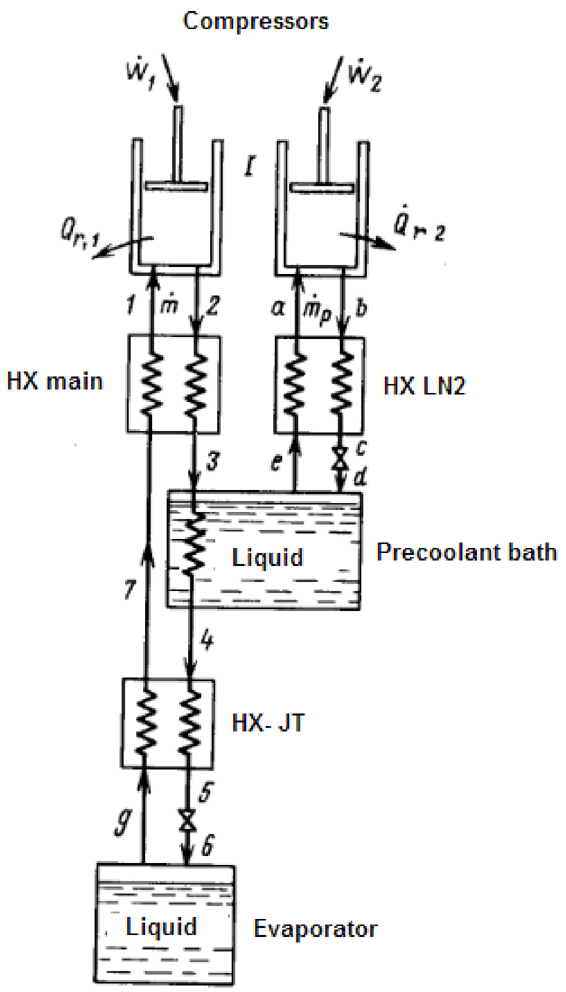
| 1..9 | $\eta_{co}$ | COP (eta_hx = 0.96) | COP (eta_hx = 0.98) | COP (eta_hx = 1) |
|---|---|---|---|---|
| Run 1 | 0.6 | 0.0511 | 0.05524 | 0.05939 |
| Run 2 | 0.65 | 0.05536 | 0.05985 | 0.06434 |
| Run 3 | 0.7 | 0.05962 | 0.06445 | 0.06929 |
| Run 4 | 0.75 | 0.06388 | 0.06905 | 0.07423 |
| Run 5 | 0.8 | 0.06813 | 0.07366 | 0.07918 |
| Run 6 | 0.85 | 0.07239 | 0.07826 | 0.08413 |
| Run 7 | 0.9 | 0.07665 | 0.08287 | 0.08908 |
| Run 8 | 0.95 | 0.08091 | 0.08747 | 0.09403 |
| Run 9 | 1 | 0.08517 | 0.09207 | 0.09898 |

**Now, plot the results:**



**Prob. 4.3.8** Write an EES Procedure to find the various parameters in a pre-cooled JT refrigeration system with H2 as the working fluid and N2 as the pre-coolant.




**EES Procedure:**

PROCEDURE Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HX_main, eta_HX_N2, eta_HX_JT: Tsat_N2, z, Qa_by_m, W_by_m, COP, COP_i, FOM)

"Gives various parameters for a Nitrogen pre-cooled JT refrign. system for Hydrogen:"

"Inputs: P1, P2, P1N2, P2N2 (kPa), T1, T1N2 (K),
eta_cN2, eta_cH2 = compressor effcy for N2 and H2., eta_HX_main = main heat exchanger effcy, eta_HX_N2 = N2 heat exchanger effcy, eta_HX_JT = JTheat exchanger effcy"

"Outputs: Tsat_N2 (K), z = (m_N2/m_H2), Qa_by_m (kJ/kg), W_by_m (kJ/kg), COP = Qa_by_W, COP_i, FOM = COP/COP_i"

```
h1:= ENTHALPY(Hydrogen, T=T1, P=P1)
s1:= ENTROPY(Hydrogen, T=T1, P=P1)

h2 := ENTHALPY(Hydrogen, T=T1, P=P2)
s2 := ENTROPY(Hydrogen, T=T1, P=P2)

hg := ENTHALPY(Hydrogen, x=1, P=P1)

ha := ENTHALPY(Nitrogen, T=T1N2, P=P1N2)

hb := ENTHALPY(Nitrogen, T=T1N2, P=P2N2)

sa := ENTROPY(Nitrogen, T=T1N2, P=P1N2)

sb := ENTROPY(Nitrogen, T=T1N2, P=P2N2)

he := Enthalpy(Nitrogen, x=1, P=P1N2)

Tsat_N2 := T_sat(Nitrogen, P = P1N2)
```

```
T7 :=Tsat_N2

h7 := ENTHALPY(Hydrogen, T=T7, P=P1)

h4 := ENTHALPY(Hydrogen, T=T7, P=P2)

Qa_by_m := (h7 – h4) – (1 – eta_HX_JT) * (h7 – hg)

z := (Qa_by_m – ((h1 – h2) – (1 – eta_HX_main) * (h1 – hg))) / ((ha – hb) – (1 -eta_HX_N2) * (ha – he))

AA := T1 * (s1 – s2) – (h1 – h2)

BB := T1N2 * (sa – sb) – (ha – hb)

W_by_m :=AA / eta_cH2 + z * BB / eta_cN2

COP := Qa_by_m / W_by_m

TL := T_sat(Hydrogen, P = P1)

COP_i:= TL / (T1 – TL)

FOM:= COP / COP_i

END

"==================================================================="
```

**Prob. 4.3.9** Determine the refrig. effect, COP, and FOM for a pre-cooled JT liquid hydrogen refrigerator, if the hydrogen is compressed between 101.3 kPa and 8.1 MPa and 300 K. The nitrogen is compressed between 101.3 kPa and 15.2 MPa and 300 K. The main HX has an effectiveness of 0.99 and the pre-coolant HX has an effectiveness of 0.98. The cold HX (or the JT HX) in the hydrogen circuit has an effectiveness of 0.96 and both the compressors may be assumed 100% efficient. [1]

**EES Solution:**

**We use the above written EES Procedure:**

P1 = 101.3 [kPa]

P2 = 8.1E03 [kPa]

T1 = 300 [K]

P1N2 = 101.3 [kPa]

P2N2 = 15.2E03 [kPa]

T1N2 = 300 [K]

eta_HX_main = 0.99

eta_HX_N2 = 0.98

eta_HX_JT = 0.96

eta_cH2 = 1

eta_cN2 = 1

CALL Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HX_main, eta_HX_N2, eta_HX_JT: Tsat_N2, z, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | | |
|---|---|---|---|
| COP = 0.01375 | $COP_i$ = 0.07292 | $\eta_{cH2}$ = 1 | $\eta_{cN2}$ = 1 |
| $\eta_{HX,JT}$ = 0.96 | $\eta_{HX,main}$ = 0.99 | $\eta_{HX,N2}$ = 0.98 | FOM = 0.1885 |
| P1 = 101.3 [kPa] | P1N2 = 101.3 [kPa] | P2 = 8100 [kPa] | P2N2 = 15200 [kPa] |
| $Qa_{by,m}$ = 133.4 [kJ/kg] | T1 = 300 [K] | T1N2 = 300 [K] | $Tsat_{N2}$ = 77.35 [K] |
| $W_{by,m}$ = 9706 [kJ/kg] | z = 9.471 | | |

**Thus:**

**Refrig. effect = Qa_by_m = 133.4 kJ/kg … ns.**

**COP = 0.01375 … Ans.**

**FOM = 0.1885 … Ans.**

---

"**Prob. 4.3.10** Determine the refrig. effect, COP, and FOM for a pre-cooled JT liquid hydrogen refrigerator, if the hydrogen is compressed between 101.3 kPa and 10.13 MPa and 300 K. The nitrogen is compressed between 101.3 kPa and 20.3 MPa and 300 K. The main HX has an effectiveness of 0.98 and the pre-coolant HX has an effectiveness of 0.97. The cold HX (or the JT HX) in the hydrogen circuit has an effectiveness of 0.95 and both the compressors may be assumed 75% efficient. [1]"

**EES Solution:**

**We use the above written EES Procedure:**

P1 = 101.3 [kPa]

P2 = 10.13E03 [kPa]

T1 = 300 [K]

P1N2 = 101.3 [kPa]

P2N2 = 20.3E03 [kPa]

T1N2 = 300 [K]

eta_HX_main = 0.98

eta_HX_N2 = 0.97

eta_HX_JT = 0.95

eta_cH2 = 0.75

eta_cN2 =0.75

CALL Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HX_main, eta_HX_N2, eta_HX_JT: Tsat_N2, z, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

**Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]**

| | | | |
|---|---|---|---|
| $COP = 0.01018$ | $COP_i = 0.07292$ | $\eta_{cH2} = 0.75$ | $\eta_{cN2} = 0.75$ |
| $\eta_{HX,JT} = 0.95$ | $\eta_{HX,main} = 0.98$ | $\eta_{HX,N2} = 0.97$ | $FOM = 0.1396$ |
| P1 = 101.3 [kPa] | P1N2 = 101.3 [kPa] | P2 = 10130 [kPa] | P2N2 = 20300 [kPa] |
| $Qa_{by,m} = 146.4$ [kJ/kg] | T1 = 300 [K] | T1N2 = 300 [K] | $Tsat_{N2} = 77.35$ [K] |
| $W_{by,m} = 14381$ [kJ/kg] | z = 10.6 | | |

**Thus:**

**Refrig. effect = Qa_by_m = 146.4 kJ/kg … Ans.**

**COP = 0.01018 … Ans.**

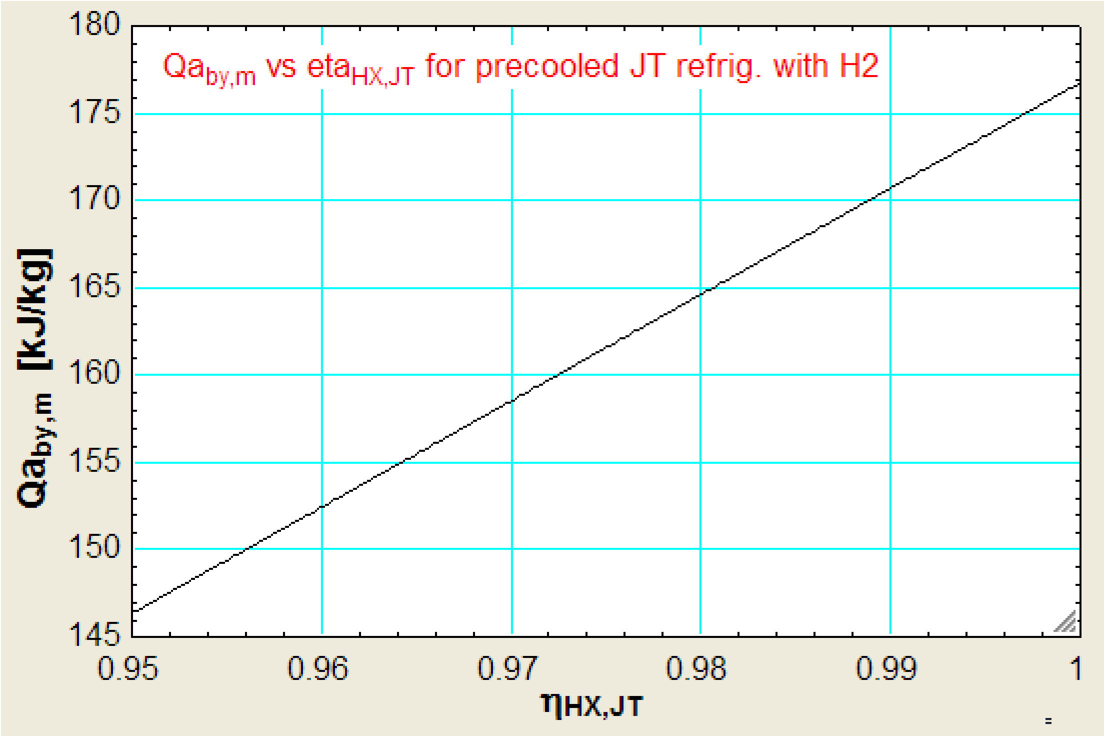
**FOM = 0.1396 … Ans.**

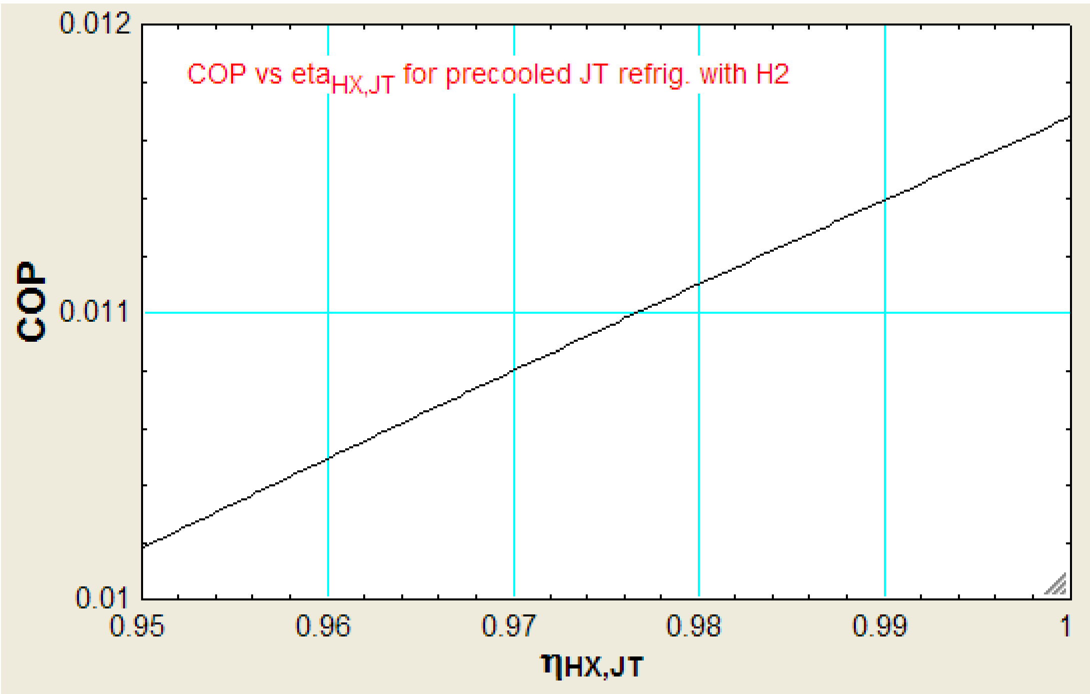
**Also, plot the effect of variation of eta_HX_JT on various parameters, if other conditions remain the same:**

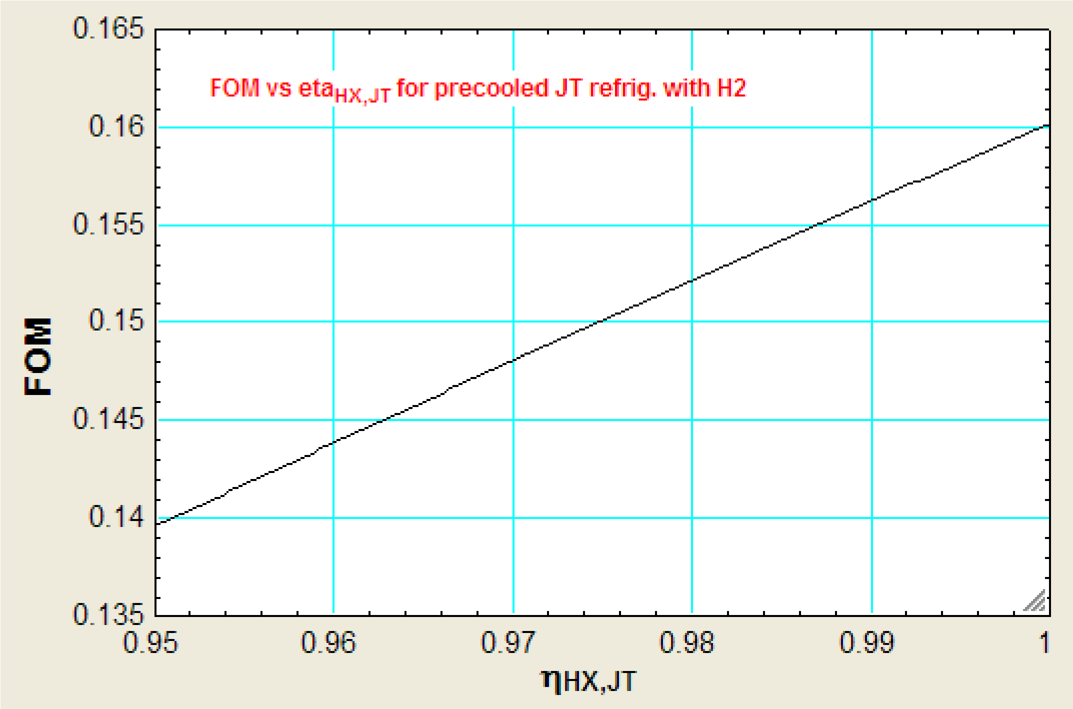
**First, compute the Parametric Table:**

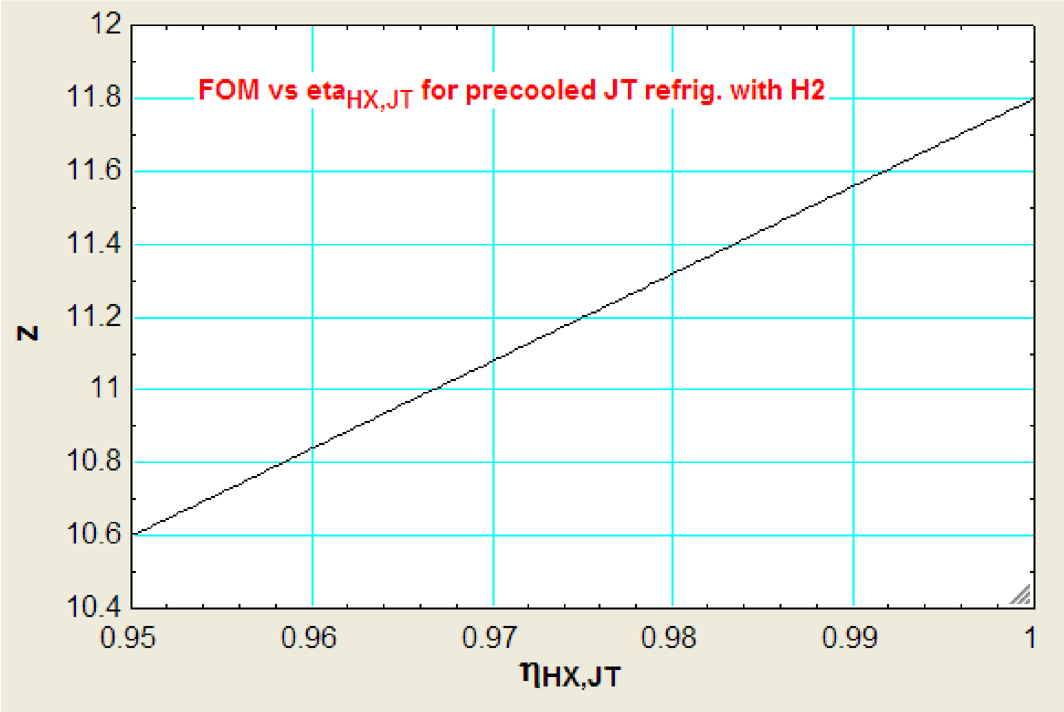
| 1..11 | $\eta_{HX,JT}$ | $Qa_{by,m}$ [kJ/kg] | COP | FOM | z |
|---|---|---|---|---|---|
| Run 1 | 0.95 | 146.4 | 0.01018 | 0.1396 | 10.6 |
| Run 2 | 0.955 | 149.4 | 0.01033 | 0.1417 | 10.72 |
| Run 3 | 0.96 | 152.4 | 0.01049 | 0.1439 | 10.84 |
| Run 4 | 0.965 | 155.5 | 0.01064 | 0.146 | 10.96 |
| Run 5 | 0.97 | 158.5 | 0.0108 | 0.148 | 11.08 |
| Run 6 | 0.975 | 161.6 | 0.01095 | 0.1501 | 11.2 |
| Run 7 | 0.98 | 164.6 | 0.0111 | 0.1522 | 11.32 |
| Run 8 | 0.985 | 167.6 | 0.01124 | 0.1542 | 11.44 |
| Run 9 | 0.99 | 170.7 | 0.01139 | 0.1562 | 11.56 |
| Run 10 | 0.995 | 173.7 | 0.01153 | 0.1582 | 11.68 |
| Run 11 | 1 | 176.8 | 0.01168 | 0.1601 | 11.8 |

**Now, plot the results:**




FOM vs etaHX,JT for precooled JT refrig. with H2
FOM
ηHX,JT
0.165
0.16
0.155
0.15
0.145
0.14
0.135
0.95
0.96
0.97
0.98
0.99
1


FOM vs etaHX,JT for precooled JT refrig. with H2
Z
ηHX,JT
12
11.8
11.6
11.4
11.2
11
10.8
10.6
10.4
0.95
0.96
0.97
0.98
0.99
1

**Prob. 4.3.11** Determine the refrig. effect, COP, and FOM for a pre-cooled JT liquid hydrogen refrigerator, if the hydrogen is compressed between 1.013 bar and 152 bar and 294 K. The nitrogen is compressed between 1.013 bar and 202.6 bar and 294 K. The nitrogen flow rate ratio (z) is 5. The main HX and the pre-coolant HX have an effectiveness of 100% each. Find the effectiveness of cold HX (or the JT HX) in the hydrogen circuit if both the compressors are 100% efficient. [1]

**EES Solution:**

**We shall use the EES Procedure for Pre-cooled HT refrig. written earlier:**

P1 = 101.3 [kPa]

P2 = 152E02 [kPa]

T1 = 294 [K]

P1N2 = 101.3 [kPa]

P2N2 = 202.6E02 [kPa]

T1N2 = 294 [K]

eta_HX_main = 1

eta_HX_N2 = 1

eta_cH2 = 1

eta_cN2 =1

z = 5

eta_HX_JT = ?

CALL Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HX_main, eta_HX_N2, eta_HX_JT: Tsat_N2, z, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

**Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]**

| | | |
|---|---|---|
| COP = 0.01077 | $COP_i$ = 0.07452 | $\eta_{cH2}$ = 1 |
| $\eta_{cN2}$ = 1 | $\eta_{HX,JT}$ = 0.8323 | $\eta_{HX,main}$ = 1 |
| $\eta_{HX,N2}$ = 1 | FOM = 0.1445 | P1 = 101.3 [kPa] |
| P1N2 = 101.3 [kPa] | P2 = 15200 [kPa] | P2N2 = 20260 [kPa] |
| $Qa_{by,m}$ = 91.55 [kJ/kg] | T1 = 294 [K] | T1N2 = 294 [K] |
| $Tsat_{N2}$ = 77.35 [K] | $W_{by,m}$ = 8502 [kJ/kg] | z = 5 |

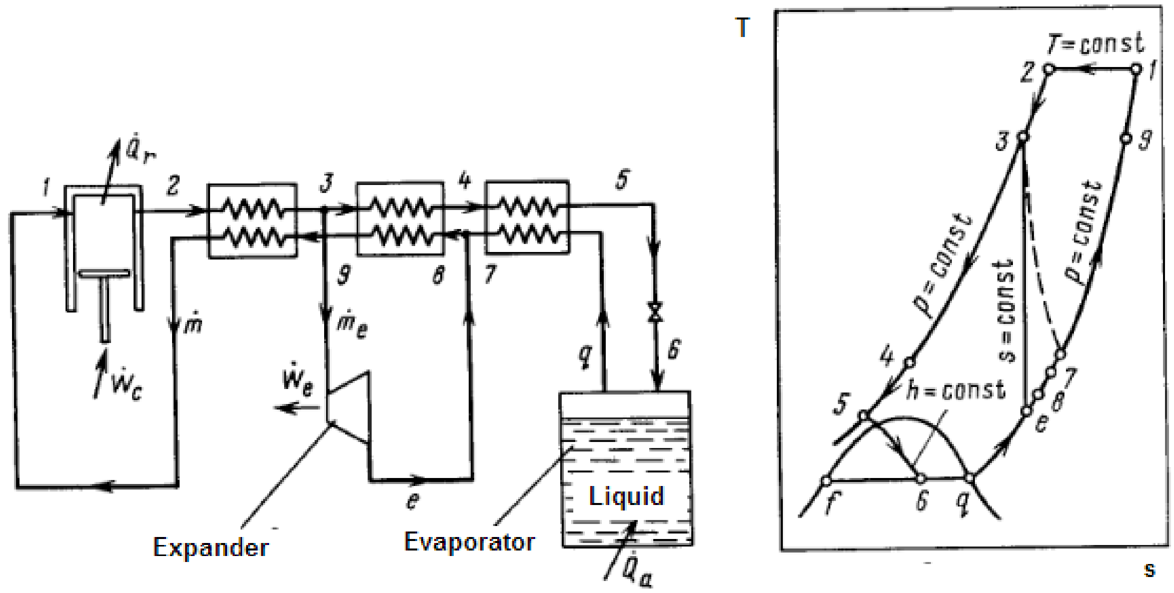
**Thus:**

**Effectiveness of JT HX = eta_HX_JT = 0.8323 … Ans.**

**Note: Note the ease with which this result is obtained in EES.**

---

**Prob. 4.3.12** Write an EES Procedure to find the various parameters in a Claude refrigeration system, where a fraction x of the gas compressed in the compressor is sent through the expander.

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**EES Procedure:**

PROCEDURE Claude_refrig(Fluid$, P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x: Tsat_fluid, Qa_by_m, W_by_m, COP, COP_i, FOM)

"Gives various parameters for a Claude refrign. system for a given Fluid"

"Inputs: P1, P2, (kPa), T1, T3 (K),
eta_co= compressor effcy ., , eta_e_ad = adab. effcy of expander, , eta_em = mech. effcy of expander, Fraction_x = fraction of the gas compressed that goes through the expander"

"Outputs: Tsat_fluid (K), Qa_by_m (kJ/kg), W_by_m (kJ/kg), COP = Qa_by_W, COP_i, FOM = COP/COP_i"

h1:= ENTHALPY(Fluid$, T=T1, P=P1)
s1:= ENTROPY(Fluid$, T=T1, P=P1)

h2 := ENTHALPY(Fluid$, T=T1, P=P2)
s2 := ENTROPY(Fluid$, T=T1, P=P2)

h3 := ENTHALPY(Fluid$, T=T3, P=P2)
s3 := ENTROPY(Fluid$, T=T3, P=P2)

```
se := s3

he := ENTHALPY(Fluid$, s=se, P=P1)

Tsat_fluid := T_sat(Fluid$, P = P1)

Qa_by_m := (h1 – h2) + Fraction_x * eta_e_ad * (h3 – he)

AA := (T1 * (s1 – s2) – (h1 – h2)) / eta_co

BB := Fraction_x * eta_e_ad * eta_em * (h3 – he)

W_by_m :=AA – BB

COP := Qa_by_m / W_by_m

TL := Tsat_fluid

COP_i:= TL / (T1 – TL)
```

FOM:= COP / COP_i

END

"============================================================="

**Prob. 4.3.13** In an ideal Claude refrigeration system using nitrogen as the working fluid, the gas enters the reversible isothermal compressor at 1.013 bar, and 300 K, and is compressed to 40.5 bar. Determine the expander mass flow rate ratio (x = me/m) required to result in a refrigeration effect of 80 kJ/kg, if the gas enters the reversible adiabatic expander at 4.5 bar and 240 K. [1]

**EES Solution:**

**Using the EES Procedure written above:**

Fluid$ = 'Nitrogen'

P1 = 101.3 [kPa]

T1 = 300 [K]

P2 = 4050 [kPa]

T3 = 240 [K]

eta_co = 1

eta_e_ad = 1

eta_em = 1

Qa_by_m = 80 [kJ/kg]

CALL Claude_refrig(Fluid$, P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x: Tsat_fluid, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | | |
|---|---|---|---|
| COP = 0.3121 | $COP_i$ = 0.3474 | $\eta_{co}$ = 1 | $\eta_{em}$ = 1 |
| $\eta_{e,ad}$ = 1 | Fluid$ = 'Nitrogen' | FOM = 0.8984 | $Fraction_x$ = 0.4675 |
| P1 = 101.3 [kPa] | P2 = 4050 [kPa] | $Qa_{by,m}$ = 80 [kJ/kg] | T1 = 300 [K] |
| T3 = 240 [K] | $Tsat_{fluid}$ = 77.35 [K] | $W_{by,m}$ = 256.3 [kJ/kg] | |

**Thus:**

**Fraction_x = 0.4675 to get a refrign of Qa_by_m = 80 kJ/kg … Ans.**

**Further:**

**COP = 0.3121 … Ans.**

**FOM = 0.8984 … Ans.**

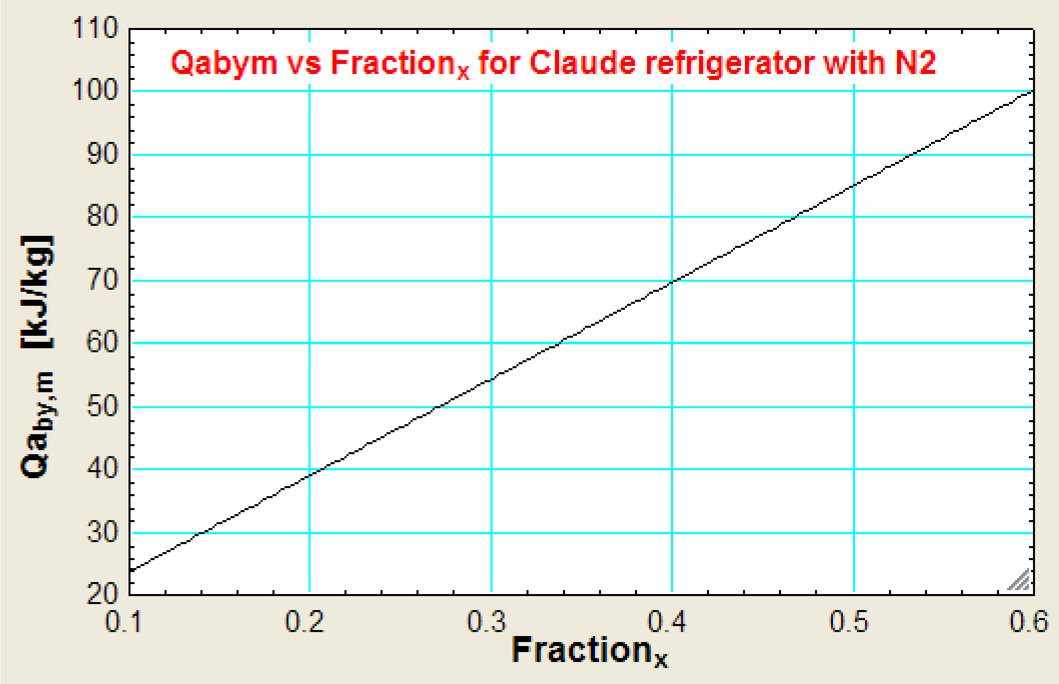
---

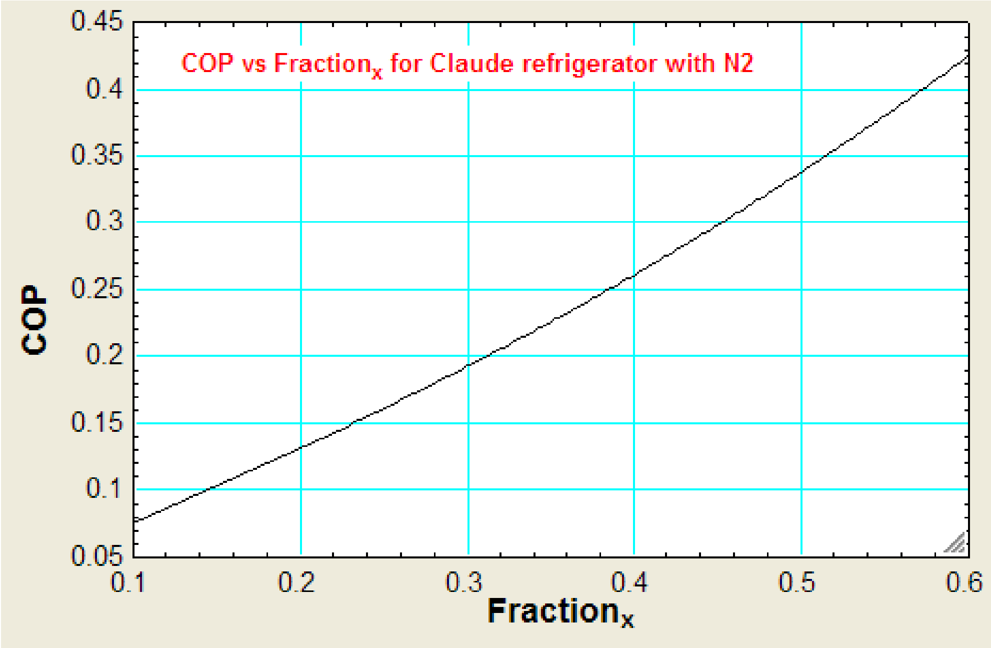
**Plot the refrig. effect and COP against Fraction_x as Fraction_x varies from 0.1 to 0.6:**

**First, compute the Parametric Table:**

| 1..11 | $Fraction_x$ | $Qa_{by,m}$ [kJ/kg] | COP |
|---|---|---|---|
| Run 1 | 0.1 | 23.68 | 0.07574 |
| Run 2 | 0.15 | 31.34 | 0.1028 |
| Run 3 | 0.2 | 39 | 0.1312 |
| Run 4 | 0.25 | 46.67 | 0.1611 |
| Run 5 | 0.3 | 54.33 | 0.1927 |
| Run 6 | 0.35 | 61.99 | 0.226 |
| Run 7 | 0.4 | 69.65 | 0.2612 |
| Run 8 | 0.45 | 77.32 | 0.2985 |
| Run 9 | 0.5 | 84.98 | 0.3381 |
| Run 10 | 0.55 | 92.64 | 0.3802 |
| Run 11 | 0.6 | 100.3 | 0.425 |

**Now, plot the results:**



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**Prob.4.3.14** In the above problem, if the compressor overall efficiency is 75%, adiabatic efficiency and mech. efficiency of expander are 80% and 90%, other quantities remaining the same, find the refrig. effect and COP.

**EES Solution:**

Fluid$ = 'Nitrogen'

P1 = 101.3 [kPa]

T1 = 300 [K]

P2 = 4050 [kPa]

T3 = 240 [K]

eta_co = 0.75

eta_e_ad = 0.8

eta_em = 0.9

Fraction_x = 0.4675 "…from previous problem, i.e. Prob.4.3.13"

CALL Claude_refrig(Fluid$, P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x: Tsat_fluid, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

**Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]**

| | | | |
|---|---|---|---|
| COP = 0.1703 | $COP_i$ = 0.3474 | $\eta_{co}$ = 0.75 | $\eta_{em}$ = 0.9 |
| $\eta_{e,ad}$ = 0.8 | Fluid$ = 'Nitrogen' | FOM = 0.4901 | $Fraction_x$ = 0.4675 |
| P1 = 101.3 [kPa] | P2 = 4050 [kPa] | $Qa_{by,m}$ = 65.67 [kJ/kg] | T1 = 300 [K] |
| T3 = 240 [K] | $Tsat_{fluid}$ = 77.35 [K] | $W_{by,m}$ = 385.7 [kJ/kg] | |

**Thus:**

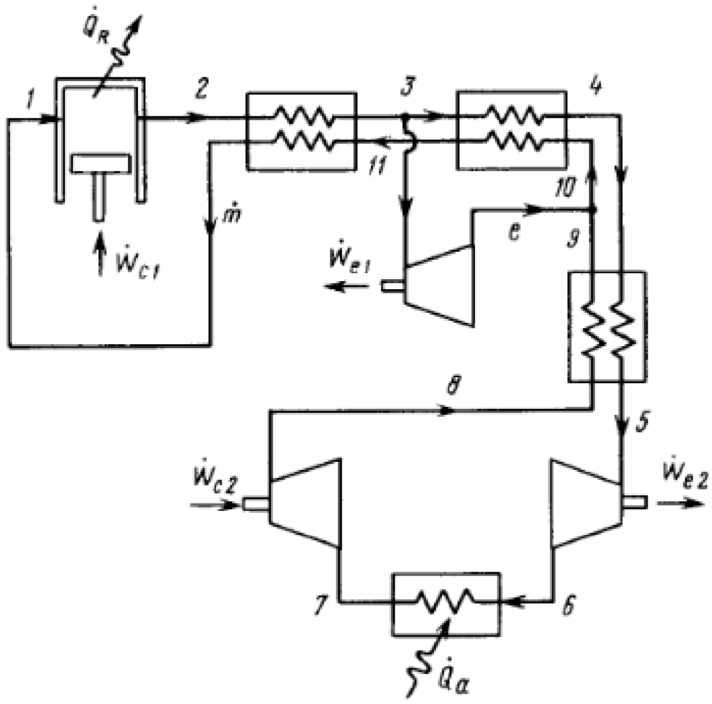
**Refrign effect = Qa_by_m = 65.67 kJ/kg … Ans.**

**COP = 0.1703 … Ans.**

**FOM = 0.4901 … Ans.**

---

**Prob.4.3.15** In a refrigeration system with a wet gas expander (see the fig.), the working fluid is helium, which enters the main compressor at 270 K and 4.053 bar. Compressors and expanders may be assumed to have efficiencies as shown below. At point 3, 30% of the main stream is by-passed to the main expander at a temp of 200 K and 40.53 bar. The helium gas leaves the main compressor at 300 K and 40.53 bar. The helium enters the low temp compressor at 1.013 bar and sat. vapor conditions and leaves at 4.053 bar. Determine the coeff. of performance and the FOM. [1]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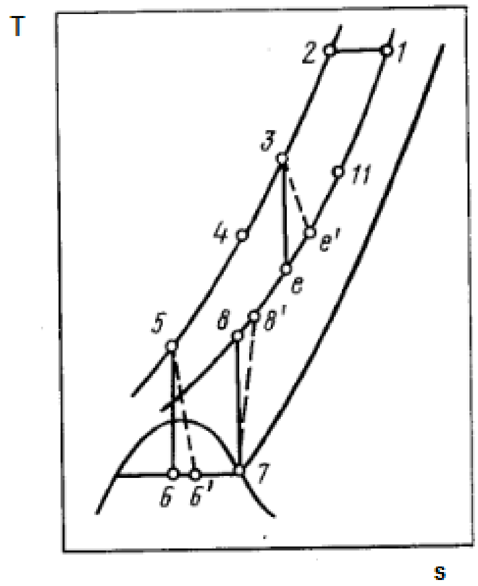
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**First, write an EES Procedure to calculate various parameters for a "Cold gas refrigeration system":**

```
PROCEDURE Coldgas_refrig_He(P1, T1, P2, T2, T3, P7, Fraction_x, eta_co, eta_c, eta_ad_e1, eta_m_e1, eta_ad_e2: T7, Qa_by_m, Wc1_by_m, Wc2_by_m,We1_by_m, We2_by_m, Wnet_by_m, COP, COP_i, FOM)

"Gives various parameters for a Cold gas refrign. system for Helium as working fluid"

"Inputs: P1, P2, P7 (kPa), T1, T2,T3 (K),
eta_co= compressor overall effcy .,eta_c = effcy of wet compressor, eta_ad_e1 = adab. effcy of expander 1, , eta_m_e1 = mech. effcy of expander 1, eta_ad_e2 = adab. effcy of wet expander, Fraction_x = fraction of the gas compressed that goes through the expander,"

"Outputs: T7 (K), Qa_by_m (kJ/kg), We1_by_m, We2_by_m, Wc1_by_m, Wc2_by_m, Wnet_by_m (kJ/kg), COP, COP_i, FOM"

h1:= ENTHALPY(Helium, T=T1, P=P1)
s1:= ENTROPY(Helium, T=T1, P=P1)

h2 := ENTHALPY(Helium, T=T2, P=P2)
s2 := ENTROPY(Helium, T=T2, P=P2)

P3 := P2
P5 := P2
P8 := P1

T7 := T_sat(Helium, P = P7)
h3 := ENTHALPY(Helium, T=T3, P=P3)
s3 := ENTROPY(Helium, T=T3, P=P3)

se := s3

he := ENTHALPY(Helium, s=se, P=P8)

h7 := Enthalpy(Helium, x=1, P=P7)
s7 := Entropy(Helium, x=1, P=P7)

sg := s7
```

```
s8:= s7

h8 := ENTHALPY(Helium, s=s8, P=P8)

sf := Entropy(Helium, x=0, P=P7)

he_prime := h3 – eta_ad_e1 * (h3 – he)

h8_prime := h7 + (h8 – h7) / eta_c

AA := (h2 – h1) – Fraction_x * (h3 -he_prime)

h5 := AA / (1 – Fraction_x) + h8_prime

s5=Entropy(Helium,h=h5,P=P5)

x6 := (sg – s5) / (sg – sf)

hg := Enthalpy(Helium,x=1,P=P7)

hf := Enthalpy(Helium,x=0,P=P7)

h6 := hg – x6 * (hg – hf)

h6_prime := h5 – eta_ad_e2 * (h5 – h6)

Qa_by_m := (1 – Fraction_x) * (h7 – h6_prime)

Wc1_by_m := (T2 * (s1 – s2) – (h1 – h2)) / eta_co "…if main expander work is utilised to aid compression"

Wc2_by_m := (1 – Fraction_x) * (h8_prime – h7)

We1_by_m := Fraction_x * eta_m_e1 * (h3 – he_prime)

We2_by_m := eta_ad_e2 * (h5 – h6)

Wnet_by_m := Wc1_by_m + Wc2_by_m – We1_by_m
```

COP := Qa_by_m / Wnet_by_m

COP_i:= T7 / (T2 – T7)

FOM:= COP / COP_i

END

"=============================================================="

**Now, solve the problem, using the above written EES Procedure:**

P1 = 4.053E02 [kPa]

T1 = 270 [K]

P2 = 40.53E02 [kPa]

T2 = 300 [K]

P3 = P2

T3 = 200[K]

Fraction_x = 0.30

P5 = P2

P7 = 101.3[kPa]

P8 = P1

eta_co = 0.7 "…effcy of main compressor"

eta_ad_e1 = 0.85 "…adiab. effcy of expander 1"

eta_m_e1 = 0.95 "…mech. effcy of expander 1"

eta_c = 0.8 "…adiab. effcy of low temp compressor"

eta_ad_e2 = 0.85 "…adiab. effcy of wet expander"

CALL Coldgas_refrig_He(P1, T1, P2, T2, T3, P7, Fraction_x, eta_co, eta_c, eta_ad_e1, eta_m_e1, eta_ad_e2: T7, Qa_by_m, Wc1_by_m, Wc2_by_m,We1_by_m, We2_by_m, Wnet_by_m, COP, COP_i, FOM)

**Results:**

**Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]**

| | | |
|---|---|---|
| COP = 0.002747 | $COP_i$ = 0.01427 | $\eta_{ad,e1}$ = 0.85 |
| $\eta_{ad,e2}$ = 0.85 | $\eta_c$ = 0.8 | $\eta_{co}$ = 0.7 |
| $\eta_{m,e1}$ = 0.95 | FOM = 0.1925 | $Fraction_x$ = 0.3 |
| P1 = 405.3 [kPa] | P2 = 4053 [kPa] | P3 = 4053 [kPa] |
| P5 = 4053 [kPa] | P7 = 101.3 [kPa] | P8 = 405.3 [kPa] |
| $Qa_{by,m}$ = 5.243 [kJ/kg] | T1 = 270 [K] | T2 = 300 [K] |
| T3 = 200 [K] | T7 = 4.222 [K] | $Wc1_{by,m}$ = 2053 [kJ/kg] |
| $Wc2_{by,m}$ = 9.993 [kJ/kg] | $We1_{by,m}$ = 154.2 [kJ/kg] | $We2_{by,m}$ = 29.21 [kJ/kg] |
| $Wnet_{by,m}$ = 1909 [kJ/kg] | | |

**Also:**

**Local variables in Procedure Coldgas_refrig_He (1 call, 0.05 sec)**

| | | | |
|---|---|---|---|
| AA =5.212 | COP=0.002747 | $COP_i$ =0.01427 | $\eta_{ad,e1}$=0.85 |
| $\eta_{ad,e2}$=0.85 | $\eta_c$=0.8 | $\eta_{co}$=0.7 | $\eta_{m,e1}$ =0.95 |
| FOM=0.1925 | $Fraction_x$ =0.3 | h1=-144.9 [kJ/kg] | h2=22.7 [kJ/kg] |
| h3=-496.5 [kJ/kg] | h5=-1511 [kJ/kg] | h6=-1545 [kJ/kg] | $h6_{prime}$ =-1540 [kJ/kg] |
| h7=-1533 [kJ/kg] | h8=-1521 [kJ/kg] | $h8_{prime}$ =-1519 [kJ/kg] | he=-1133 [kJ/kg] |
| $he_{prime}$ =-1038 [kJ/kg] | hf =-1554 [kJ/kg] | hg=-1533 [kJ/kg] | P1 =405.3 [kPa] |
| P2 =4053 [kPa] | P3 =4053 [kPa] | P5 =4053 [kPa] | P7 =101.3 [kPa] |
| P8 =405.3 [kPa] | $Qa_{by,m}$ =5.243 [kJ/kg] | s1=-3.394 [kJ/kg-K] | s2=-7.625 [kJ/kg-K] |
| s3=-9.731 [kJ/kg-K] | s5=-26.07 [kJ/kg-K] | s7=-23.08 [kJ/kg-K] | s8=-23.08 [kJ/kg-K] |
| se=-9.731 [kJ/kg-K] | sf =-27.98 [kJ/kg-K] | sg=-23.08 [kJ/kg-K] | T1 =270 [K] |
| T2 =300 [K] | T3 =200 [K] | T7 =4.222 [K] | $Wc1_{by,m}$=2053 [kJ/kg] |
| $Wc2_{by,m}$=9.993 [kJ/kg] | $We1_{by,m}$=154.2 [kJ/kg] | $We2_{by,m}$=29.21 [kJ/kg] | $Wnet_{by,m}$ =1909 [kJ/kg] |
| x6=0.6102 | | | |

**Thus:**

**COP = 0.002747 … Ans.**

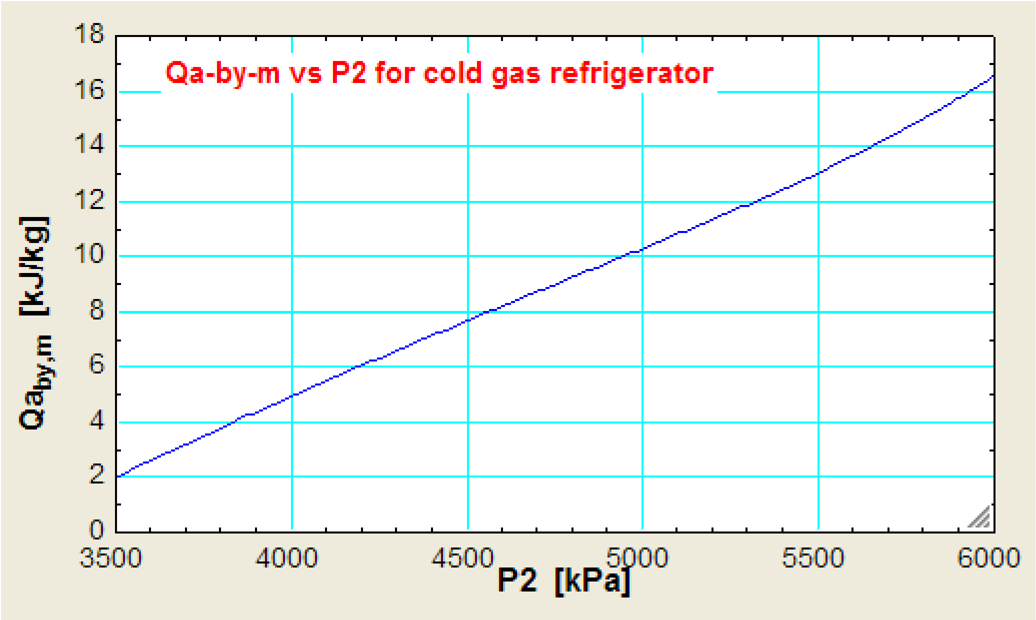
**FOM = 0.1925 … Ans.**

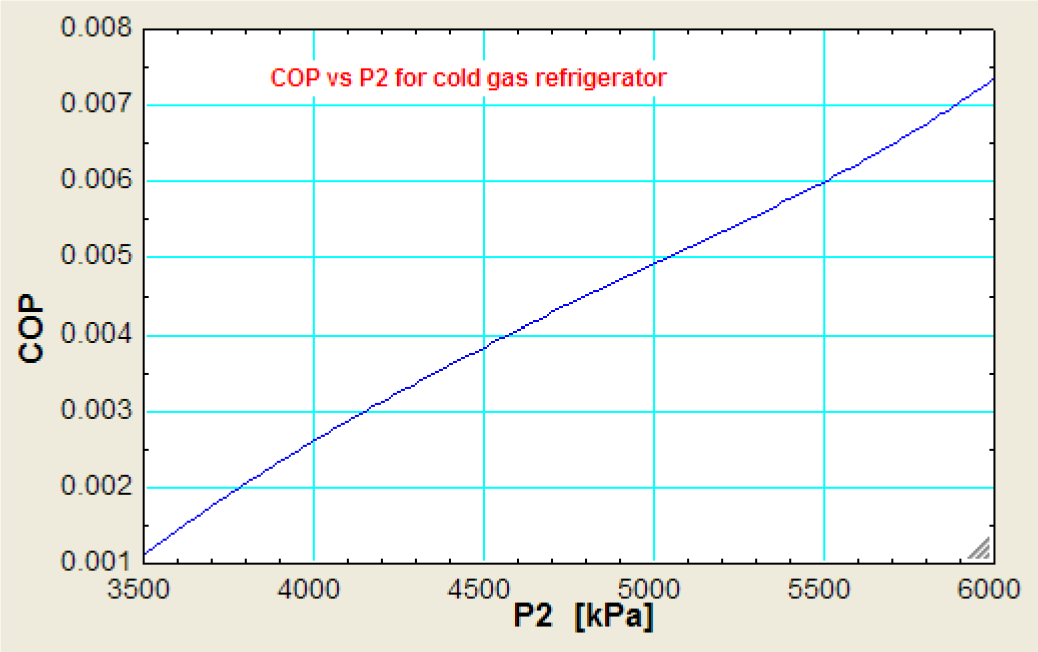
**Also, Plot Refrig. effect, COP and FOM as P2 varies from 35 to 60 bar for expander flow rate ratios of 0.3, other factors remaining the same:**

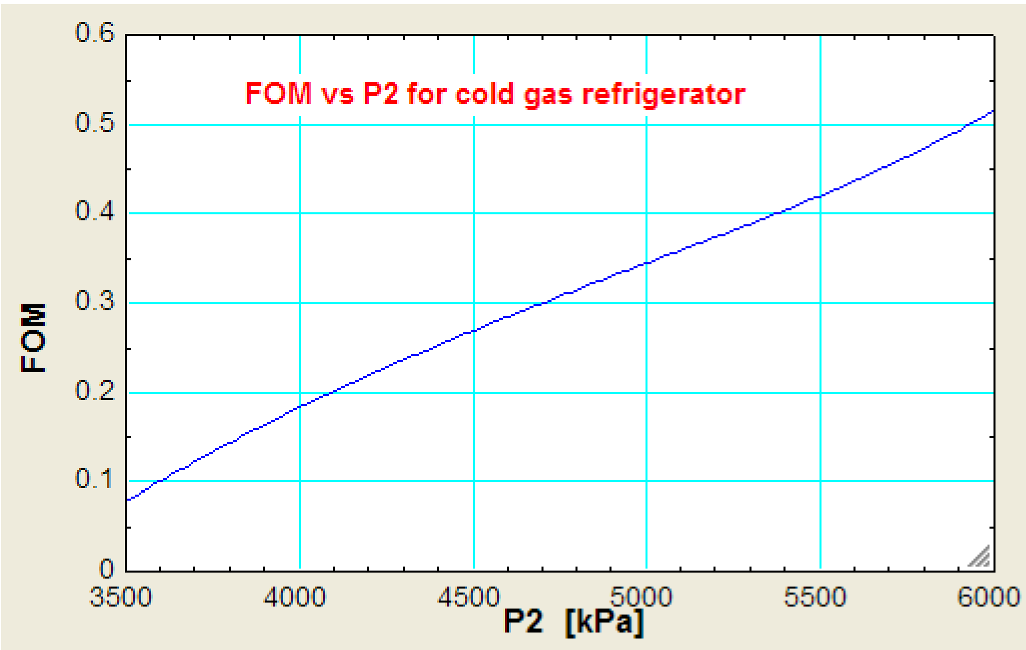
**First, compute the Parametric Table:**

| 1..6 | P2 [kPa] | $Qa_{by,m}$ [kJ/kg] | COP | FOM |
|---|---|---|---|---|
| Run 1 | 3500 | 1.983 | 0.001113 | 0.07795 |
| Run 2 | 4000 | 4.946 | 0.002607 | 0.1827 |
| Run 3 | 4500 | 7.662 | 0.003833 | 0.2685 |
| Run 4 | 5000 | 10.29 | 0.004921 | 0.3448 |
| Run 5 | 5500 | 13.03 | 0.005996 | 0.4201 |
| Run 6 | 6000 | 16.55 | 0.007355 | 0.5153 |

**Now, plot the results:**



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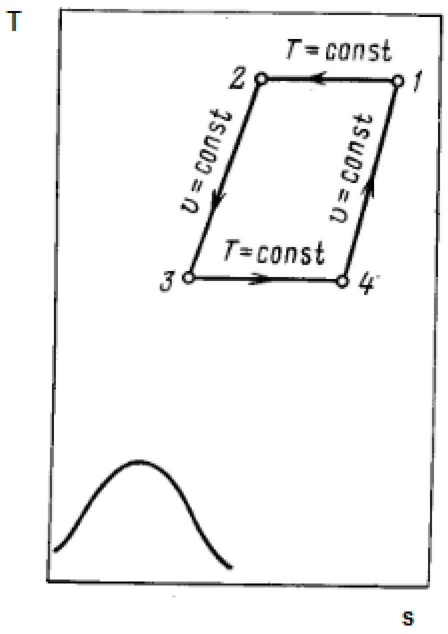
**Prob.4.3.16** Determine the fraction of the ideal refrigeration effect (Δ Q/Q_ideal) that is lost in a Philips refrigerator having a regenerator which is 98% efficient. Hydrogen gas is the working substance (cp/cv = 1.4), and the volume ratio (v4/v3) is 1.85. The refrigerator operates between the temp limits of 300 K and 80 K. [1]




**EES Solution:**

"Data:"

epsilon = 0.98

gamma = 1.4

T2 = 300 [K]

T3 = 80 [K]

v4_by_v3 = 1.85

"Then, we have:"

"Fraction = DELTA_Q / QA_ideal"

Fraction = ((1 – epsilon) / (gamma – 1)) * (T2 / T3 – 1) / ln(v4_by_v3)

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

| | | |
|---|---|---|
| $\varepsilon = 0.98$ | Fraction = 0.2235 | $\gamma = 1.4$ |
| T2 = 300 [K] | T3 = 80 [K] | $v4_{by,v3} = 1.85$ |

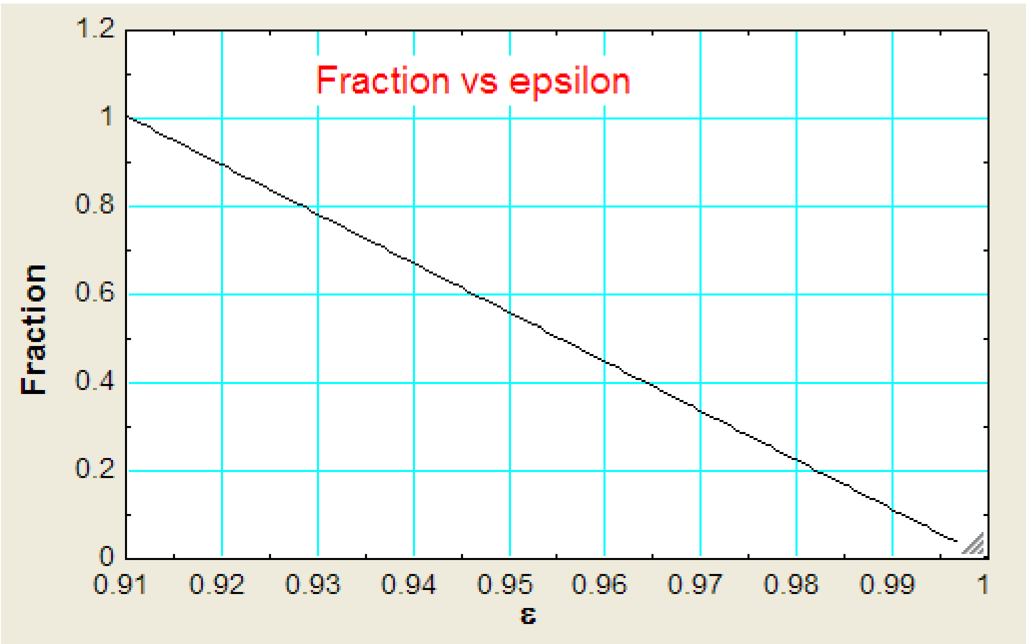
**Thus: Fraction of refrigeration lost due to regenerator inefficiency = 0.2235 = 22.35% … Ans.**

**Plot Fraction as epsilon varies from 0.91 to 1:**

**First, compute the Parametric Table:**

| 1..10 | ε | Fraction |
|---|---|---|
| Run 1 | 0.91 | 1.006 |
| Run 2 | 0.92 | 0.894 |
| Run 3 | 0.93 | 0.7823 |
| Run 4 | 0.94 | 0.6705 |
| Run 5 | 0.95 | 0.5588 |
| Run 6 | 0.96 | 0.447 |
| Run 7 | 0.97 | 0.3353 |
| Run 8 | 0.98 | 0.2235 |
| Run 9 | 0.99 | 0.1118 |
| Run 10 | 1 | 0 |

**Now, plot:**

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**Prob. 4.3.17** Based on Schmidt's theory for analysis of Stirling cycle (Alpha type), write an EES Program to plot the P-v diagram and also to calculate other performance parameters with air as the working fluid, under following conditions [19]:

Swept volume of an expansion piston: 0.628 cm3, swept volume of a compression piston: 0.628 cm3, dead volume of the expansion space: 0.2cm3, dead volume of the compression space: 0.2cm3, regenerator volume: 0.2cm3, phase angle: 90deg, mean pressure: 101.3 kPa, expansion gas temperature: 400 deg C, compression gas temperature: 30 deg C, engine speed: 2000 rpm.

**Alpha type Stirling engine:**

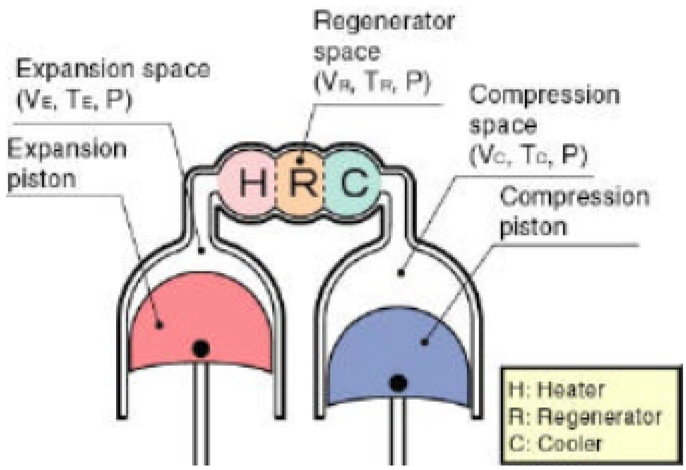



Fig. Alpha-type Stirling Engine

In the program given below:

**Inputs:** x (crank angle, deg.), dx (phase angle = 90 deg.), TC (comprn. space gas temp, K), TE(expn. space gas temp, K), VSE (swept vol. of expn. piston or displacer, m^3), VSc (swept vol. of comprn. piston or power piston, m^3), VDE (dead vol of expn. space, m^3), VDC(dead vol of comprn. space, m^3), VR(regenerator volume, m^3), N (rpm), Rgas (gas constant, J/kg.K), Pmean (mean pressure, kPa).

**Outputs:** x (crank angle, deg.), P (engine pressure, kPa), V (total momental volume, m^3),

VE(expn. space momental vol, m^3), VC (comprn. space momental vol, m^3), LE (indicated expn power, kW), LC (indicated comprn power, kW), Li (indicated power, kW), m (total mass of working gas, kg).

**EES Program:**

"Data:"

V_SE=0.628E-06[m3]"… swept vol. of expn. space"

V_SC=0.628E-06[m3]"… swept vol. of comprn. space"

V_DE=0.2E-06[m3]"… dead vol. of expn. space"

V_DC=0.2E-06[m3]"…. dead vol. of comprn. space"

V_R=0.2E-06[m3]"… Regenerator Volume"

{x=0[deg]"deg, crank angle"}

dx=90[deg]"deg, phase angle"

P_mean=101.3[kPa]"kPa, mean pressure"

```
T_E=400+273[K]"K, expn. space temp."

T_C=30+273[K]"K, comprn. space temp."

N=2000"RPM, Engine speed"

R=0.287[kJ/kg-K]"kJ/kg.K, … Gas const. for air"

"-------------------------------------------------------"

t=T_C/T_E"temp. ratio"

vratio=V_SC/V_SE"swept vol. ratio"

T_R=(T_E+T_C)/2 [K]"K, Regen. temp."

"Vol. ratios:"

X_DE=V_DE/V_SE"dead vol. ratio of expn. space"

X_DC=V_DC/V_SE"dead vol. ratio of comprn. space"

X_R=V_R/V_SE"dead vol. ratio of regen. space"

"coefficients:"

a=arctan((vratio*sin(dx))/(t+cos(dx)))

S=t+2*t*X_DE+(4*t*X_R)/(1+t)+vratio+2*X_DC

B=sqrt((t^2+2*t*vratio*cos(dx)+vratio^2))

c=B/S

"Instantaneous Engine Pressure:"

P=(P_mean*sqrt(1-c^2))/(1-c*cos(x-a)) "kPa"
```

```
"Instantaneous volumes:"

V_E=(V_SE/2)*(1-cos(x))+V_DE "m3 …for expn. vol."

V_C=(V_SC/2)*(1-cos(x-dx))+V_DC"m3…for comprn. vol."

V=V_E+V_R+V_C"m3….total instantaneous vol."

"mass flow:"

m=(P*V_SE)/(2*R*T_C)*(S-B*cos(x-a))"kg"

P_min=P_mean*sqrt((1-c)/(1+c))"kPa… min. pressure"

P_max=P_mean*sqrt((1+c)/(1-c))"kPa.. max. pressure"

"Indicated expn. work:"

W_E=(P_mean*V_SE*pi*c*sin(a))/(1+sqrt(1-c^2))"kJ"

"Indicated comprn. work:"

W_C=-(P_mean*V_SE*pi*c*t*sin(a))/(1+sqrt(1-c^2))"kJ"

"Indicated work per cycle"

W_i=W_E+W_C"[kJ/cycle]"

"Indicated expn. power:"

L_E=W_E*N/60"kW…heat to be supplied from heat source"

"Indicated comprn. power:"

L_C=W_C*N/60"kW…heat to be rejected in cooler"

"Indicated Power of the engine:"

L_i=W_i*N/60"kW….indicated power of engine"
```

"Cycle effcy.:"

eta=W_i/W_e

"Carnot effcy.:"

eta_carnot=1-T_C/T_E

"========================================"

**Results:**

**Variation of various parameters as the crank angle x varies from 0 to 360 deg. Is shown below in the Parametric Table:**

| x [deg] | P [kPa] | V [m3] | $V_E$ [m3] | $V_C$ [m3] |
|---|---|---|---|---|
| 0 | 111.1 | 9.140E-07 | 2.000E-07 | 5.140E-07 |
| 2 | 112.8 | 9.032E-07 | 2.002E-07 | 5.030E-07 |
| 4 | 114.5 | 8.929E-07 | 2.008E-07 | 4.921E-07 |
| 6 | 116.2 | 8.829E-07 | 2.017E-07 | 4.812E-07 |
| 8 | 117.9 | 8.734E-07 | 2.031E-07 | 4.703E-07 |
| 10 | 119.7 | 8.642E-07 | 2.048E-07 | 4.595E-07 |
| 12 | 121.4 | 8.556E-07 | 2.069E-07 | 4.487E-07 |
| 14 | 123.2 | 8.474E-07 | 2.093E-07 | 4.380E-07 |
| 16 | 125 | 8.396E-07 | 2.122E-07 | 4.274E-07 |
| 18 | 126.8 | 8.323E-07 | 2.154E-07 | 4.170E-07 |
| 20 | 128.5 | 8.255E-07 | 2.189E-07 | 4.066E-07 |
| 22 | 130.3 | 8.192E-07 | 2.229E-07 | 3.964E-07 |
| 24 | 132 | 8.134E-07 | 2.271E-07 | 3.863E-07 |
| 26 | 133.7 | 8.081E-07 | 2.318E-07 | 3.764E-07 |
| 28 | 135.4 | 8.033E-07 | 2.368E-07 | 3.666E-07 |
| 30 | 137.1 | 7.991E-07 | 2.421E-07 | 3.570E-07 |
| 32 | 138.7 | 7.953E-07 | 2.477E-07 | 3.476E-07 |
| 34 | 140.2 | 7.921E-07 | 2.537E-07 | 3.384E-07 |
| 36 | 141.7 | 7.894E-07 | 2.600E-07 | 3.294E-07 |
| 38 | 143.2 | 7.872E-07 | 2.666E-07 | 3.207E-07 |
| 40 | 144.6 | 7.856E-07 | 2.735E-07 | 3.122E-07 |
| 42 | 145.9 | 7.845E-07 | 2.807E-07 | 3.039E-07 |
| 44 | 147.1 | 7.840E-07 | 2.881E-07 | 2.959E-07 |
| 46 | 148.3 | 7.840E-07 | 2.959E-07 | 2.881E-07 |
| 48 | 149.3 | 7.845E-07 | 3.039E-07 | 2.807E-07 |
| 50 | 150.3 | 7.856E-07 | 3.122E-07 | 2.735E-07 |
| 52 | 151.2 | 7.872E-07 | 3.207E-07 | 2.666E-07 |
| 54 | 151.9 | 7.894E-07 | 3.294E-07 | 2.600E-07 |
| 56 | 152.6 | 7.921E-07 | 3.384E-07 | 2.537E-07 |
| 58 | 153.1 | 7.953E-07 | 3.476E-07 | 2.477E-07 |

| x [deg] | P [kPa] | V [m$^3$] | $V_E$ [m$^3$] | $V_C$ [m$^3$] |
|---|---|---|---|---|
| 60 | 153.5 | 7.991E-07 | 3.570E-07 | 2.421E-07 |
| 62 | 153.8 | 8.033E-07 | 3.666E-07 | 2.368E-07 |
| 64 | 154 | 8.081E-07 | 3.764E-07 | 2.318E-07 |
| 66 | 154 | 8.134E-07 | 3.863E-07 | 2.271E-07 |
| 68 | 153.9 | 8.192E-07 | 3.964E-07 | 2.229E-07 |
| 70 | 153.7 | 8.255E-07 | 4.066E-07 | 2.189E-07 |
| 72 | 153.4 | 8.323E-07 | 4.170E-07 | 2.154E-07 |
| 74 | 153 | 8.396E-07 | 4.274E-07 | 2.122E-07 |
| 76 | 152.4 | 8.474E-07 | 4.380E-07 | 2.093E-07 |
| 78 | 151.7 | 8.556E-07 | 4.487E-07 | 2.069E-07 |
| 80 | 151 | 8.642E-07 | 4.595E-07 | 2.048E-07 |
| 82 | 150.1 | 8.734E-07 | 4.703E-07 | 2.031E-07 |
| 84 | 149.1 | 8.829E-07 | 4.812E-07 | 2.017E-07 |
| 86 | 148 | 8.929E-07 | 4.921E-07 | 2.008E-07 |
| 88 | 146.8 | 9.032E-07 | 5.030E-07 | 2.002E-07 |
| 90 | 145.6 | 9.140E-07 | 5.140E-07 | 2.000E-07 |
| 92 | 144.3 | 9.251E-07 | 5.250E-07 | 2.002E-07 |
| 94 | 142.9 | 9.367E-07 | 5.359E-07 | 2.008E-07 |
| 96 | 141.4 | 9.485E-07 | 5.468E-07 | 2.017E-07 |
| 98 | 139.9 | 9.608E-07 | 5.577E-07 | 2.031E-07 |
| 100 | 138.3 | 9.733E-07 | 5.685E-07 | 2.048E-07 |
| 102 | 136.7 | 9.861E-07 | 5.793E-07 | 2.069E-07 |
| 104 | 135 | 9.993E-07 | 5.900E-07 | 2.093E-07 |
| 106 | 133.3 | 0.000001013 | 6.006E-07 | 2.122E-07 |
| 108 | 131.6 | 0.000001026 | 6.110E-07 | 2.154E-07 |

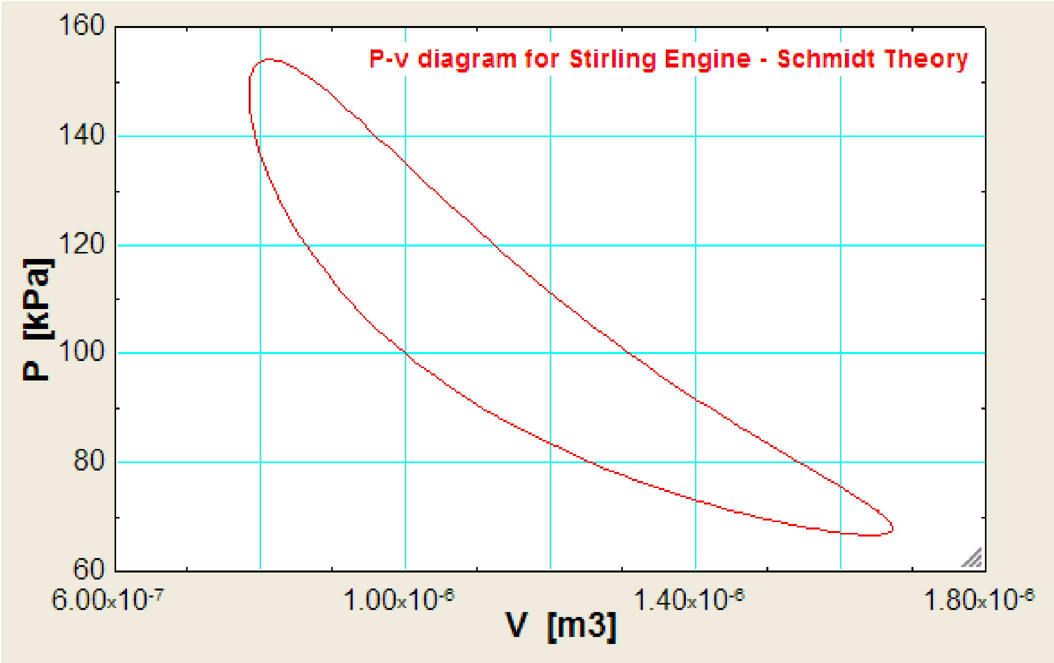
| x [deg] | P [kPa] | V [m$^3$] | $V_E$ [m$^3$] | $V_C$ [m$^3$] |
|---|---|---|---|---|
| 110 | 129.9 | 0.00000104 | 6.214E-07 | 2.189E-07 |
| 112 | 128.1 | 0.000001054 | 6.316E-07 | 2.229E-07 |
| 114 | 126.3 | 0.000001069 | 6.417E-07 | 2.271E-07 |
| 116 | 124.6 | 0.000001083 | 6.516E-07 | 2.318E-07 |
| 118 | 122.8 | 0.000001098 | 6.614E-07 | 2.368E-07 |
| 120 | 121 | 0.000001113 | 6.710E-07 | 2.421E-07 |
| 122 | 119.3 | 0.000001128 | 6.804E-07 | 2.477E-07 |
| 124 | 117.5 | 0.000001143 | 6.896E-07 | 2.537E-07 |
| 126 | 115.8 | 0.000001159 | 6.986E-07 | 2.600E-07 |
| 128 | 114.1 | 0.000001174 | 7.073E-07 | 2.666E-07 |
| 130 | 112.4 | 0.000001189 | 7.158E-07 | 2.735E-07 |
| 132 | 110.7 | 0.000001205 | 7.241E-07 | 2.807E-07 |
| 134 | 109 | 0.00000122 | 7.321E-07 | 2.881E-07 |
| 136 | 107.4 | 0.000001236 | 7.399E-07 | 2.959E-07 |
| 138 | 105.8 | 0.000001251 | 7.473E-07 | 3.039E-07 |
| 140 | 104.2 | 0.000001267 | 7.545E-07 | 3.122E-07 |
| 142 | 102.7 | 0.000001282 | 7.614E-07 | 3.207E-07 |
| 144 | 101.2 | 0.000001297 | 7.680E-07 | 3.294E-07 |
| 146 | 99.71 | 0.000001313 | 7.743E-07 | 3.384E-07 |
| 148 | 98.27 | 0.000001328 | 7.803E-07 | 3.476E-07 |
| 150 | 96.87 | 0.000001343 | 7.859E-07 | 3.570E-07 |
| 152 | 95.5 | 0.000001358 | 7.912E-07 | 3.666E-07 |
| 154 | 94.17 | 0.000001373 | 7.962E-07 | 3.764E-07 |
| 156 | 92.87 | 0.000001387 | 8.009E-07 | 3.863E-07 |
| 158 | 91.6 | 0.000001402 | 8.051E-07 | 3.964E-07 |

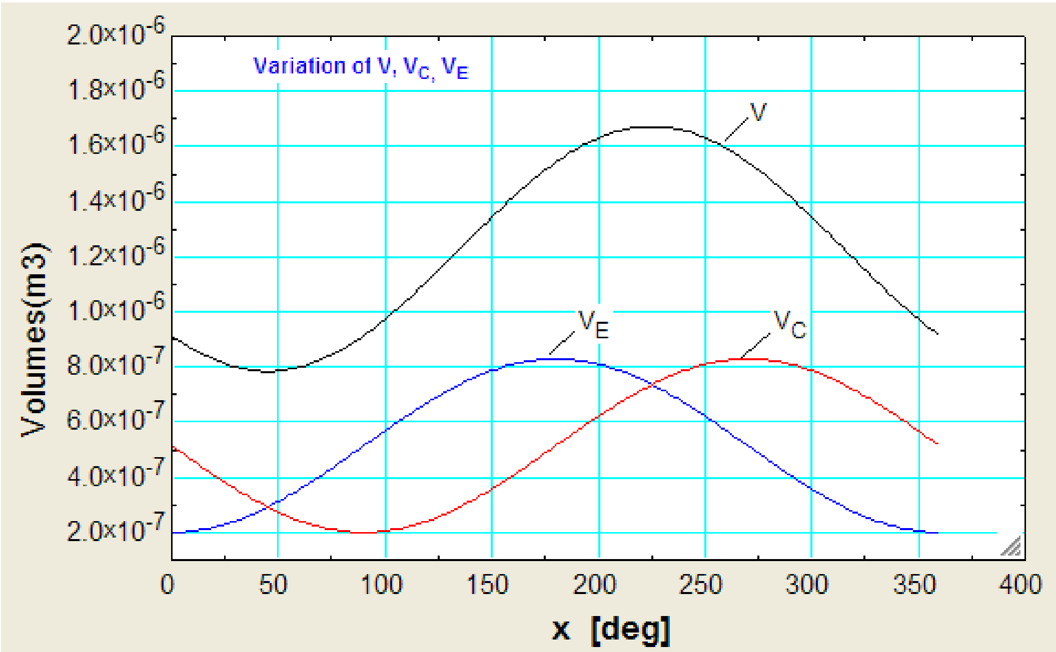
| x [deg] | P [kPa] | V [m$^3$] | $V_E$ [m$^3$] | $V_C$ [m$^3$] |
|---|---|---|---|---|
| 160 | 90.37 | 0.000001416 | 8.091E-07 | 4.066E-07 |
| 162 | 89.18 | 0.00000143 | 8.126E-07 | 4.170E-07 |
| 164 | 88.02 | 0.000001443 | 8.158E-07 | 4.274E-07 |
| 166 | 86.9 | 0.000001457 | 8.187E-07 | 4.380E-07 |
| 168 | 85.82 | 0.00000147 | 8.211E-07 | 4.487E-07 |
| 170 | 84.76 | 0.000001483 | 8.232E-07 | 4.595E-07 |
| 172 | 83.75 | 0.000001495 | 8.249E-07 | 4.703E-07 |
| 174 | 82.76 | 0.000001507 | 8.263E-07 | 4.812E-07 |
| 176 | 81.81 | 0.000001519 | 8.272E-07 | 4.921E-07 |
| 178 | 80.9 | 0.000001531 | 8.278E-07 | 5.030E-07 |
| 180 | 80.01 | 0.000001542 | 8.280E-07 | 5.140E-07 |
| 182 | 79.16 | 0.000001553 | 8.278E-07 | 5.250E-07 |
| 184 | 78.34 | 0.000001563 | 8.272E-07 | 5.359E-07 |
| 186 | 77.55 | 0.000001573 | 8.263E-07 | 5.468E-07 |
| 188 | 76.8 | 0.000001583 | 8.249E-07 | 5.577E-07 |
| 190 | 76.07 | 0.000001592 | 8.232E-07 | 5.685E-07 |
| 192 | 75.37 | 0.0000016 | 8.211E-07 | 5.793E-07 |
| 194 | 74.71 | 0.000001609 | 8.187E-07 | 5.900E-07 |
| 196 | 74.07 | 0.000001616 | 8.158E-07 | 6.006E-07 |
| 198 | 73.46 | 0.000001624 | 8.126E-07 | 6.110E-07 |
| 200 | 72.88 | 0.00000163 | 8.091E-07 | 6.214E-07 |
| 202 | 72.33 | 0.000001637 | 8.051E-07 | 6.316E-07 |
| 204 | 71.81 | 0.000001643 | 8.009E-07 | 6.417E-07 |
| 206 | 71.31 | 0.000001648 | 7.962E-07 | 6.516E-07 |
| 208 | 70.84 | 0.000001653 | 7.912E-07 | 6.614E-07 |

| x [deg] | P [kPa] | V [m$^3$] | $V_E$ [m$^3$] | $V_C$ [m$^3$] |
|---|---|---|---|---|
| 210 | 70.4 | 0.000001657 | 7.859E-07 | 6.710E-07 |
| 212 | 69.98 | 0.000001661 | 7.803E-07 | 6.804E-07 |
| 214 | 69.59 | 0.000001664 | 7.743E-07 | 6.896E-07 |
| 216 | 69.22 | 0.000001667 | 7.680E-07 | 6.986E-07 |
| 218 | 68.88 | 0.000001669 | 7.614E-07 | 7.073E-07 |
| 220 | 68.57 | 0.00000167 | 7.545E-07 | 7.158E-07 |
| 222 | 68.27 | 0.000001671 | 7.473E-07 | 7.241E-07 |
| 224 | 68.01 | 0.000001672 | 7.399E-07 | 7.321E-07 |
| 226 | 67.76 | 0.000001672 | 7.321E-07 | 7.399E-07 |
| 228 | 67.55 | 0.000001671 | 7.241E-07 | 7.473E-07 |
| 230 | 67.35 | 0.00000167 | 7.158E-07 | 7.545E-07 |
| 232 | 67.18 | 0.000001669 | 7.073E-07 | 7.614E-07 |
| 234 | 67.03 | 0.000001667 | 6.986E-07 | 7.680E-07 |
| 236 | 66.91 | 0.000001664 | 6.896E-07 | 7.743E-07 |
| 238 | 66.81 | 0.000001661 | 6.804E-07 | 7.803E-07 |
| 240 | 66.73 | 0.000001657 | 6.710E-07 | 7.859E-07 |
| 242 | 66.67 | 0.000001653 | 6.614E-07 | 7.912E-07 |
| 244 | 66.64 | 0.000001648 | 6.516E-07 | 7.962E-07 |
| 246 | 66.63 | 0.000001643 | 6.417E-07 | 8.009E-07 |
| 248 | 66.65 | 0.000001637 | 6.316E-07 | 8.051E-07 |
| 250 | 66.68 | 0.00000163 | 6.214E-07 | 8.091E-07 |
| 252 | 66.74 | 0.000001624 | 6.110E-07 | 8.126E-07 |
| 254 | 66.83 | 0.000001616 | 6.006E-07 | 8.158E-07 |
| 256 | 66.93 | 0.000001609 | 5.900E-07 | 8.187E-07 |
| 258 | 67.06 | 0.0000016 | 5.793E-07 | 8.211E-07 |

| x [deg] | P [kPa] | V [m3] | $V_E$ [m3] | $V_C$ [m3] | x [deg] | P [kPa] | V [m3] | $V_E$ [m3] | $V_C$ [m3] |
|---|---|---|---|---|---|---|---|---|---|
| 260 | 67.22 | 0.000001592 | 5.685E-07 | 8.232E-07 | 310 | 79.36 | 0.000001267 | 3.122E-07 | 7.545E-07 |
| 262 | 67.4 | 0.000001583 | 5.577E-07 | 8.249E-07 | 312 | 80.22 | 0.000001251 | 3.039E-07 | 7.473E-07 |
| 264 | 67.6 | 0.000001573 | 5.468E-07 | 8.263E-07 | 314 | 81.11 | 0.000001236 | 2.959E-07 | 7.399E-07 |
| 266 | 67.82 | 0.000001563 | 5.359E-07 | 8.272E-07 | 316 | 82.04 | 0.00000122 | 2.881E-07 | 7.321E-07 |
| 268 | 68.07 | 0.000001553 | 5.250E-07 | 8.278E-07 | 318 | 82.99 | 0.000001205 | 2.807E-07 | 7.241E-07 |
| 270 | 68.34 | 0.000001542 | 5.140E-07 | 8.280E-07 | 320 | 83.98 | 0.000001189 | 2.735E-07 | 7.158E-07 |
| 272 | 68.64 | 0.000001531 | 5.030E-07 | 8.278E-07 | 322 | 85.01 | 0.000001174 | 2.666E-07 | 7.073E-07 |
| 274 | 68.96 | 0.000001519 | 4.921E-07 | 8.272E-07 | 324 | 86.07 | 0.000001159 | 2.600E-07 | 6.986E-07 |
| 276 | 69.31 | 0.000001507 | 4.812E-07 | 8.263E-07 | 326 | 87.17 | 0.000001143 | 2.537E-07 | 6.896E-07 |
| 278 | 69.68 | 0.000001495 | 4.703E-07 | 8.249E-07 | 328 | 88.3 | 0.000001128 | 2.477E-07 | 6.804E-07 |
| 280 | 70.08 | 0.000001483 | 4.595E-07 | 8.232E-07 | 330 | 89.46 | 0.000001113 | 2.421E-07 | 6.710E-07 |
| 282 | 70.5 | 0.00000147 | 4.487E-07 | 8.211E-07 | 332 | 90.66 | 0.000001098 | 2.368E-07 | 6.614E-07 |
| 284 | 70.95 | 0.000001457 | 4.380E-07 | 8.187E-07 | 334 | 91.9 | 0.000001083 | 2.318E-07 | 6.516E-07 |
| 286 | 71.43 | 0.000001443 | 4.274E-07 | 8.158E-07 | 336 | 93.17 | 0.000001069 | 2.271E-07 | 6.417E-07 |
| 288 | 71.93 | 0.00000143 | 4.170E-07 | 8.126E-07 | 338 | 94.48 | 0.000001054 | 2.229E-07 | 6.316E-07 |
| 290 | 72.46 | 0.000001416 | 4.066E-07 | 8.091E-07 | 340 | 95.82 | 0.00000104 | 2.189E-07 | 6.214E-07 |
| 292 | 73.02 | 0.000001402 | 3.964E-07 | 8.051E-07 | 342 | 97.2 | 0.000001026 | 2.154E-07 | 6.110E-07 |
| 294 | 73.61 | 0.000001387 | 3.863E-07 | 8.009E-07 | 344 | 98.61 | 0.000001013 | 2.122E-07 | 6.006E-07 |
| 296 | 74.22 | 0.000001373 | 3.764E-07 | 7.962E-07 | 346 | 100.1 | 9.993E-07 | 2.093E-07 | 5.900E-07 |
| 298 | 74.86 | 0.000001358 | 3.666E-07 | 7.912E-07 | 348 | 101.5 | 9.861E-07 | 2.069E-07 | 5.793E-07 |
| 300 | 75.54 | 0.000001343 | 3.570E-07 | 7.859E-07 | 350 | 103.1 | 9.733E-07 | 2.048E-07 | 5.685E-07 |
| 302 | 76.24 | 0.000001328 | 3.476E-07 | 7.803E-07 | 352 | 104.6 | 9.608E-07 | 2.031E-07 | 5.577E-07 |
| 304 | 76.97 | 0.000001313 | 3.384E-07 | 7.743E-07 | 354 | 106.2 | 9.485E-07 | 2.017E-07 | 5.468E-07 |
| 306 | 77.74 | 0.000001297 | 3.294E-07 | 7.680E-07 | 356 | 107.8 | 9.367E-07 | 2.008E-07 | 5.359E-07 |
| 308 | 78.53 | 0.000001282 | 3.207E-07 | 7.614E-07 | 358 | 109.4 | 9.251E-07 | 2.002E-07 | 5.250E-07 |
| 310 | 79.36 | 0.000001267 | 3.122E-07 | 7.545E-07 | 360 | 111.1 | 9.140E-07 | 2.000E-07 | 5.140E-07 |

**Now, plot the results:**



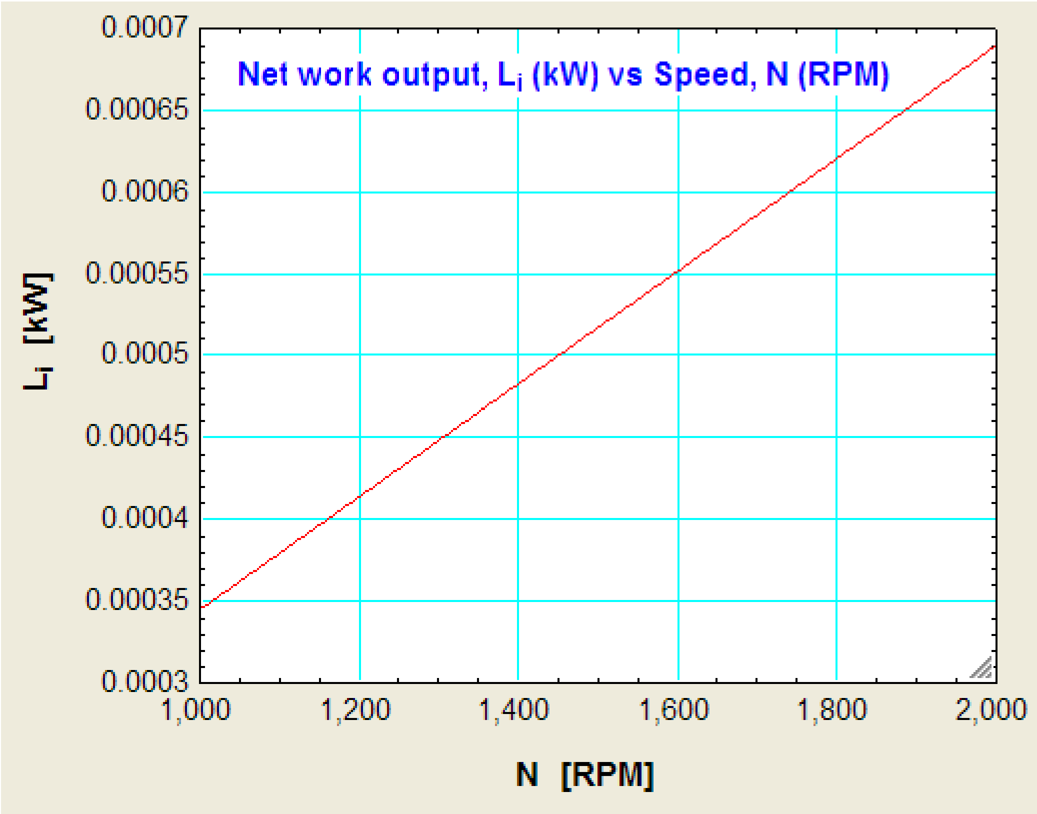



**Plot L_i, the net work (kW) as Speed, N varies from 1000 to 2000 RPM:**

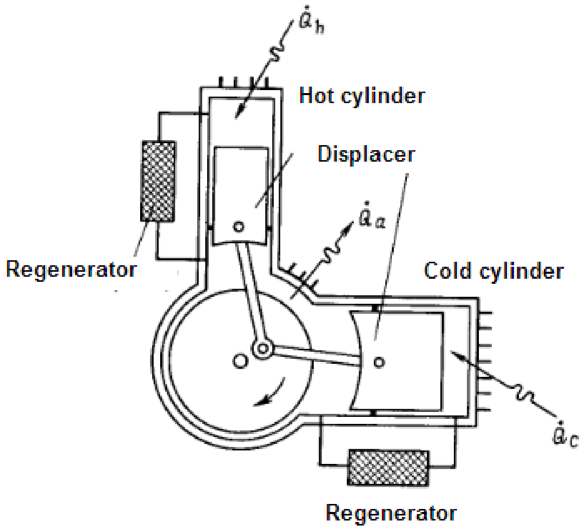
**The Parametric Table:**

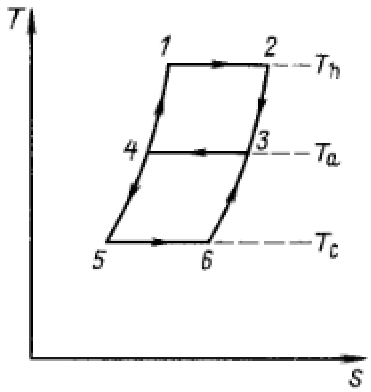
| 1..11 | N [RPM] | $L_i$ [kW] |
|---|---|---|
| Run 1 | 1,000 | 0.0003447 |
| Run 2 | 1,100 | 0.0003792 |
| Run 3 | 1,200 | 0.0004137 |
| Run 4 | 1,300 | 0.0004481 |
| Run 5 | 1,400 | 0.0004826 |
| Run 6 | 1,500 | 0.0005171 |
| Run 7 | 1,600 | 0.0005515 |
| Run 8 | 1,700 | 0.000586 |
| Run 9 | 1,800 | 0.0006205 |
| Run 10 | 1,900 | 0.000655 |
| Run 11 | 2,000 | 0.0006894 |

**Now, plot:**



**Prob. 4.3.18** An ideal Vuilleumier refrigerator absorbs energy as heat from a high temp source at 600 K and rejects heat to a sink at 300 K. The refrigerator absorbs heat from the low temp region at 20 K. If the heat transfer rate at the hot end of the refrigerator is 2 kW, determine the refrigeration rate at the cold end. [1]






**EES Solution:**

"Data:"

Th = 600 [K]

Ta = 300 [K]

Tc = 20 [K]

Qh = 2 [kW]

COP = Qc / Qh "..finds Qc (kW)"

COP = (Tc / Th) * ((Th – Ta) / (Ta – Tc)) "…finds COP"

**Results:**

Unit Settings: SI K kPa kJ mass deg

COP = 0.03571 Qc = 0.07143 [kW] Qh = 2 [kW] Ta = 300 [K] Tc = 20 [K]
Th = 600 [K]

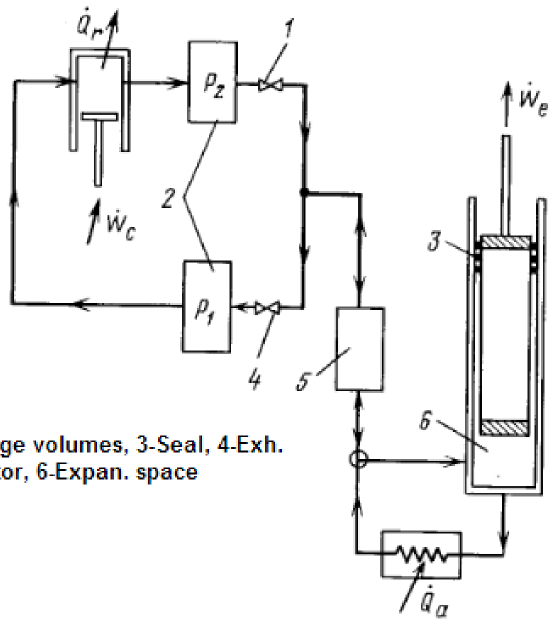
**Thus:**

**COP = 0.03571 … Ans.**

**Qc = 0.07143 kW … Ans.**

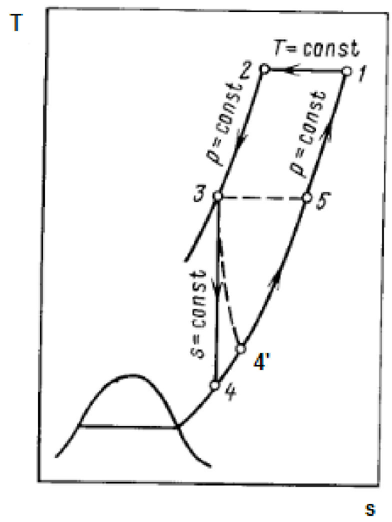
**Prob.4.3.19** A Solvay refrigerator uses neon as the working fluid, with the expander work utilized to aid in compression. Neon is compressed from 152 kPa to 1.013 MPa and 294 K in a compressor having an overall isothermal effcy of 75%. The adiabatic effcy of the expander is 90% and the mechanical effcy of expander is 96%. At the beginning of the expansion process, the temp of neon gas is 85.6 K and the pressure is 1.013 MPa. The neon gas enters the regenerator at the cold end at 152 kPa and 85.6 K. Determine the COP of the refrigerator. [1]

Solvay refrigerator.




1-Inlet valve, 2-Surge volumes, 3-Seal, 4-Exh. valve, 5-Regenerator, 6-Expan. space

T-s diagram




**EES Solution:**

"Data:"

"Fluid = Neon"

P1 = 152 [kPa]

P2 = 1.013E03 [kPa]

T1 = 294 [K]

T2 = T1

P3 = P2

```
P5 = P1

T3 = 85.6 [K]

T5 = T3

eta_co = 0.75 “…overall effcy of compressor”

eta_ad = 0.9 “… adiab. effcy of expander”

eta_e_m = 0.96 “… mech. effcy of expander”

“Then, we have:”

h1 = Enthalpy(Neon,T=T1,P=P1) “kJ/kg”

s1 = Entropy(Neon,T=T1,P=P1) “kJ/kg-K”

h2 = Enthalpy(Neon,T=T2,P=P2) “kJ/kg”
```

```
s2 = Entropy(Neon,T=T2,P=P2) "kJ/kg-K"

h3 = Enthalpy(Neon,T=T3,P=P3) "kJ/kg"

s3 = Entropy(Neon,T=T3,P=P3) "kJ/kg-K"

s4 = s3 "… for isentropic expn."

h4 = Enthalpy(Neon,P=P1,s=s4) "kJ/kg"

h5 = Enthalpy(Neon,T=T5,P=P5) "kJ/kg"

"Refrigeration effect:"

h4_prime = h3 – eta_ad * (h3 – h4)

Qa_by_m = h5 – h4_prime

"Net compressor work:"

Wnet_by_m = ((T2 * (s1 – s2) – (h1 – h2)) / eta_co) – eta_e_m * eta_ad * (h3 – h4)

COP = Qa_by_m / Wnet_by_m

"==========================================="
```

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| COP = 0.1616 | $\eta_{ad}$ = 0.9 | $\eta_{co}$ = 0.75 |
| $\eta_{e,m}$ = 0.96 | h1 = -4.217 [kJ/kg] | h2 = -3.958 [kJ/kg] |
| h3 = -221.2 [kJ/kg] | h4 = -267.1 [kJ/kg] | h4' = -262.5 [kJ/kg] |
| h5 = -219.3 [kJ/kg] | P1 = 152 [kPa] | P2 = 1,013 [kPa] |
| P3 = 1,013 [kPa] | P5 = 152 [kPa] | $Qa_{by,m}$ = 43.2 [kJ/kg] |
| s1 = -0.1817 [kJ/kg-K] | s2 = -0.964 [kJ/kg-K] | s3 = -2.255 [kJ/kg-K] |
| s4 = -2.255 [kJ/kg-K] | T1 = 294 [K] | T2 = 294 [K] |
| T3 = 85.6 [K] | T5 = 85.6 [K] | $Wnet_{by,m}$ = 267.4 [kJ/kg] |

**Thus:**

**Refrig. effect = Qa_by_m = 43.2 kJ/kg … Ans.**

**Net compressor work = Wnet_by_m = 267.4 kJ/kg …. Ans.**

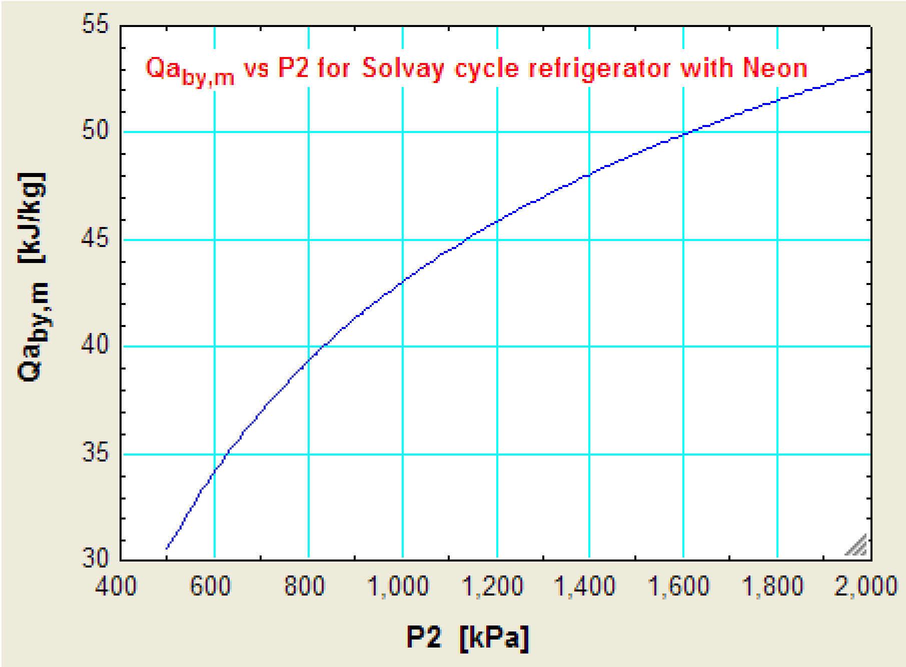
**COP = 0.1616 … Ans.**

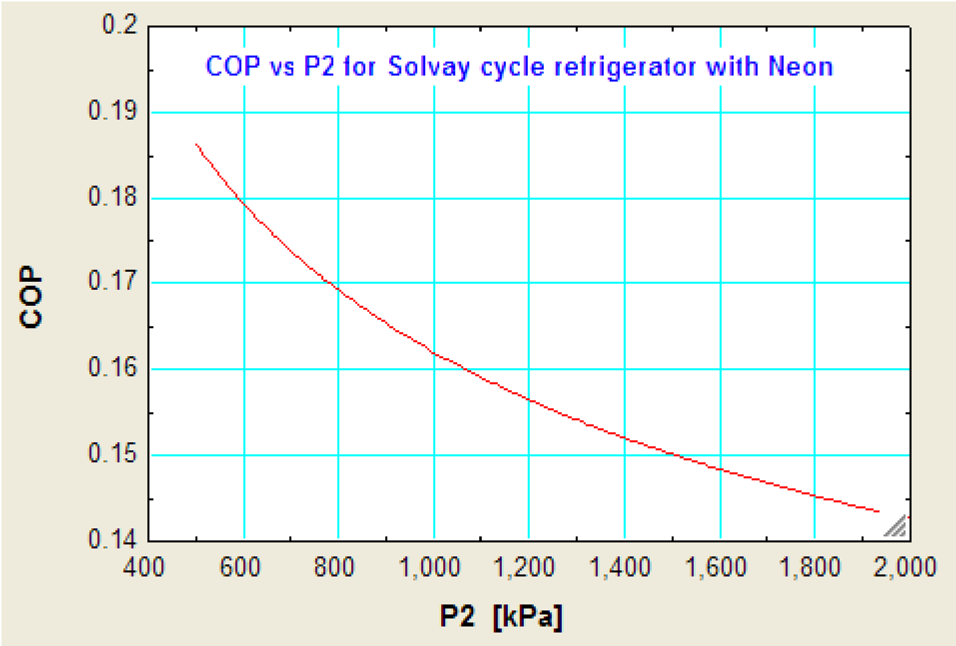
**Plot Refrig. effect and COP as high pressure P2 varies from 5 to 20 bar:**

**First, compute the Parametric Table:**

| 1..16 | P2 [kPa] | $Qa_{by,m}$ [kJ/kg] | $Wnet_{by,m}$ [kJ/kg] | COP |
|---|---|---|---|---|
| Run 1 | 500 | 30.55 | 164 | 0.1863 |
| Run 2 | 600 | 34.13 | 190.3 | 0.1793 |
| Run 3 | 700 | 36.98 | 212.8 | 0.1738 |
| Run 4 | 800 | 39.32 | 232.4 | 0.1692 |
| Run 5 | 900 | 41.3 | 249.8 | 0.1653 |
| Run 6 | 1,000 | 43 | 265.4 | 0.162 |
| Run 7 | 1,100 | 44.49 | 279.7 | 0.159 |
| Run 8 | 1,200 | 45.81 | 292.8 | 0.1565 |
| Run 9 | 1,300 | 46.99 | 304.9 | 0.1541 |
| Run 10 | 1,400 | 48.05 | 316.1 | 0.152 |
| Run 11 | 1,500 | 49.03 | 326.6 | 0.1501 |
| Run 12 | 1,600 | 49.92 | 336.4 | 0.1484 |
| Run 13 | 1,700 | 50.74 | 345.7 | 0.1468 |
| Run 14 | 1,800 | 51.5 | 354.5 | 0.1453 |
| Run 15 | 1,900 | 52.21 | 362.9 | 0.1439 |
| Run 16 | 2,000 | 52.88 | 370.8 | 0.1426 |

**Now, plot:**


Qa_by,m vs P2 for Solvay cycle refrigerator with Neon
Qa_by,m [kJ/kg]
P2 [kPa]
55
50
45
40
35
30
400
600
800
1,000
1,200
1,400
1,600
1,800
2,000

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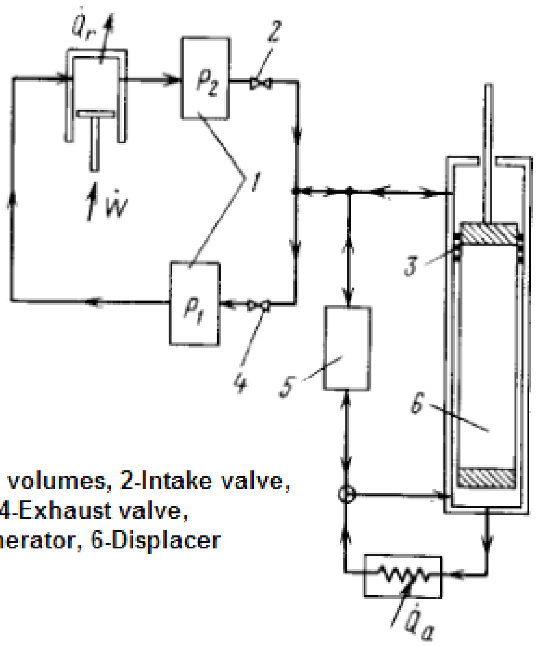
**Prob. 4.3.20** Write an EES Procedure to find the various parameters for a Gifford-McMahon (G-M) cycle refrigerator, using a given working gas.

A G-M refrigerator operates between the pressure limits of 1.013 bar and 10.13 bar using helium as working medium. The max. temp in the space to be cooled is 70 K and the temp of the gas leaving the compressor is 300 K. Assume that regenerator efficiency is 100%, compressor overall efficiency = 60%, adiabatic efficiency of expansion in expansion space = 90%. Determine the refrigeration effect, the COP and the FOM for the system. [1]

Also, plot:

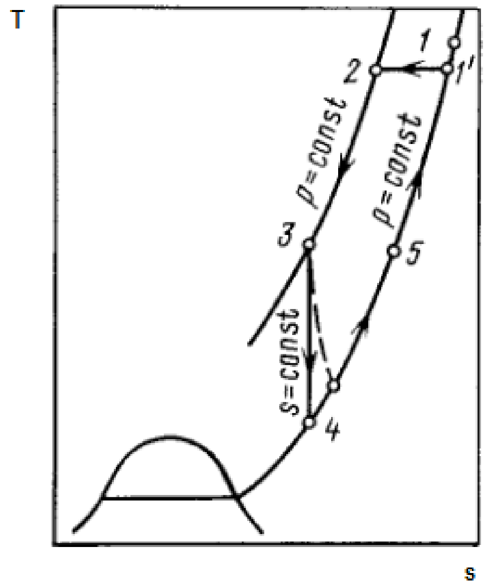
a) refrig. effect vs regen. effcy for T5 = 70, 100 and 120 K
b) the effect on COP and FOM by varying regen. effectiveness, for compressor efficiencies of 0.6, 0.8 and 1.

G-M cycle refrigerator.




1-Surge volumes, 2-Intake valve,
3-Seal, 4-Exhaust valve,
5-Regenerator, 6-Displacer

T-s diagram




**EES Procedure:**

PROCEDURE GM_cycle_refrig(Fluid$, P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen: T1_prime, T3, T4_prime, v3,v4_prime, Qa_by_m, W_by_m, COP, COP_i, FOM)

"Gives various parameters for a GM cycle refrign. system for given working fluid"

"Inputs:
Fluid$, Low and High pressures:P1, P2 (kPa),
T2 = Temp at exit of after cooler of compressor (K),
T5 = refrign. termp. (K),
eta_co = compressor overall effcy,
eta_ad = adiab. effcy of expn. space,
epsilon_regen = regenerator effcy."

"Outputs:
T1_prime = temp of gas exiting the refrigerator
T3 = temp of high pressure gas at the exit of regenerator in to the expansion space
T4_prime = temp of gas after expansion in the expansion space, taking in to account adiabatic effcy of expansion
v3 = sp. vol. at state 3; v4_prime = sp. vol. at state 4_prime

Qa_by_m = refrig effect per kg gas compressed; W_by_m =work done per kg of gas compressed
COP = coeff of performance; COP_i = ideal COP for refrign at constant pressure
FOM = figure of merit for the cycle"

h2:= ENTHALPY(Fluid$, T=T2, P=P2)
s2 := ENTROPY(Fluid$, T=T2, P=P2)

h5:= ENTHALPY(Fluid$, T=T5, P=P1)

T3 := T5

h3:= ENTHALPY(Fluid$, T=T3, P=P2)

s3 := ENTROPY(Fluid$, T=T3, P=P2)

s4 := s3

h4:= ENTHALPY(Fluid$, s=s4, P=P1)

```
T4 :=Temperature(Fluid$,P=P1,h=h4)

h4_prime := h4 + (1 – eta_ad) * (h3 – h4)

T4_prime :=Temperature(Fluid$,P=P1,h=h4_prime)

v3 := Volume(Fluid$,T=T3,P=P2)

v4_prime := Volume(Fluid$,T=T4_prime,P=P1)

DELTA_Q := (1 – epsilon_regen) * (h2 – h3)

Qa_by_m := (v3 / v4_prime) * (h5 – h4_prime) – DELTA_Q

h1_prime := h2 + Qa_by_m

T1_prime := Temperature(Fluid$,P=P1,h=h1_prime)

h1 := ENTHALPY(Fluid$, T=T2, P=P1)

s1 := ENTROPY(Fluid$, T=T2, P=P1)

W_by_m := (T2 * (s1 – s2) – (h1 – h2)) / eta_co

COP := Qa_by_m / W_by_m

COP_i := (T5 – T4_prime) / (T2 * ln(T5 / T4_prime) – (T5 – T4_prime))

FOM := COP / COP_i

END

"============================================================"
```

**Now, solve the problem:**

```
P1 = 101.3 [kPa]

P2 = 1013 [kPa]
```

T2 = 300 [K]

T5 = 70 [K]

eta_co = 0.6

eta_ad = 0.9

epsilon_regen = 1

Fluid$ = 'Helium'

CALL GM_cycle_refrig(Fluid$, P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen : T1_prime, T3, T4_prime, v3,v4_prime, Qa_by_m, W_by_m, COP, COP_i, FOM)

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| COP = 0.01824 | $COP_i$ = 0.1934 | $\varepsilon_{regen}$ = 1 |
| $\eta_{ad}$ = 0.9 | $\eta_{co}$ = 0.6 | Fluid$ = 'Helium' |
| FOM = 0.09429 | P1 = 101.3 [kPa] | P2 = 1,013 [kPa] |
| $Qa_{by,m}$ = 43.7 [kJ/kg] | T1' = 309 [K] | T2 = 300 [K] |
| T3 = 70 [K] | T4' = 32.12 [K] | T5 = 70 [K] |
| v3 = 0.1464 [$m^3$/kg] | v4' = 0.6603 [$m^3$/kg] | $W_{by,m}$ = 2,396 [kJ/kg] |

**Thus:**

**Refrig. effect = Qa_by_m = 43.7 kJ/kg … Ans.**

**COP = 0.01824 … Ans.**

**FOM = 0.09429 … Ans.**

**Also, plot:**

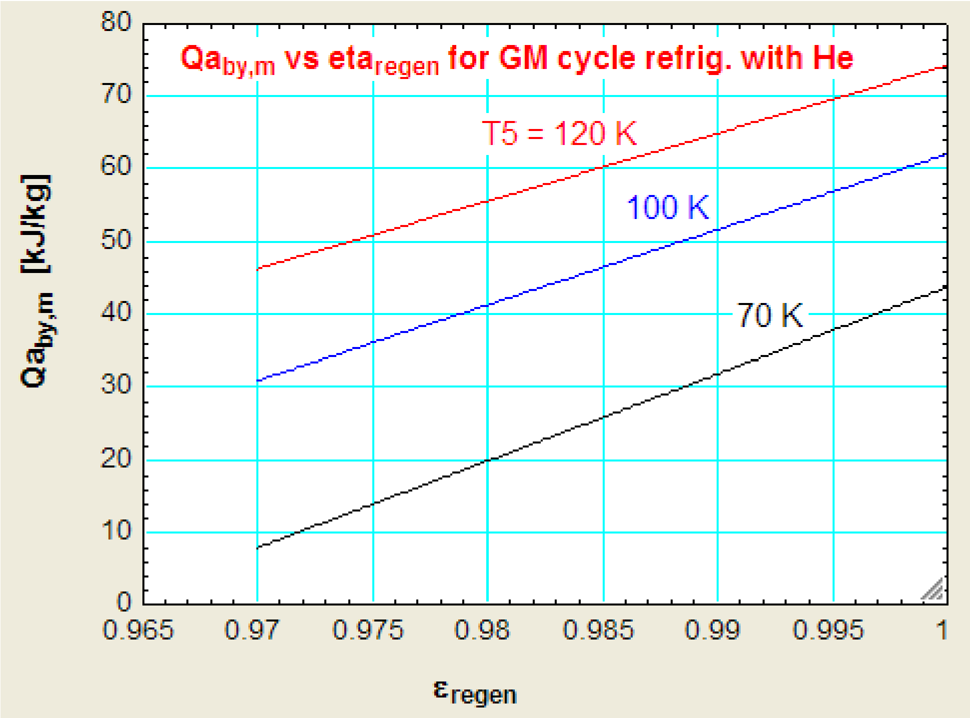
a) **refrig. effect vs regen. effcy for T5 = 70, 100 and 120 K**
b) **the effect on COP and FOM by varying regen. effectiveness, for compressor efficiencies of 0.6, 0.8 and 1:**

a) **refrig. effect vs regen. effcy for T5 = 70, 100 and 120 K:**

**Parametric Table:**

| | T5 = 70 K | 100 K | 120 K |
|---|---|---|---|
| $e_{regen}$ | $Qa_{by,m}$ [kJ/kg] | $Qa_{by,m}$ [kJ/kg] | $Qa_{by,m}$ [kJ/kg] |
| 0.97 | 7.835 | 30.85 | 46.2 |
| 0.975 | 13.81 | 36.04 | 50.88 |
| 0.98 | 19.79 | 41.24 | 55.55 |
| 0.985 | 25.77 | 46.43 | 60.23 |
| 0.99 | 31.74 | 51.63 | 64.9 |
| 0.995 | 37.72 | 56.82 | 69.58 |
| 1 | 43.7 | 62.02 | 74.25 |

**Plot:**

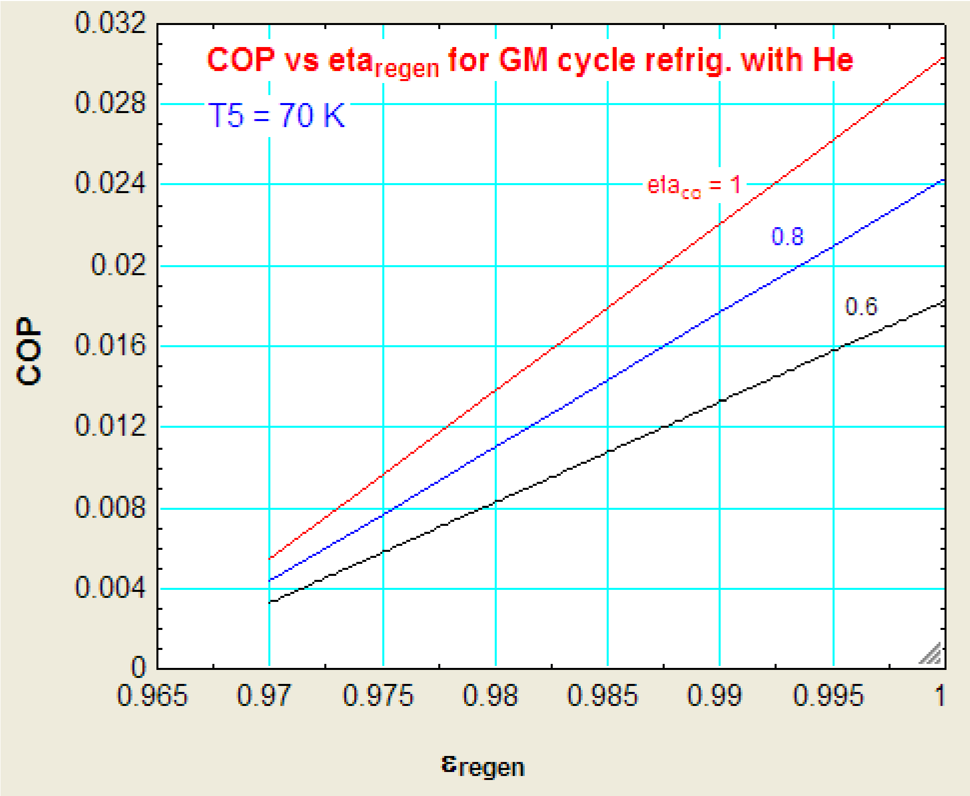


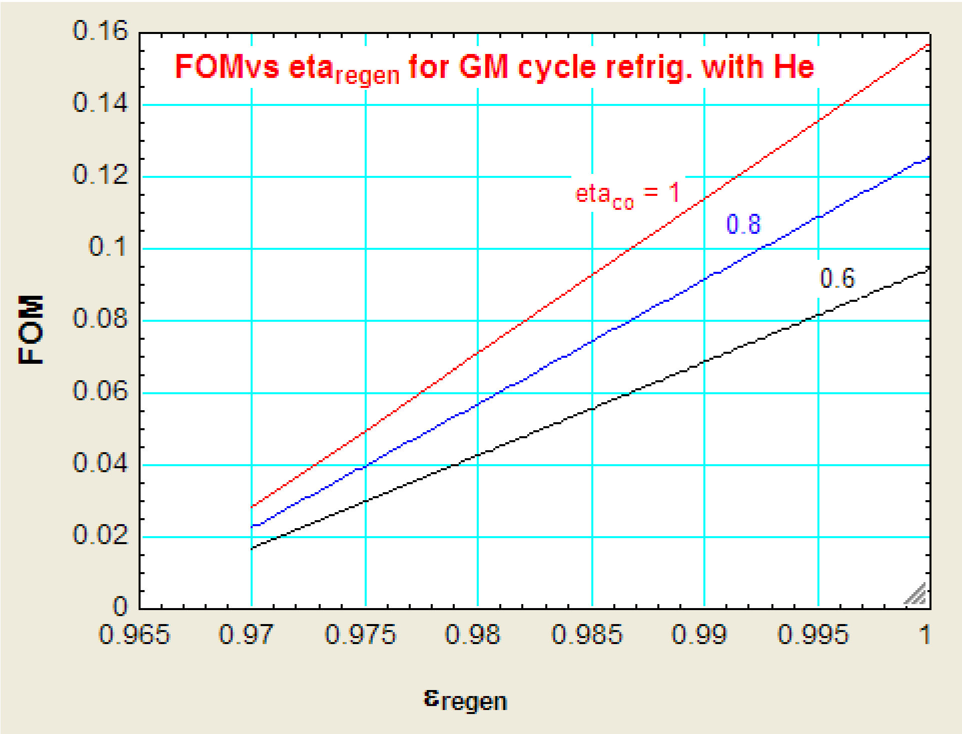

b) **the effect on COP and FOM by varying regen. effectiveness, for compressor efficiencies of 0.6, 0.8 and 1:**

**Parametric Table:**

| | eta_co = 0.6 | 0.6 | 0.8 | 0.8 | 1 | 1 |
|---|---|---|---|---|---|---|
| $\varepsilon_{regen}$ | COP | FOM | COP | FOM | COP | FOM |
| 0.97 | 0.00327 | 0.01691 | 0.004361 | 0.02254 | 0.005451 | 0.02818 |
| 0.975 | 0.005765 | 0.0298 | 0.007687 | 0.03974 | 0.009609 | 0.04967 |
| 0.98 | 0.00826 | 0.0427 | 0.01101 | 0.05694 | 0.01377 | 0.07117 |
| 0.985 | 0.01075 | 0.0556 | 0.01434 | 0.07413 | 0.01792 | 0.09266 |
| 0.99 | 0.01325 | 0.0685 | 0.01767 | 0.09133 | 0.02208 | 0.1142 |
| 0.995 | 0.01574 | 0.08139 | 0.02099 | 0.1085 | 0.02624 | 0.1357 |
| 1 | 0.01824 | 0.09429 | 0.02432 | 0.1257 | 0.0304 | 0.1572 |

**Plot:**


COP vs eta$_{regen}$ for GM cycle refrig. with He
T5 = 70 K
eta$_{co}$ = 1
0.8
0.6
COP
0.032
0.028
0.024
0.02
0.016
0.012
0.008
0.004
0
0.965
0.97
0.975
0.98
0.985
0.99
0.995
1
εregen


FOMvs eta$_{regen}$ for GM cycle refrig. with He
eta$_{co}$ = 1
0.8
0.6
FOM
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.965
0.97
0.975
0.98
0.985
0.99
0.995
1
εregen

**Prob. 4.3.21** Write a EES Procedure to determine the effectiveness of a practical regenerator of a GM cycle refrigerator, consisting of copper wire screen matrix housed in a perspex displacer, when a given fluid is working gas.

**Frst, we need the porosity of the regenerator.**

This is calculated knowing the physical dimensions of the regenerator and the mass of copper screens.

porosity = (Total vol. - metal vol.)/Total vol.

i.e.

$$p := \frac{\frac{\pi \cdot DR^2}{4} \cdot LR - \frac{m\_cu}{\rho\_cu}}{\frac{\pi \cdot DR^2}{4} \cdot LR}$$

**Now, write the EES Procedure:**

PROCEDURE Regen_GM(Fluid$, PH, PL, TH, TL, N, d_i, d_0, l_cyl, d_disp, l_disp, LR, DR, dw, p : Q_ideal, m_fluid, A_ht, Re, h, NTU, I, f, DELTAP, Qloss_reg)

"Gives various parameters for a regenerator of GM cycle refrign. system for given working fluid"

"Inputs:
Fluid$, Low and High pressures:PH, PL (kPa), TH = High Temp (K), , TL = Low termp. (K), d_i = cylinder ID, d_0 = cyl. OD, l_cyl = cyl. length, d_disp = dia of displacer, l_disp = length of displacer, LR = length of regen, DR = dia of regen., dw = wire dia, (m), p = porosity, as defined."

"Outputs:
Q_ideal = ideal refrign., m_fluid = mass flow rate of fluid (kg/s), A_ht = heat transfer area of mesh (m^2), Re = Reynolds No., h = heat transfer coeff.(kW/m^2.K), NTU = No. of Transfer Units, I = regen. ineffcy., f = friction factor., DELTAP = pressure drop (Pa), Qloss_reg = regen. heat loss (kW)"

MW := MolarMass(Fluid$) "…Mol. wt. of fluid"

R_u := 8.314 "kJ/kg-K … universal gas const."

R_fluid := R_u / MW "kJ/kg-K … Particular gas const."

cp_fluid :=Cp(Fluid$,T=TH,P=PH)"kJ/kg-K"

cv_fluid :=Cv(Fluid$,T=TH,P=PH)"kJ/kg-K"

gamma := cp_fluid / cv_fluid "… ratio of sp. heats"

rho_cu := 8874 "kg/m^3 … density of copper"

T_m = (TH + TL) / 2 "mean temp., K"

mu_fluid := Viscosity(Fluid$,T=T_m,P=PL) "… viscosity of fluid, kg/m.s"

A_ht := (4 * (1 – p) / dw) * pi * DR^2 * LR / 4 "… heat transfer area" d_eq := 4 * LR * p * pi * DR^2 / (4 * A_ht) "m … Equiv. dia"

V_stroke := (pi * d_i^2 / 4) * (l_cyl – l_disp) "… stroke vol."

m_fluid :=((PH * V_stroke) / (R_fluid * TL)) * (N / 60) "kg/s …. Fluid flow rate "

G := m_fluid / (p * pi * DR^2 / 4)

Re := G * d_eq / mu_fluid"…Reynolds No."

Pr := Prandtl(Fluid$,T=T_m,P=PL) "…Prandtl No."

St := 0.68 * (Re^(-0.407)) * Pr^(-2/3)"… Stanton No."

cp := Cp(Fluid$,T=T_m,P=PL)"kJ/kg-K"

h := St * G * cp "…heat transfer coeff."

NTU := h * A_ht / (2 * cp * m_fluid)

m_s := (pi * DR^2 / 4) * LR * (1 – p) * rho_cu "….mass of solid, i.e. copper"

tau := (1 / (N / 60)) * (1/2) "s… duration of each blow (hot or cold)"

CR := m_s * c_('Copper', T_m) / (cp * m_fluid * tau) "….capacity rate ratio"

DELTAT := m_fluid * (TH – TL) * tau * cp / (m_s * c_('Copper', T_m)) "…. matrix temp swing"

I := 1 / (1 + NTU) "…. Regen. Inefficiency"

Qloss_reg := I * m_fluid * (TH – TL) * cp / 2 "Rgen. Heat loss… factor 2 in the denominator, since loss occurs only during the pressurising period"

f := (860 * (1 – p) / Re) + (2.2 * p / Re^0.1) "… friction factor"

rho_fluid := Density(Fluid$,T=T_m,P=PH) "… density of fluid"

Q_ideal := (PH – PL) * V_stroke * N / 60 "kW …. Ideal refrign. produced"

DELTAP := (f * LR / d_eq) * (G^2 / (2* rho_fluid)) "….pressure drop"

END

"==============================================================="

**Prob. 4.3.22** In a practical GM cycle refrigerator using Helium, following are the data. Apply the EES Procedure written above and determine the various parameters of interest.

**Data:**

PH = 900 kPa; PL = 101.3 kPa; N = 130 RPM; TH = 310 K; TL = 80 K;

d_i = 0.0486 m; d_0 = 0.05 m; l_cyl = 0.24 m; d_disp = 0.0473 m; l_disp = 0.22 m;

LR = 0.101 m; DR = 0.042 m; dw = 8E-05 m; p = 0.749 …porosity.

Fluid$ = 'Helium'

**EES Solution:**

**Applying the EES Procedure written above:**

CALL Regen_GM(Fluid$, PH, PL, TH, TL, N, d_i, d_0, l_cyl, d_disp, l_disp, LR, DR, dw, p : Q_ideal, m_fluid, A_ht, Re, h, NTU, I, f, DELTAP, Qloss_reg)

**Results:**

Unit Settings: SI K kPa kJ mass deg

| | | |
|---|---|---|
| $A_{ht}$ = 1.756 [m^2] | ΔP = 564.3 [Pa] | DR = 0.042 [m] |
| dw = 0.00008 [m] | $d_0$ = 0.05 [m] | $d_{disp}$ = 0.0473 [m] |
| $d_i$ = 0.0486 [m] | f = 33.45 | Fluid$ = 'Helium' |
| h = 0.8936 [kW/m^2-K] | I = 0.002873 | LR = 0.101 [m] |
| $l_{cyl}$ = 0.24 [m] | $l_{disp}$ = 0.22 [m] | $m_{fluid}$ = 0.0004354 [kg/s] |
| N = 130 [RPM] | NTU = 347 | p = 0.749 |
| PH = 900 [kPa] | PL = 101.3 [kPa] | $Qloss_{reg}$ = 0.0007471 [kW] |
| $Q_{ideal}$ = 0.0642 [kW] | Re = 6.728 | TH = 310 [K] |
| TL = 80 [K] | | |

**Thus:**

**Ideal refrign. = Q_ideal = 0.0642 kW = 64.2 W … Ans.**

**Overall heat transfer coeff. in regenerator = h = 0.8936 kW/m^2.K = 893.6 W/m^2.K … Ans.**

**Regen. ineffcy = I = 0.002873 = 0.2873% …. Ans.**

**Regen. heat loss = Qloss_reg = 0.0007471 kW = 0.7471 W … Ans.**

## Other details of calculations:

**Local variables in Procedure Regen_GM (1 call, 0.05 sec)**

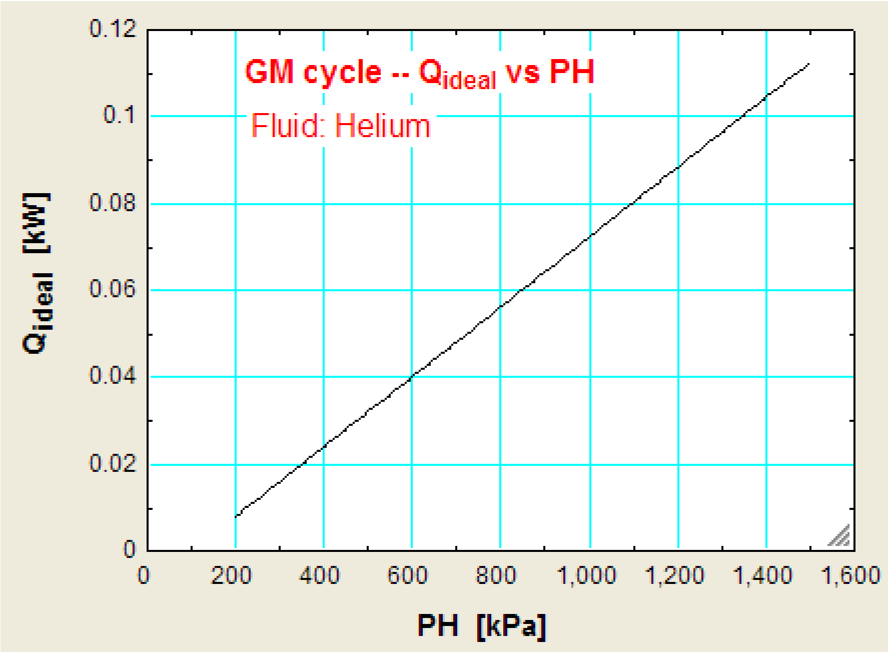
| | | |
|---|---|---|
| $A_{ht}$ =1.756 [m$^2$] | cp=5.193 [kJ/kg-K] | $cp_{fluid}$=5.192 [kJ/kg-K] |
| $cv_{fluid}$=3.117 [kJ/kg-K] | $\Delta$P =564.3 [Pa] | $\Delta$T =1.098 [K] |
| dw =0.00008 [m] | $d_0$=0.05 [m] | $d_{disp}$=0.0473 [m] |
| $d_i$ =0.0486 [m] | f =33.45 | Fluid$='Helium' |
| $\gamma$=1.666 | h=0.8936 [kW/m$^2$-K] | I =0.002873 |
| $l_{cyl}$ =0.24 [m] | $l_{disp}$=0.22 [m] | $\mu_{fluid}$=0.00001489 [kg/m-s] |
| $m_{fluid}$ =0.0004354 [kg/s] | $m_s$ =0.3117 [kg] | N =130 [RPM] |
| p=0.749 | PH =900 [kPa] | PL =101.3 [kPa] |
| $Qloss_{reg}$=0.0007471 [kW] | $Q_{ideal}$ =0.0642 [kW] | Re =6.728 |
| $\rho_{fluid}$=2.207 [kg/m$^3$] | $R_{fluid}$=2.077 [kJ/kg-K] | $R_u$ =8.314 [kJ/kgmol-K] |
| $\tau$=0.2308 [s] | TH =310 [K] | TL =80 [K] |
| $V_{stroke}$ =0.0000371 [m$^3$] | | |

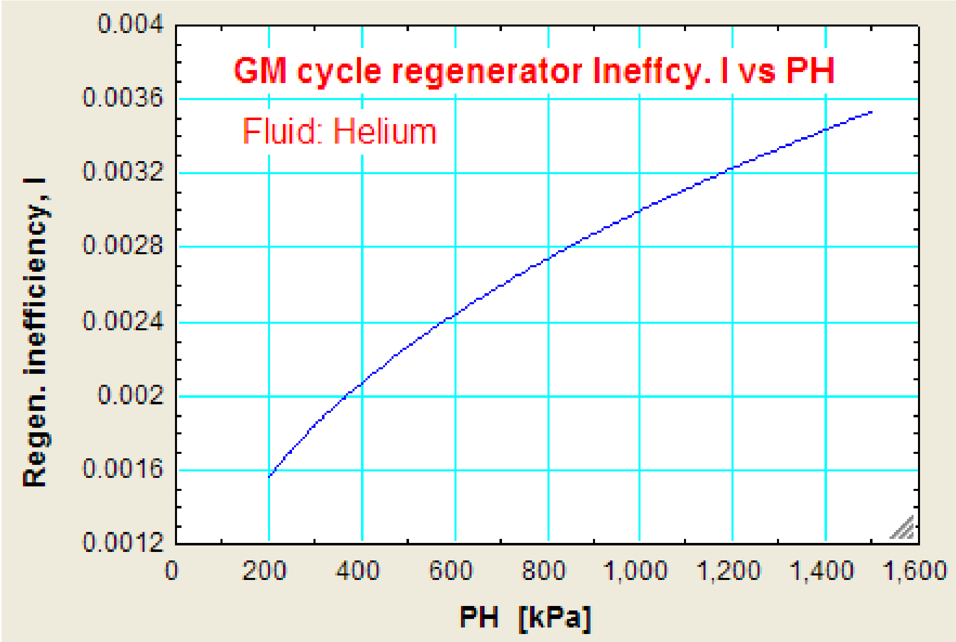
## Effect of changing PH:

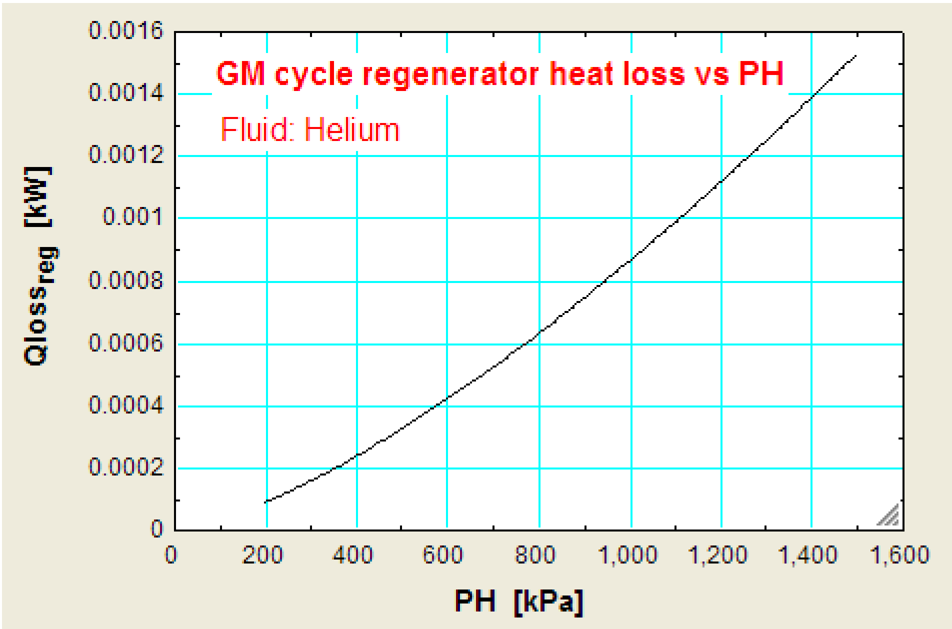
## Parametric Table:

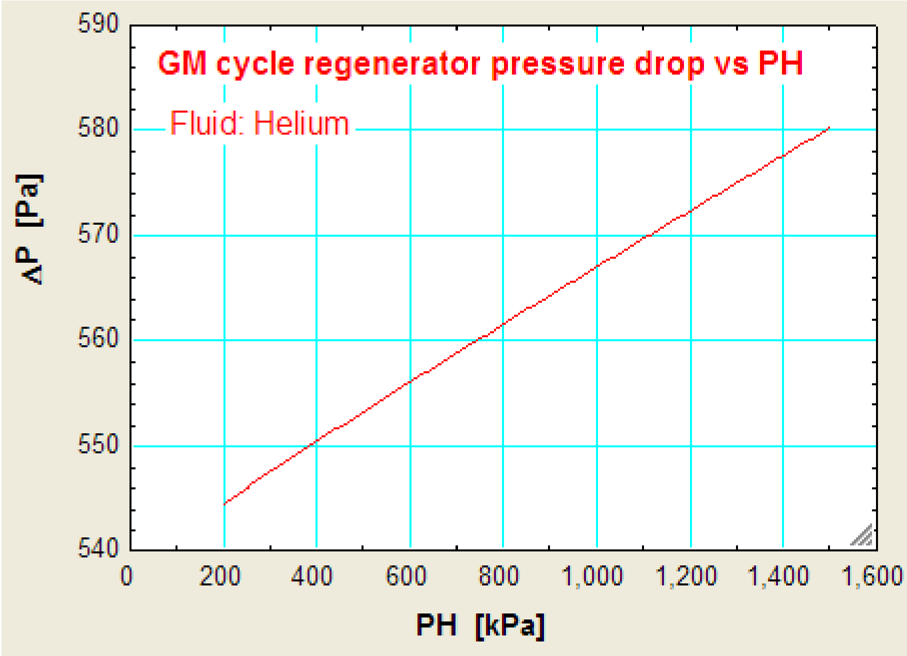
| 1..14 | PH [kPa] | $Q_{ideal}$ [kW] | I | $Qloss_{reg}$ [kW] | $\Delta$P [Pa] |
|---|---|---|---|---|---|
| Run 1 | 200 | 0.007934 | 0.00156 | 0.00009013 | 544.4 |
| Run 2 | 300 | 0.01597 | 0.001839 | 0.0001594 | 547.4 |
| Run 3 | 400 | 0.02401 | 0.002067 | 0.0002389 | 550.3 |
| Run 4 | 500 | 0.03205 | 0.002263 | 0.0003269 | 553.2 |
| Run 5 | 600 | 0.04009 | 0.002437 | 0.0004225 | 556 |
| Run 6 | 700 | 0.04813 | 0.002595 | 0.0005247 | 558.8 |
| Run 7 | 800 | 0.05617 | 0.002739 | 0.0006331 | 561.6 |
| Run 8 | 900 | 0.0642 | 0.002873 | 0.0007471 | 564.3 |
| Run 9 | 1,000 | 0.07224 | 0.002999 | 0.0008664 | 567 |
| Run 10 | 1,100 | 0.08028 | 0.003117 | 0.0009906 | 569.7 |
| Run 11 | 1,200 | 0.08832 | 0.003229 | 0.001119 | 572.3 |
| Run 12 | 1,300 | 0.09636 | 0.003336 | 0.001253 | 575 |
| Run 13 | 1,400 | 0.1044 | 0.003437 | 0.00139 | 577.6 |
| Run 14 | 1,500 | 0.1124 | 0.003535 | 0.001532 | 580.2 |

**Plots:**


GM cycle -- Qideal vs PH
Fluid: Helium
Qideal [kW]
PH [kPa]
0.12
0.1
0.08
0.06
0.04
0.02
0
0
200
400
600
800
1,000
1,200
1,400
1,600


GM cycle regenerator Ineffcy. I vs PH
Fluid: Helium
Regen. inefficiency, I
0.004
0.0036
0.0032
0.0028
0.0024
0.002
0.0016
0.0012
0
200
400
600
800
1,000
1,200
1,400
1,600
PH [kPa]


GM cycle regenerator heat loss vs PH
Fluid: Helium
Qlossreg [kW]
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
0
200
400
600
800
1,000
1,200
1,400
1,600
PH [kPa]

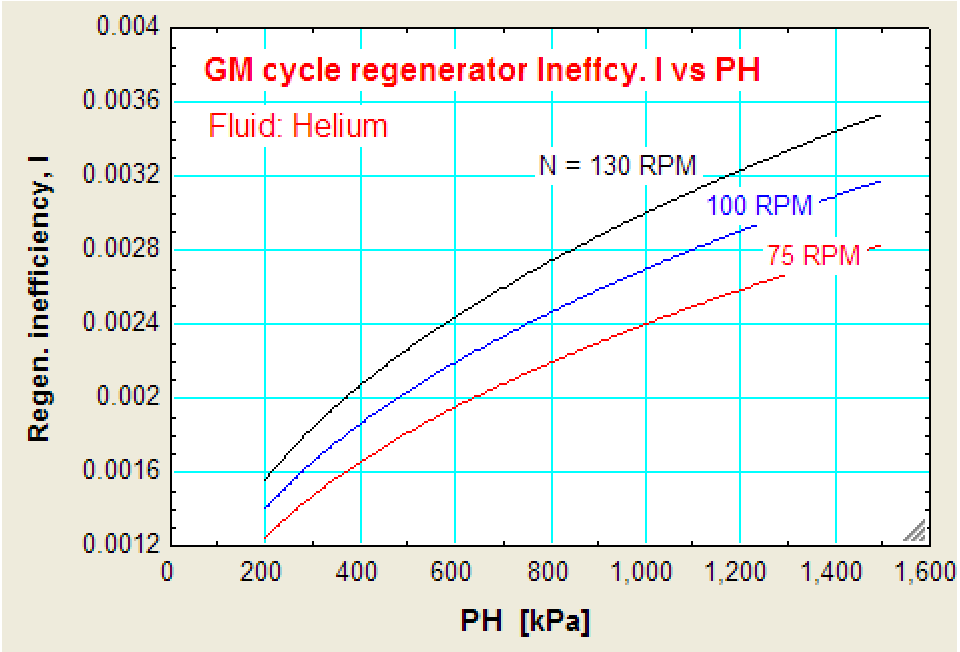



**Plot Regen. Ineffcy I vs PH for Speeds N = 130, 100 and 75 RPM:**

**Parametric Table:**

| PH [kPa] | N = 130 RPM I | 100 RPM I | 75 RPM I |
|---|---|---|---|
| 200 | 0.00156 | 0.001402 | 0.001247 |
| 300 | 0.001839 | 0.001653 | 0.001471 |
| 400 | 0.002067 | 0.001858 | 0.001653 |
| 500 | 0.002263 | 0.002035 | 0.00181 |
| 600 | 0.002437 | 0.002191 | 0.001949 |
| 700 | 0.002595 | 0.002332 | 0.002075 |
| 800 | 0.002739 | 0.002462 | 0.002191 |
| 900 | 0.002873 | 0.002583 | 0.002298 |
| 1,000 | 0.002999 | 0.002696 | 0.002399 |
| 1,100 | 0.003117 | 0.002802 | 0.002493 |
| 1,200 | 0.003229 | 0.002903 | 0.002583 |
| 1,300 | 0.003336 | 0.002999 | 0.002668 |
| 1,400 | 0.003437 | 0.00309 | 0.00275 |
| 1,500 | 0.003535 | 0.003178 | 0.002828 |

**Plot:**


GM cycle regenerator Ineffcy. I vs PH
Fluid: Helium
N = 130 RPM
100 RPM
75 RPM
Regen. inefficiency, I
PH [kPa]
0.004
0.0036
0.0032
0.0028
0.0024
0.002
0.0016
0.0012
0
200
400
600
800
1,000
1,200
1,400
1,600

**Prob. 4.3.23:** Write a EES Function to find out the effectiveness of a regenerator by Λ-π method.

We shall use the built-in 2D Interpolation Function INTERPOLATE2DM in EES.

First, copy the Table of Effcy values for the regenerator for different values of Λ and ∏, given in Barron, in to a Lookout Table in EES, and name that Table as: 'Regen-Lambda_PI'. See below:

Regen-Lambda_PI

| | Λ | Effcy(PI = 0) | Effcy(PI = 5) | Effcy(PI = 10) | Effcy(PI = 15) | Effcy(PI = 20) | Effcy(PI = 25) | Effcy(PI = 30) | Effcy(PI = 35) | Effcy(PI = 40) | Effcy(PI = 45) | Effcy(PI = 50) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Row 1 | | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| Row 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Row 3 | 5 | 0.7143 | 0.639 | 0.469 | 0.326 | 0.25 | 0.202 | 0.167 | 0.143 | 0.125 | 0.112 | 0.1 |
| Row 4 | 10 | 0.8333 | 0.809 | 0.738 | 0.61 | 0.494 | 0.402 | 0.333 | 0.284 | 0.25 | 0.222 | 0.2 |
| Row 5 | 15 | 0.8824 | 0.866 | 0.84 | 0.784 | 0.693 | 0.585 | 0.498 | 0.43 | 0.375 | 0.332 | 0.3 |
| Row 6 | 20 | 0.9091 | 0.9 | 0.886 | 0.856 | 0.811 | 0.756 | 0.651 | 0.578 | 0.5 | 0.443 | 0.4 |
| Row 7 | 25 | 0.9259 | 0.92 | 0.911 | 0.895 | 0.871 | 0.827 | 0.77 | 0.698 | 0.62 | 0.555 | 0.5 |
| Row 8 | 30 | 0.9375 | 0.933 | 0.927 | 0.918 | 0.903 | 0.881 | 0.845 | 0.797 | 0.727 | 0.654 | 0.598 |
| Row 9 | 35 | 0.9459 | 0.943 | 0.939 | 0.932 | 0.922 | 0.907 | 0.888 | 0.856 | 0.81 | 0.754 | 0.692 |
| Row 10 | 40 | 0.9524 | 0.95 | 0.947 | 0.942 | 0.935 | 0.925 | 0.912 | 0.891 | 0.865 | 0.82 | 0.773 |
| Row 11 | 45 | 0.9574 | 0.955 | 0.953 | 0.949 | 0.944 | 0.938 | 0.928 | 0.914 | 0.898 | 0.871 | 0.835 |
| Row 12 | 50 | 0.9615 | 0.961 | 0.958 | 0.955 | 0.951 | 0.946 | 0.939 | 0.929 | 0.915 | 0.897 | 0.875 |
| Row 13 | 100 | 0.9804 | 0.98 | 0.979 | 0.979 | 0.978 | 0.977 | 0.976 | 0.975 | 0.973 | 0.972 | 0.97 |
| Row 14 | 200 | 0.9901 | 0.99 | 0.989 | 0.989 | 0.989 | 0.989 | 0.989 | 0.988 | 0.988 | 0.988 | 0.988 |

Now, write the Function to get Effcy of regenerator when PI and LAMBDA are given, using the built-in INTERPOLATE2DM function in EES:

```
FUNCTION RegenEffcyLambdaPAI(PAI, LAMBDA)

   IF ((PAI < 0) OR (PAI > 50)) THEN

      CALL ERROR ('PAI should be between 0 and 50!')

   ENDIF

   IF ((LAMBDA < 0) OR (LAMBDA> 200)) THEN

      CALL ERROR ('LAMBDA should be between 0 and 200!')

   ENDIF

RegenEffcyLambdaPAI = INTERPOLATE2DM('Regen-Lambda_PI', PAI, LAMBDA)

END

"====================================================================="
```

**Prob. 4.3.24:** A regenerator is constructed of a matrix material with sp. heat = 1 kJ/kg.K, and a mass of 5.5 kg. The fluid flowing through the regenerator is a gas with sp. heat of 1.005 kJ/kg.K and a mass flow rate of 32 g/s for both heating and cooling periods. The heat transfer coeff for both the cooling and heating periods is 402 W/m^2.K and the surface area is 2.00 m^2. The heating and cooling periods are 2.5 minutes, each. Find the effectiveness of regenerator using the Lambda-PI method. [1]

**EES Solution:**

"Data:"

ms = 5.5 [kg] "…mass of solid"

cs = 1 [kJ/kg-K] "…. sp. heat of solid"

cpgas = 1.005 [kJ/kg-K] "… sp. heat of gas"

mdot = 0.032 [kg/s] "…mass flow rate of helium gas"

P = 2.5 * 60 "s…heating and cooling periods"

hc = 0.402[kW/m^2-K] "…heat transfer coeff."

As = 2.00 [m^2] "..surface area"

"Calculations:"

LAMBDA = hc * As / (mdot * cpgas) "…dimensionless length"

PAI = hc * As * P / (ms * cs) "…dimensionless period"

"Now, calculate the effcy by calling the Function:"

epsilon = RegenEffcyLambdaPAI(PAI, LAMBDA)

**Results:**

Unit Settings: SI K kPa kJ mass deg

| | | |
|---|---|---|
| As = 2 [m^2] | cpgas = 1.005 [kJ/kg-K] | cs = 1 [kJ/kg-K] |
| ε = 0.854 | hc = 0.402 [kW/m^2-K] | Λ = 25.0000 |
| mdot = 0.032 [kg/s] | ms = 5.5 [kg] | P = 150 [s] |
| PAI = 21.93 | | |

**Thus:**

**PAI = 21.93**

**LAMBDA = 25.0**

**Effcy = 0.854 = 85.4% … Ans.**

---

**Prob.4.3.25** Write a EES Function to calculate the effectiveness of a regenerator by NTU-ε method, when CR = (Cmin/Cmax) = 1 (i.e. balanced regenerator).

**We have:**

$$NTU_0 := \frac{1}{C_{min}} \cdot \left( \frac{1}{hh \cdot Ah} + \frac{1}{hc \cdot Ac} \right)^{-1}$$

$$C_R := \frac{C_{min}}{C_{max}}$$

$$C_m := \frac{ms \cdot cs}{P0 \cdot C_{min}}$$

$P_0 = P_h + P_c$

$f = 1/ P_0$

$\varepsilon = Q\_dot /[C_{min}. (T_{h1} - T_{c1})]$

**When CR = 1:**

**Effcy values are given in Table 5.2 of Ref.[1] for a balanced regenerator (i.e. CR = 1), as a function of Cm and NTU_0.**

**When CR is not equal to 1:**

**First, find the 'effective values' of NTU and Cm:**

$$NTU_e := \frac{2 \cdot NTU_0 \cdot C_R}{(1 + C_R)}$$

$$C_{me} := \frac{2 \cdot C_m \cdot C_R}{(1 + C_R)}$$

**Using these effective values, find the effectiveness (ε1) for a 'balanced' regenerator from Table.**

**Then, determine following parameter 'X':**

$$X := \frac{1 + C_R}{2 \cdot C_R} \cdot \frac{\varepsilon_1}{1 - \varepsilon_1} \cdot (1 - C_R)$$

**Finally, regenerator effectiveness is calculated as:**

$$\varepsilon := \frac{1 - \exp(-X)}{1 - C_R \cdot \exp(-X)}$$

**Now, copy the Table 5.2 in Ref.[1], in to a EES Lookout Table, with the Title: 'BalancedRegen_Effcy_NTU' as shown below:**

| $NTU_0$ | EFFCY-Cm = 0.8 | EFFCY-Cm = 1 | EFFCY-Cm = 1.25 | EFFCY-Cm = 1.5 | EFFCY-Cm = 2 | EFFCY-Cm = 3 | EFFCY-Cm = 5 | EFFCY-Cm = 1000000 |
|---|---|---|---|---|---|---|---|---|
| | 0.8 | 1 | 1.25 | 1.5 | 2 | 3 | 5 | 1,000,000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.5 | 0.315 | 0.322 | 0.326 | 0.328 | 0.33 | 0.332 | 0.333 | 0.3333 |
| 1 | 0.449 | 0.467 | 0.478 | 0.485 | 0.491 | 0.496 | 0.499 | 0.5 |
| 1.5 | 0.521 | 0.548 | 0.566 | 0.576 | 0.586 | 0.594 | 0.598 | 0.6 |
| 2 | 0.566 | 0.601 | 0.623 | 0.636 | 0.649 | 0.659 | 0.664 | 0.6667 |
| 2.5 | 0.599 | 0.639 | 0.664 | 0.679 | 0.694 | 0.705 | 0.711 | 0.7143 |
| 3 | 0.622 | 0.667 | 0.696 | 0.712 | 0.728 | 0.74 | 0.746 | 0.75 |
| 3.5 | 0.642 | 0.69 | 0.721 | 0.738 | 0.755 | 0.767 | 0.774 | 0.7778 |
| 4 | 0.659 | 0.709 | 0.741 | 0.759 | 0.776 | 0.789 | 0.796 | 0.8 |
| 4.5 | 0.673 | 0.724 | 0.758 | 0.776 | 0.794 | 0.807 | 0.814 | 0.8182 |
| 5 | 0.685 | 0.738 | 0.772 | 0.791 | 0.809 | 0.822 | 0.829 | 0.8333 |
| 5.5 | 0.696 | 0.749 | 0.785 | 0.803 | 0.821 | 0.834 | 0.842 | 0.8462 |
| 6 | 0.705 | 0.759 | 0.796 | 0.814 | 0.832 | 0.845 | 0.853 | 0.8571 |
| 6.5 | 0.713 | 0.768 | 0.805 | 0.824 | 0.842 | 0.855 | 0.862 | 0.8667 |
| 7 | 0.721 | 0.776 | 0.814 | 0.833 | 0.85 | 0.863 | 0.87 | 0.875 |
| 7.5 | 0.728 | 0.784 | 0.822 | 0.84 | 0.858 | 0.871 | 0.878 | 0.8824 |
| 8 | 0.734 | 0.79 | 0.829 | 0.847 | 0.865 | 0.877 | 0.884 | 0.8889 |
| 8.5 | 0.74 | 0.796 | 0.835 | 0.854 | 0.871 | 0.883 | 0.89 | 0.8947 |
| 9 | 0.746 | 0.802 | 0.841 | 0.859 | 0.876 | 0.888 | 0.895 | 0.9 |
| 9.5 | 0.751 | 0.807 | 0.846 | 0.864 | 0.881 | 0.893 | 0.9 | 0.9048 |
| 10 | 0.756 | 0.811 | 0.851 | 0.869 | 0.886 | 0.898 | 0.904 | 0.9091 |
| 20 | 0.796 | 0.865 | 0.906 | 0.922 | 0.935 | 0.943 | 0.948 | 0.9524 |
| 30 | 0.816 | 0.889 | 0.928 | 0.943 | 0.947 | 0.961 | 0.964 | 0.9677 |
| 40 | 0.826 | 0.903 | 0.942 | 0.955 | 0.963 | 0.971 | 0.974 | 0.9756 |
| 50 | 0.834 | 0.914 | 0.951 | 0.962 | 0.97 | 0.975 | 0.978 | 0.9804 |
| 60 | 0.844 | 0.92 | 0.957 | 0.968 | 0.974 | 0.98 | 0.982 | 0.9836 |
| 90 | 0.862 | 0.935 | 0.969 | 0.977 | 0.982 | 0.986 | 0.987 | 0.989 |
| 100 | 0.867 | 0.939 | 0.971 | 0.979 | 0.984 | 0.987 | 0.989 | 0.9901 |
| 500 | 0.891 | 0.974 | 0.993 | 0.995 | 0.996 | 0.997 | 0.998 | 0.998 |

**Note: The last column gives Effcy values for Cm = Infinity (taken as 10^6 here).**

**Now, write the desired Function to find effectiveness for any given NTU_0 and Cm, by 2D-linear interpolation, using the EES built-in function: INTERPLOATE2DM:**

```
FUNCTION EffcyBalancedRegan_NTUmethod(Cm, NTU_0)

   IF ((Cm < 0.8) OR (Cm > 1000000)) THEN

      CALL ERROR ('Cm should be between 0.8 and 10^6!')

   ENDIF
```

```
    IF ((NTU_0 < 0) OR (NTU_0> 500)) THEN

        CALL ERROR ('NTU_0 should be between 0 and 500!')

    ENDIF

EffcyBalancedRegan_NTUmethod = INTERPOLATE2DM('BalancedRegen_Effcy_NTU', Cm, NTU_0)

END

"=================================================================="
```

**Prob.4.3.26** Using the NTU-effectiveness method, determine the effectiveness for a valved type regenerator having a matrix material with a sp. heat of 0.48 kJ/kg.K and a total mass of 2688 kg. The mass flow rate of gas during the heating period is 2 kg/s and the mass flow rate during the cooling period is 1.8 kg/s. The sp. heat of gas for both the periods is 1.006 kJ/kg.K. The total period for the regenerator is 9 min. The convective heat transfer coeff for the heating period is 416 W/m^2.K and that for the cooling period is 388 W/m^2.K. The surface area for the heating and cooling periods is equal to 108 m^2 for each period. [1].

**EES Solution:**

```
"Data:"

ms = 2688[kg]

cs = 0.48 [kJ/kg-K]

mdot_h = 2 [kg/s]

mdot_c = 1.8 [kg/s]

cph = 1.006 [kJ/kg-K]

cpc = 1.006 [kJ/kg-K]

P0 = 540 [s]

hh = 0.416 [kW/m^2-K]

hc = 0.388 [kW/m^2-K]

Ah = 108 [m^2]

Ac = 108 [m^2]

"Calculations:"

"Capacity rates:"

C_c = mdot_c * cpc

C_h = mdot_h * cph

"Since C_c < C_h, we have:"

C_min = C_c

C_max = C_h
```

```
"Therefore: Capacity rate ratio"

C_R = C_min / C_max

"Since C_R = 0.9, i.e. not equal to 1, it is an 'Unbalanced Regenerator."

"Then:"

NTU_0 = (1 / C_min) * (1/(hh * Ah) + 1 / (hc * Ac))^(-1)

Cm = (ms * cs) / (P0 * C_min) ".. matrix capacity ratio"

"Calculate 'effective NTU' and 'effective Cm':"

NTU_e = (2 * NTU_0 * C_R) / (1 + C_R) "....effective NTU"

Cm_e = (2 * Cm * C_R) / (1 + C_R) "... effective Cm"

"For these effective values, find regen. effectiveness from the EES Function written above:"

epsilon_1 = EffcyBalancedRegan_NTUmethod(Cm_e, NTU_e)

"Now, find parameter X:"

X = ((1 + C_R) / (2 * C_R)) * (epsilon_1 / (1 – epsilon_1)) * (1 – C_R)

"Therefore: Regen. effectiveness:"

epsilon = (1 – exp(-X)) / (1 – C_R * exp(-X))

"==========================================================="
```

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| Ac = 108 [$m^2$] | Ah = 108 [$m^2$] | Cm = 1.319 |
| $Cm_e$ = 1.25 | cpc = 1.006 [kJ/kg-K] | cph = 1.006 [kJ/kg-K] |
| cs = 0.48 [kJ/kg-K] | $C_c$ = 1.811 [kW/K] | $C_h$ = 2.012 [kW/K] |
| $C_{max}$ = 2.012 [kW/K] | $C_{min}$ = 1.811 [kW/K] | $C_R$ = 0.9 |
| $\varepsilon$ = 0.8996 | $\varepsilon_1$ = 0.8584 | hc = 0.388 [kW/$m^2$-K] |
| hh = 0.416 [kW/$m^2$-K] | $mdot_c$ = 1.8 [kg/s] | $mdot_h$ = 2 [kg/s] |
| ms = 2,688 [kg] | $NTU_0$ = 11.97 | $NTU_e$ = 11.34 |
| P0 = 540 [s] | X = 0.6398 | |

**Thus:**

**Regen. effectiveness = epsilon = 0.8996 … Ans.**

**Prob.4.3.27** A regenerator matrix is made of stacked wire screens having an equivalent dia of 0.4 mm. The free flow area through the screens is 0.003 m^2. Porosity of matrix is 0.675. The mean temp of air flowing through the regenerator is 200 K, and the gas properties are: μ = 13.3 μ Pa.s, cp = 1.006 kJ/kg.K and Pr = 0.739. The mass flow rate of air through the regenerator = 50 g/s. Determine the convective heat transfer coeff between the air and matrix material. [1].

Also, plot the convective heat transfer coeff as the mass flow rate varies from 30 g/s to 100 g/s

**EES Solution:**

**For stacked wire screens, we have, for Colburn j-factor and friction factor [1]:**

$$n = 0.37 + \frac{0.0136 \cdot e_v}{1 - e_v} \qquad jH = \frac{0.2 \cdot Re^{-n}}{1 - e_v}$$ ...Colburn j-factor

$$f = \frac{860 \cdot (1 - e_v)}{Re} + \frac{2.2 \cdot e_v}{Re^{0.1}}$$ ...friction factor

**"Data:"**

d_e = 0.0004 [m] "..equiv. dia"

A_ff = 0.003 [m^2] "… free flow area"

e_v = 0.675 "…porosity"

T_m = 200 [K]

{mdot = 0.05 [kg/s]}

"Gas props at T_m = 200 K:"

mu = 13.3E-06 [Pa-s]

cp = 1.006 [kJ/kg-K]

Pr = 0.739

"Calculations:"

G = mdot / A_ff "…kg/m^2-s… mass velocity"

Re = G * d_e / mu "… Reynolds No."

"Colburn j factor:"

n = 0.37 + (0.0136 * e_v) / (1 – e_v)

jH = (0.2 * Re^(-n)) / (1 – e_v)

"But, jH = (hc / (cp * G)) / * Pr^(2/3) "

"Therefore: heat transfer coeff, hc:"

hc = (jH * cp * G) / Pr^(2/3)

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| $A_{ff}$ = 0.003 [m^2] | cp = 1.006 [kJ/kg-K] | $d_e$ = 0.0004 [m] |
| $e_v$ = 0.675 | G = 16.67 [kg/m^2-s] | hc = 1.061 [kW/m^2-K] |
| jH = 0.05174 | mdot = 0.05 [kg/s] | μ = 0.0000133 [Pa-s] |
| n = 0.3982 | Pr = 0.739 | Re = 501.3 |
| $T_m$ = 200 [K] | | |

**Thus:**

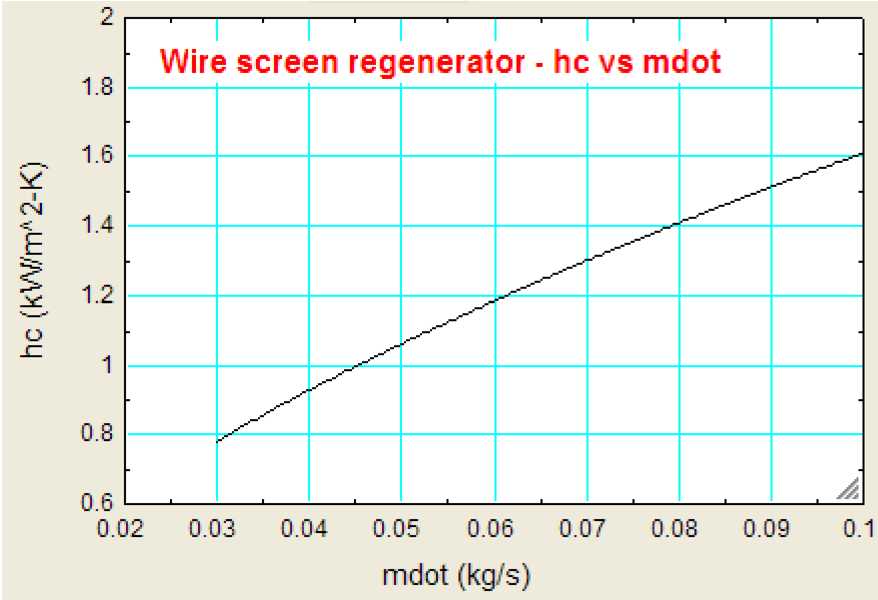
**Convective heat transfer coeff. = hc = 1.061 kW/m^2-K … Ans.**

**Also, plot the convective heat transfer coeff as the mass flow rate varies from 30 g/s to 100 g/s:**

**First, compute the Parametric Table:**

| 1..8 | mdot [kg/s] | jH | hc [kW/m2-K] |
|---|---|---|---|
| Run 1 | 0.03 | 0.06342 | 0.7805 |
| Run 2 | 0.04 | 0.05655 | 0.928 |
| Run 3 | 0.05 | 0.05174 | 1.061 |
| Run 4 | 0.06 | 0.04812 | 1.184 |
| Run 5 | 0.07 | 0.04525 | 1.3 |
| Run 6 | 0.08 | 0.04291 | 1.408 |
| Run 7 | 0.09 | 0.04094 | 1.512 |
| Run 8 | 0.1 | 0.03926 | 1.611 |

**Now, plot:**




**Prob.4.3.28** Nitrogen gas at an average temp of 200 K and an average pressure of 506.6 kPa flows through a regenerator matrix made of 3 mm dia random stacked spheres at a mass flow rate of 20 g/s. Porosity of matrix is 0.4 and the matrix is contained in a tube having an inside dia of 80 mm and a length of 750 mm. The properties of nitrogen gas are: μ = 12.95 μ Pa.s, ρ = 8.56 kg/m^3. Determine the pressure drop of nitrogen flowing through the regenerator. [1].

Also, plot ΔP vs mdot as mdot varies from 20 g/s to 70 g/s.

**For random stacked spheres, we have, for Colburn j-factor and friction factor [1]:**

$$jH = \frac{h_c \cdot Pr^{\frac{2}{3}}}{G \cdot cp} = 0.255 \cdot (e_v \cdot Re)^{\frac{-1}{3}}$$

$$G = \frac{mdot}{A_{ff}} \qquad A_{ff} = \frac{e_v \cdot V_0}{L}$$

$$f = \frac{1 - e_v}{e_v} \cdot \left[\frac{570 \cdot (1 - e_v)^2}{e_v \cdot Re} + 3.5\right]$$

$$A = \frac{6 \cdot (1 - e_v) \cdot V_0}{D_s}$$

$$\Delta P = \left(\frac{f \cdot L}{D_s}\right) \cdot \left(\frac{G^2}{2 \cdot \rho}\right)$$

**EES Solution:**

```
"Data:"

D_s = 0.003 [m] "…dia of spheres"

mdot = 0.02 [kg/s] "…mass flow rate of N2 gas"

e_v = 0.4 " … porosity"

d_i = 0.08 [m] "…inside dia of tube"

L = 0.75 [m] "…length of tube"

"Gas properties:"

mu = 12.95E-06 [Pa-s] "…dyn. viscosity of gas"

rho = 8.56 [kg/m^3] "… density of gas"

"Calculations:"

V_0 = (pi * d_i^2/4) * L "m^3 …. tube volume"

A_ff = e_v * V_0 / L "m^2 .. frree flow area"

G = mdot / A_ff "…kg/m^2-s … mass velocity"

Re = G * D_s / mu " .. Reynolds No."

f = ((1 – e_v) / e_v) * (((570 * (1 – e_v)^2) / (e_v * Re)) + 3.5) "….friction factor"

"Pressure drop:"

DELTAP = (f * L / D_s) * (G^2 /(2 * rho)) "Pa"
```

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| $A_{ff}$ = 0.002011 [m$^2$] | $\Delta P$ = 8,068 [Pa] | $d_i$ = 0.08 [m] |
| $D_s$ = 0.003 [m] | $e_v$ = 0.4 | f = 5.584 |
| G = 9.947 [kg/m$^2$-s] | L = 0.75 [m] | mdot = 0.02 [kg/s] |
| $\mu$ = 0.00001295 [Pa-s] | Re = 2,304 | $\rho$ = 8.56 [kg/m$^3$] |
| $V_0$ = 0.00377 [m$^3$] | | |

**Thus:**

**Reynolds No., Re = 2304 … Ans.**

**Friction factor, f = 5.584 … Ans.**

**Pressure drop, ΔP = 8068 Pa … Ans.**

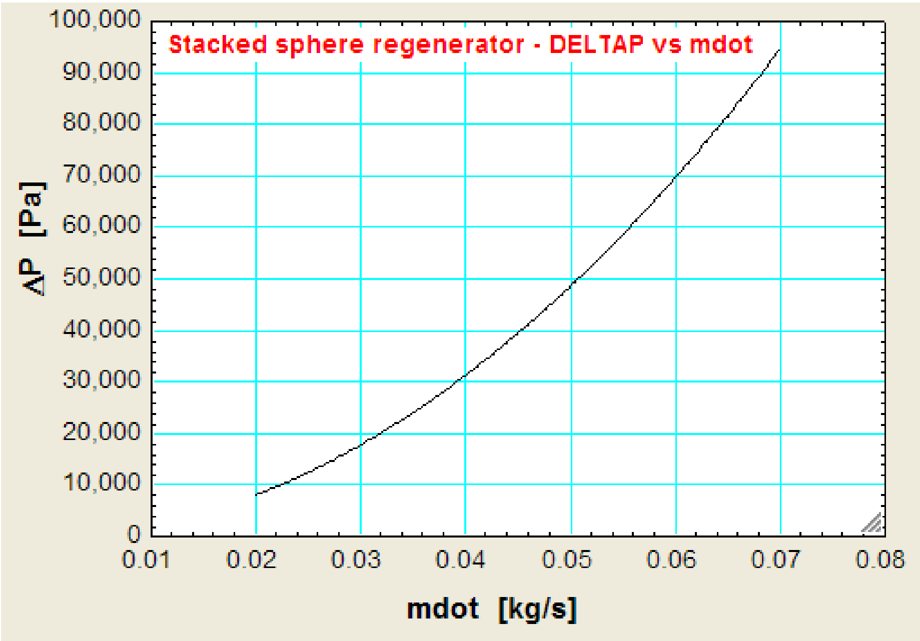
---

**Also, plot ΔP vs mdot as mdot varies from 20 g/s to 70 g/s:**

**First, compute the Parametric Table:**

| 1..6 | mdot [kg/s] | Re | f | ΔP [Pa] |
|---|---|---|---|---|
| Run 1 | 0.02 | 2,304 | 5.584 | 8,068 |
| Run 2 | 0.03 | 3,457 | 5.473 | 17,792 |
| Run 3 | 0.04 | 4,609 | 5.417 | 31,308 |
| Run 4 | 0.05 | 5,761 | 5.384 | 48,617 |
| Run 5 | 0.06 | 6,913 | 5.361 | 69,719 |
| Run 6 | 0.07 | 8,065 | 5.345 | 94,614 |

**Now, plot:**




---

**Prob.4.3.29** Write a EES Procedure for Brillouin function and entropy as a function of Brillouin constant ξ and atomic constant J. Tabulate the results and draw the graphs.

**Solution:**

**Brillouin function and entropy for paramagnetic materials:**

**Brillouin function:**

$$B(\xi, J) := (2 \cdot J + 1) \cdot \coth((2 \cdot J + 1) \cdot \xi) - \coth(\xi)$$

where

$$\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$ ... Brillouin dimensionless parameter

**Entropy:**

$$\frac{s}{R} = \ln(2 \cdot J + 1)$$ ...for H = 0

$$sbyR(\xi, J) := \ln\left[\frac{\sinh((2 \cdot J + 1) \cdot \xi)}{\sinh(\xi)}\right] - \xi \cdot B(\xi, J)$$

**EES Procedure:**

PROCEDURE Brillouin_Function_entropy(Xi, J: B, sbyR)

"Gives Brillouin Function, B and s/R for a paramagnetic material"

"Inputs: Brillouin dimensionless parameter, Xi, Atomic constant, J"

"Outputs: Brillouin function, B, and sbyR where s = entropy, R = gas const."

B:= (2 * J + 1) * (1 / tanh((2*J + 1) * Xi)) – 1/ tanh(Xi)

sbyR = ln(Sinh((2*J + 1) * Xi) / Sinh(Xi)) – Xi * B

END

"================================================================"

**Program:**

Xi = 0.3

J = 1/2

CALL Brillouin_Function_entropy(Xi, J: B, sbyR)

"================================================================"

**Results:**

Unit Settings: SI K kPa kJ mass deg

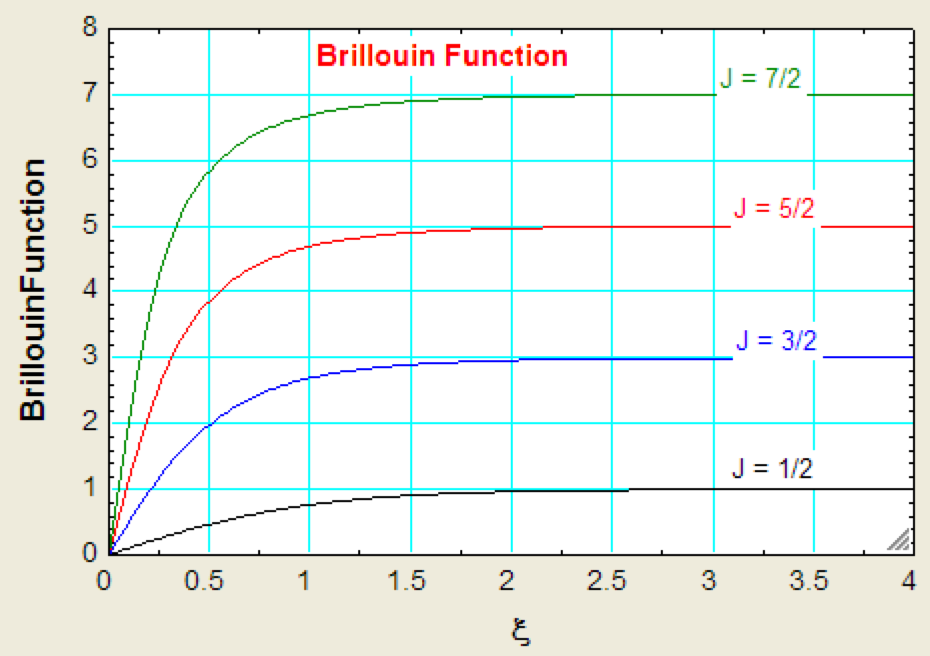
| | | | |
|---|---|---|---|
| B = 0.2913 | J = 0.5 | sbyR = 0.6501 | ξ = 0.3 |

**To plot the graphs:**

**The Parametric Table:**

| | J = 1/2 | J = 1/2 | J = 3/2 | J = 3/2 | J = 5/2 | J = 5/2 | J = 7/2 | J = 7/2 |
|---|---|---|---|---|---|---|---|---|
| ξ | BrillouinFunction | sbyR | BrillouinFunction | sbyR | BrillouinFunctio | sbyR | BrillouinFunction | sbyR |
| 1.000E-06 | -0.00002626 | 0.6931 | -0.000006378 | 1.386 | -0.00001026 | 1.792 | -0.00001013 | 2.079 |
| 0.1 | 0.09967 | 0.6882 | 0.4944 | 1.362 | 1.139 | 1.736 | 2.014 | 1.981 |
| 0.2 | 0.1974 | 0.6735 | 0.9573 | 1.293 | 2.131 | 1.588 | 3.613 | 1.745 |
| 0.3 | 0.2913 | 0.6501 | 1.365 | 1.191 | 2.904 | 1.397 | 4.7 | 1.478 |
| 0.4 | 0.3799 | 0.6191 | 1.708 | 1.072 | 3.468 | 1.201 | 5.395 | 1.237 |
| 0.5 | 0.4621 | 0.5822 | 1.985 | 0.9475 | 3.866 | 1.023 | 5.841 | 1.038 |
| 0.6 | 0.537 | 0.5411 | 2.204 | 0.8275 | 4.147 | 0.8695 | 6.139 | 0.8749 |
| 0.7 | 0.6044 | 0.4974 | 2.375 | 0.7169 | 4.348 | 0.7393 | 6.346 | 0.7412 |
| 0.8 | 0.664 | 0.4527 | 2.507 | 0.618 | 4.495 | 0.6296 | 6.494 | 0.6302 |
| 0.9 | 0.7163 | 0.4083 | 2.61 | 0.531 | 4.604 | 0.5369 | 6.604 | 0.5371 |
| 1 | 0.7616 | 0.3653 | 2.69 | 0.4554 | 4.687 | 0.4584 | 6.687 | 0.4584 |
| 1.1 | 0.8005 | 0.3245 | 2.752 | 0.3901 | 4.751 | 0.3916 | 6.751 | 0.3916 |
| 1.2 | 0.8337 | 0.2865 | 2.801 | 0.3338 | 4.8 | 0.3345 | 6.8 | 0.3345 |
| 1.3 | 0.8617 | 0.2514 | 2.84 | 0.2854 | 4.84 | 0.2858 | 6.84 | 0.2858 |
| 1.4 | 0.8854 | 0.2195 | 2.871 | 0.2439 | 4.871 | 0.244 | 6.871 | 0.244 |
| 1.5 | 0.9051 | 0.1909 | 2.895 | 0.2082 | 4.895 | 0.2083 | 6.895 | 0.2083 |
| 1.6 | 0.9217 | 0.1653 | 2.915 | 0.1776 | 4.915 | 0.1776 | 6.915 | 0.1776 |
| 1.7 | 0.9354 | 0.1426 | 2.931 | 0.1513 | 4.931 | 0.1513 | 6.931 | 0.1513 |
| 1.8 | 0.9468 | 0.1227 | 2.944 | 0.1288 | 4.944 | 0.1288 | 6.944 | 0.1288 |
| 1.9 | 0.9562 | 0.1053 | 2.954 | 0.1096 | 4.954 | 0.1096 | 6.954 | 0.1096 |
| 2 | 0.964 | 0.09009 | 2.963 | 0.09311 | 4.963 | 0.09311 | 6.963 | 0.09311 |
| 2.5 | 0.9866 | 0.04018 | 2.986 | 0.04068 | 4.986 | 0.04068 | 6.986 | 0.04068 |
| 3 | 0.9951 | 0.01731 | 2.995 | 0.01739 | 4.995 | 0.01739 | 6.995 | 0.01739 |
| 3.5 | 0.9982 | 0.007289 | 2.998 | 0.007301 | 4.998 | 0.007301 | 6.998 | 0.007301 |
| 4 | 0.9993 | 0.003018 | 2.999 | 0.00302 | 4.999 | 0.00302 | 6.999 | 0.00302 |

**Now, plot the graphs:**

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**Prob.4.3.30** Write a EES Procedure to get properties of given paramagnetic material:

PROCEDURE Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

"Gives properties of a paramagnetic material"

"Inputs: Material:
1 = Cerium ethyl sulfate
2 = Cerium magnesium nitrate
3 = Chromium methylammonium alum
4 = Chromium potassium alum
5 = Copper potassium sulfate
6 = Gadolinium sulfate
7 = Iron ammonium alum
8 = Manganese ammonium sulfate
9 = Titanium cesium alum "

"Outputs:
Materal Name,
Lande splitting factor, g,
Ionic const. J,

```
Ionic wt., M (kg/mol),
Gas const. R, (J/kg.K) and
Curie const., C (K-m3/kg)"

"Write the properties in a (9 × 6) Matrix: "

"Material No., g J M R C"

Props[1,1..6] = [1, 1.7,1.00, 0.678, 12.263, 2.51E-05]

Props[2,1..6] = [2, 1.84, 0.50, 0.765, 10.868, 5.22E-06]

Props[3,1..6] = [3, 2, 1.50, 0.492, 16.899, 3.05E-05]

Props[4,1..6] = [4, 1.97, 1.50, 0.499, 16.662, 4.58E-05]

Props[5,1..6] = [5, 2.14, 0.50, 0.442, 18.811, 1.221E-05]

Props[6,1..6] = [6, 1.992, 3.50, 0.373, 22.29, 2.633E-04]

Props[7,1..6] = [7, 2, 2.50, 0.482, 17.25, 1.141E-04]

Props[8,1..6] = [8, 2.06, 2.50, 0.391, 21.264, 1.492E-04]

Props[9,1..6] = [9, 1.89, 0.50, 0.589, 14.116, 4.21E-06]

IF (Material = 1) THEN MaterialName$ = 'Cerium_ethyl_sulfate'

IF (Material = 2) THEN MaterialName$ = 'Cerium magnesium nitrate'

IF (Material = 3) THEN MaterialName$ = 'Chromium methylammonium alum'

IF (Material = 4) THEN MaterialName$ = 'Chromium potassium alum'

IF (Material = 5) THEN MaterialName$ = 'Copper potassium sulfate'

IF (Material = 6) THEN MaterialName$ = 'Gadolinum sulfate'
```

```
IF (Material = 7) THEN MaterialName$ = 'Iron ammonium alum'

IF (Material = 8) THEN MaterialName$ = 'Manganese ammonium sulfate'

IF (Material = 9) THEN MaterialName$ = 'Titanium cesium alum'

    Lande_g := Props[Material,2]

    J := Props[Material,3]

    M := Props[Material,4]

    R := Props[Material,5]

    C := Props[Material,6]

END

"=============================="
```

**"Example:"**

Material = 3

CALL Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

C = 3.050E-05 [K-m$^3$/kg]
$Lande_g$ = 2
Material = 3
R = 16.899 [J/kg-K]
J = 1.5
M = 0.492 [kg/mol]
MaterialName$ = 'Chromium methylammonium alum'

**Prob.4.3.31** Determine the magnetic moment per unit mass and entropy for gadolinium sulfate at 0.1 K in a magnetic field of 60 kA/m (75.4 mT) assuming that Brillouin expression is valid at this condition.[1]

**Solution:**

We have, for Magnetic moment:

$I = (1/2).n.g.\ \mu_B.B(\xi)$

**Props of Gadolinium sulfate:**

Using the EES Procedure written above:

Material = 6 … for Gadolinium sulfate

We have:

Material = 6

CALL Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

We get:

**Unit Settings: SI K kPa kJ mass deg**

C = 2.6330E-04 [K-m$^3$/kg]
$Lande_g$ = 1.992
Material = 6
R = 22.290 [J/kg-K]
J = 3.5
M = 0.373 [kg/mol]
MaterialName$ = 'Gadolinum sulfate'

"Therefore, we have:"

M = 0.373 [kg/mol] "…Ionic weight,"

Lande_g = 1.992 "….Lande splitting factor"

J = 7/2

"Also:"

H = 60 * 10^3 [A/m]

N_0 = 6.023 * 10^23 "per mol … Avogadro's No."

mu_B = 0.9273 * 10^(-23) "[A-m^2] … Bohr magneton"

mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] …permeability of free space"

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

n = N_0 / M "…no. of ions per unit mass"

T = 0.1 [K] "Temp (K)"

"Now, we have, Brillouin parameter:"

Xi = Lande_g * mu_B * mu_0 * H / (2 * k * T)

"Therefore: Brillouin function:"

CALL Brillouin_Function_entropy(Xi, J: B, sbyR)

We get:

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 5.857 | H = 60,000 [A/m] | J = 3.5 |
| k = 1.380E-23 [J/K] | $Lande_g$ = 1.992 | M = 0.373 [kg/mol] |
| $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] | n = 1.615E+24 [per kg] |
| $N_0$ = 6.023E+23 | sbyR = 1.03 | T = 0.1 [K] |
| $\xi$ = 0.5044 | | |

"i.e. We get:

Brillouin_Function, B = 5.8572"

"And, Magnetic moment, I:"

B = 5.8572

I = (1/2) * n * Lande_g * mu_B * B

We get:

**Unit Settings: SI K kPa kJ mass deg**

| | |
|---|---|
| B = 5.857 | H = 60,000 [A/m] |
| I = 87.3522 [A-m$^2$/kg] | J = 3.5 |
| k = 1.380E-23 [J/K] | $Lande_g$ = 1.992 |
| M = 0.373 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m$^2$] |
| $\mu_B$ = 9.273E-24 [A-m$^2$] | n = 1.615E+24 [per-kg] |
| $N_0$ = 6.023E+23 | T = 0.1 [K] |
| $\xi$ = 0.5044 | |

i.e.

**Magnetic moment, I = 87.3522 A-m^2/kg … Ans.**

---

**"To find entropy:"**

"We have:"

R = 22.29 [J/kg-K]

sbyR = 1.03

"Therefore, entropy:"

s = R * sbyR

We get:

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 5.857 | H = 60,000 [A/m] | I = 87.3522 [A-m^2/kg] |
| J = 3.5 | k = 1.380E-23 [J/K] | $Lande_g$ = 1.992 |
| M = 0.373 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m^2] | $\mu_B$ = 9.273E-24 [A-m^2] |
| n = 1.615E+24 [per kg] | $N_0$ = 6.023E+23 | R = 22.290 [J/kg-K] |
| s = 22.96 [J/kg-K] | sbyR = 1.03 | T = 0.1 [K] |
| ξ = 0.5044 | | |

**i.e. Entropy = s = 22.96 J/kg-K … Ans.**

---

**Prob.4.3.32** Determine the magnetic moment per unit mass and entropy for cerium magnesium nitrate at 0.1 K in a magnetic field of 189 kA/m (0.2375 T) assuming that Brillouin expression is valid at this condition. What would be the value of magnetic moment per unit mass according to the Curie law?[1]

**Solution:**

We have:

$$\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$ ... Brillouin dimensionless parameter

**And for CMN, we have:**

Using the EES Procedure written above:

Material = 2 … for Cerium magnesium nitrate

We get:

Material = 2

CALL Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

**Unit Settings: SI K kPa kJ mass deg**

| | |
|---|---|
| C = 5.2200E-06 [K-m$^3$/kg] | J = 0.5 |
| $Lande_g$ = 1.84 | M = 0.765 [kg/mol] |
| Material = 2 | MaterialName$ = 'Cerium magnesium nitrate' |
| R = 10.868 [J/kg-K] | |

"Therefore, we have:"

M = 0.765 [kg/mol] "…Ionic weight,"

Lande_g = 1.84 "….Lande splitting factor"

J = 1/2

"Also:"

H = 189 * 10^3 [A/m]

N_0 = 6.023 * 10^23 "per mol …. Avogadro's No."

mu_B = 0.9273 * 10^(-23) "[A-m^2] .. Bohr magneton"

mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] …permeability of free space"

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

n = N_0 / M "…no. of ions per unit mass"

T = 0.1 [K] "Temp (K)"

"Now, we have, Brillouin parameter:"

Xi = Lande_g * mu_B * mu_0 * H / (2 * k * T)

"Therefore: Brillouin function:"

CALL Brillouin_Function_entropy(Xi, J: B, sbyR)

**We get:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 0.8991 | H = 189,000 [A/m] | J = 0.5 |
| k = 1.380E-23 [J/K] | $Lande_g$ = 1.84 | M = 0.765 [kg/mol] |
| $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] | n = 7.873E+23 [per·kg] |
| $N_0$ = 6.023E+23 | sbyR = 0.1998 | T = 0.1 [K] |
| $\xi$ = 1.468 | | |

"i.e. We get:

Brillouin_Function, B = 0.8991"

"And, Magnetic moment, I:"

B = 0.8991

I = (1/2) * n * Lande_g * mu_B * B

"To find entropy:"

"We have:"

R = 10.868 [J/kg-K]

sbyR = 0.1998

"Therefore, entropy:"

s = R * sbyR

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | |
|---|---|
| B = 0.8991 | H = 189,000 [A/m] |
| I = 6.0390 [A-m^2/kg] | J = 0.5 |
| k = 1.380E-23 [J/K] | $Lande_g$ = 1.84 |
| M = 0.765 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m^2] |
| $\mu_B$ = 9.273E-24 [A-m^2] | n = 7.873E+23 [per kg] |
| $N_0$ = 6.023E+23 | R = 10.868 [J/kg-K] |
| s = 2.171 [J/kg-K] | sbyR = 0.1998 |
| T = 0.1 [K] | $\xi$ = 1.468 |

**Thus:**

**Magnetic moment per unit mass = I = 6.039 A-m^2/kg … Ans.**

**Entropy = s = 2.171 J/kg-K … Ans.**

---

"If the Curie Law is valid:"

"We have, Curie const. for CMN:"

C = 5.22 * 10^(-6) [k-m^3/kg]

"Then, Magnetic momet, I:"

I_curie = C * H / T

**We get:**

**Unit Settings: SI K kPa kJ mass deg**

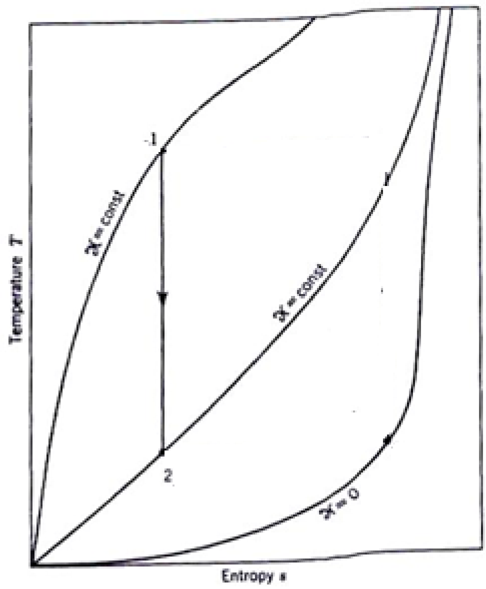
| | | |
|---|---|---|
| B = 0.8991 | C = 5.2200E-06 [K-m^3/kg] | H = 189,000 [A/m] |
| I = 6.0390 [A-m^2/kg] | $I_{curie}$ = 9.866 [A-m^2/kg] | J = 0.5 |
| k = 1.380E-23 [J/K] | $Lande_g$ = 1.84 | M = 0.765 [kg/mol] |
| $\mu_0$ = 0.000001257 [J/A-m^2] | $\mu_B$ = 9.273E-24 [A-m^2] | n = 7.873E+23 [per kg] |
| $N_0$ = 6.023E+23 | R = 10.868 [J/kg-K] | s = 2.171 [J/kg-K] |
| sbyR = 0.1998 | T = 0.1 [K] | $\xi$ = 1.468 |

**i.e. Magnetic moment, when Curie Law is valid:**

**I_curie = 9.866 A-m^2/kg … Ans.**

---

**Prob.4.3.33** Magnetic field intensity around a cerium magnesium nitrate (CMN) pellet is decreased reversibly and adiabatically from 320 kA/m (0.402 T) to 32 kA/m (40.2 mT). If the initial temp is 2 K, determine the final temp. [1]




**EES Solution:**

"We have, for CMN:"

M = 0.765 [kg/mol] "…Ionic weight,"

Lande_g = 1.84 "….Lande splitting factor"

J = 1/2

"Also:"

H1 = 320 * 10^3 [A/m]

H2 = 32 * 10^3[A/m]

N_0 = 6.023 * 10^23 "per mol … Avogadro's No."

mu_B = 0.9273 * 10^(-23) "[A-m^2] … Bohr magneton"

mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] … permeability of free space"

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

n = N_0 / M "…no. of ions per unit mass"

T1 = 2 [K] "Temp (K)"

"Now, we have, Brillouin parameter:"

Xi = Lande_g * mu_B * mu_0 * H1 / (2 * k * T1)

"Now, we have, Brillouin parameter, and entropy:"

Xi = Lande_g * mu_B * mu_0 * H1 / (2 * k * T1)

"Therefore: Brillouin function:"

CALL Brillouin_Function_entropy(Xi, J: B, sbyR)

**We get:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 0.1236 | H1 = 320,000 [kJ/kg] | H2 = 32,000 [kJ/kg] |
| J = 0.5 | k = 1.380E-23 [J/K] | $Lande_g$ = 1.84 |
| M = 0.765 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] |
| n = 7.873E+23 [per-kg] | $N_0$ = 6.023E+23 | sbyR = 0.6855 |
| T1 = 2 [K] | $\xi$ = 0.1243 | |

"i.e. We have:

Brillouin_Function, B = 0.1236"

"And, Entropy:"

"We have:"

R = 10.868 [J/kg-K]

sbyR = 0.6855

"Therefore, entropy before demagnetization:"

s1 = R * sbyR

**"Let T2 be the temp at the end of adiabatic demagnetization. And, s2 = s1."**

Xi_2 = Lande_g * mu_B * mu_0 * H2 / (2 * k * T2) "..Brillouin dimensionless parameter for the case after demagnetization:"

"Then, we get, using the EES Procedure for Brillouin Function and Entropy:"

CALL Brillouin_Function_entropy(Xi_2, J: B, sbyR)

**We get:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 0.1235 | H1 = 320,000 [kJ/kg] | H2 = 32,000 [A/m] |
| J = 0.5 | k = 1.380E-23 [J/K] | $Lande_g$ = 1.84 |
| M = 0.765 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] |
| n = 7.873E+23 [per kg] | $N_0$ = 6.023E+23 | R = 10.868 [J/kg-K] |
| s1 = 7.45 [J/kg-K] | sbyR = 0.6855 | T1 = 2 [K] |
| T2 = 0.2002 [K] | $\xi$ = 0.1243 | $\xi_2$ = 0.1241 |

**Thus:**

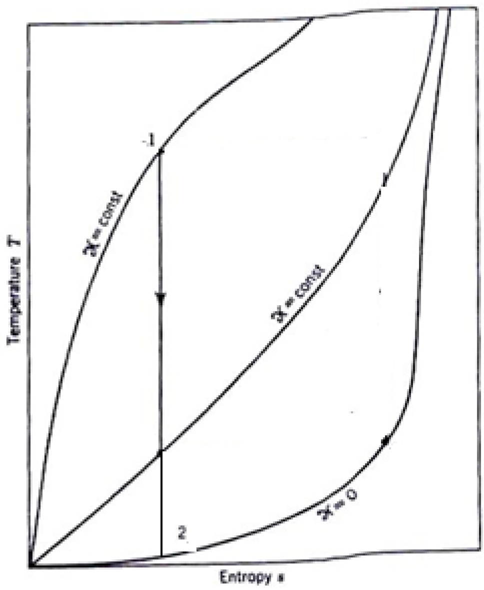
**Temp. before demagnetization, T1 = 2 K**

**Temp. after demagnetization, T2 = 0.02002 K … Ans.**

**Entropy = s1 = s2 = 7.45 J/kg.K … Ans.**

---

**Prob.4.3.34** Magnetic field intensity around an iron ammonium alum pellet is decreased reversibly and adiabatically from 500 kA/m (0.628 T) to zero (external field). If the "internal field" for iron ammonium alum is 35 kA/m (44 mT), and the initial temp of the pellet is 1.5 K, determine the final temp. of the pellet. [1]




**EES Solution:**

"Props of Iron ammonium alum:

Material = 7 … for Iron ammonium alum

Using the EES Procedure written above, we have:"

Material = 7

CALL Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

We get:

**Unit Settings: SI K kPa kJ mass deg**

| | |
|---|---|
| C = 1.1410E-04 [K-m^3/kg] | J = 2.5 |
| Lande_g = 2 | M = 0.482 [kg/mol] |
| Material = 7 | MaterialName$ = 'Iron ammonium alum' |
| R = 17.250 [J/kg-K] | |

"i.e. we get:"

J = 5/2

Lande_g = 2

R = 17.25 [J/kg-K]

M = 0.482 [kg/mol]

T1 = 1.5 [K]

H1 = 500 * 10^3 [A/m]

H2 = 0 [A/m]

H_int = 35 * 10^3 [A/m] "…Internal magnetic field"

"Also:"

N_0 = 6.023 * 10^23 " per mol … Avogadro's No."

mu_B = 0.9273 * 10^(-23) "[A-m^2] … Bohr magneton"

mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] …permeability of free space "

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

n = N_0 / M "…no. of ions per unit mass"

"Clculations:"

"Effective magn. fields:"

H1_eff = (H1^2 + H_int^2)^(1/2) "A/m"

H2_eff = (H2^2 + H_int^2)^(1/2) "A/m"

"Then:"

Xi_1 = Lande_g * mu_B * mu_0 * H1_eff / (2 * k * T1)

"Then, find sbyR:"

CALL Brillouin_Function_entropy(Xi_1, J: B, sbyR)

"Then, Entropy:"

s1 = R * sbyR

We get:

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 2.782 | H1 = 500,000 [kJ/kg] | $H1_{eff}$ = 501,224 |
| H2 = 0 [A/m] | $H2_{eff}$ = 35,000 | $H_{int}$ = 35,000 |
| J = 2.5 | k = 1.380E-23 [J/K] | $Lande_g$ = 2 |
| M = 0.482 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] |
| n = 1.250E+24 [per kg] | $N_0$ = 6.023E+23 | R = 17.250 [J/kg-K] |
| s1 = 24.71 [J/kg-K] | sbyR = 1.432 | T1 = 1.5 [K] |
| $\xi_1$ = 0.2821 | | |

Note that initial entropy, before demagnetization, s1 = 24.71 J/kg-K.

**After demagentizaton:**

"Let T2 be the temp at the end of adiabatic demagnetization. And, s2 = s1. i.e. sbyR remains the same."

"We get:"

sbyR = 1.432

CALL Brillouin_Function_entropy(Xi_2, J: B, sbyR) "… finds Xi_2"

Xi_2 = Lande_g * mu_B * mu_0 * H2_eff / (2 * k * T2) "… finds T2"

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

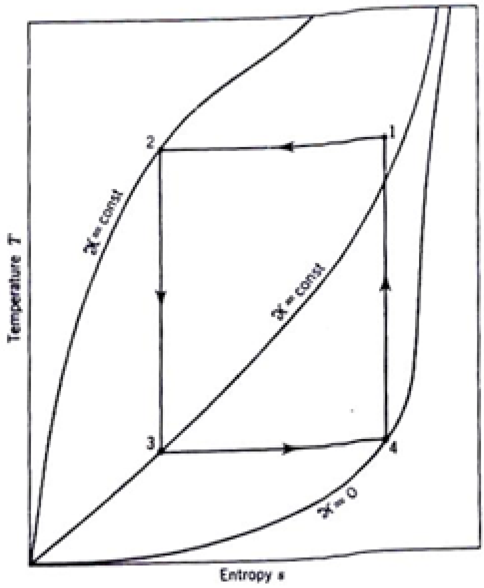
| | |
|---|---|
| B = 2.784 | H1 = 500,000 [kJ/kg] |
| $H1_{eff}$ = 501,224 | H2 = 0 [A/m] |
| $H2_{eff}$ = 35,000 | $H_{int}$ = 35,000 |
| J = 2.5 | k = 1.380E-23 [J/K] |
| $Lande_g$ = 2 | M = 0.482 [kg/mol] |
| $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] |
| n = 1.250E+24 [per kg] | $N_0$ = 6.023E+23 |
| R = 17.250 [J/kg-K] | sbyR = 1.432 |
| T1 = 1.5 [K] | T2 = 0.1047 [K] |
| $\xi_2$ = 0.2823 | |

**Thus:**

**Final temp after demagnetization = T2 = 0.1047 K … Ans.**

---

**Prob.4.3.35** A magnetic refrigerator operates between 0 kA/m and 231.3 kA/m (0.291 T) during the reversible isothermal heat addition process. Heat is added to the salt from the low temp region at 0.22 K and heat is rejected to a helium bath at 2.5 K. The mass of the working salt is 25 g. and the refrigerator operates at 30 cycles per hour. Determine the heat transfer rate from the low temp source to the refrigerator, assuming the Brillouin expression is valid. The working salt is Chromium potassium alum. [1]




**EES Solution:**

"Props of Chromium potassium alum:"

"Use the EES Procedure written earlier:"

Material = 4

CALL Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

**We get:**

**Unit Settings: SI K kPa kJ mass deg**

C = 4.5800E-05 [K-m^3/kg]

J = 1.5

$Lande_g$ = 1.97

M = 0.499 [kg/mol]

Material = 4

MaterialName$ = 'Chromium potassium alum'

R = 16.662 [J/kg-K]

"Further:"

m = 0.025 [kg] ".. mass of pellet"

N_0 = 6.023 * 10^23 "per mol …. Avogadro's No."

mu_B = 0.9273 * 10^(-23) "[A-m^2] … Bohr magneton"

mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] …permeability of free space"

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

```
n = N_0 / M "…no. of ions per unit mass"

T4 = 0.22[K]

H2 = 231.3 * 10^3 [A/m]

H4 = 0 [A/m]

T2 = 2.5 [K]

Lande_g = 1.97

J = 3/2

R = 16.662 [J/kg-K]

"Therefore:"

Xi_2 = Lande_g * mu_B * mu_0 * H2 / (2 * k * T2)

"And, get s2byR using the EES Procedure for Brillouin Function:"

CALL Brillouin_Function_entropy(Xi_2, J: B, s2byR)

"Therefore:"

s2 = R * s2byR "J/kg-K"

"Now, s3 = s2 for isentropic process 2-3:"

"i.e."

s3 = s2 "….for isentropic process 2-3"

"At zero magnetic field, we have entropy:"

s4 = R * ln(2*J + 1)

"Therefore, heat transfer to refrigerator, Qpercycle:"
```

Qpercycle = m * T4 * (s4 – s3) "J / cycle"

"And, heat transfer rate, Q for 30 cycles/h:"

Q = Qpercycle * (30 / 3600) "W"

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

B = 0.3821
H4 = 0 [A/m]
k = 1.380E-23 [J/K]
m = 0.025 [kg]
$\mu_B$ = 9.273E-24 [A-m$^2$]
$N_0$ = 6.023E+23
Qpercycle = 0.001342 [J/cycle]
s2 = 22.85 [kJ/kg-K]
s3 = 22.85 [kJ/kg-K]
T2 = 2.5 [K]
$\xi_2$ = 0.07692

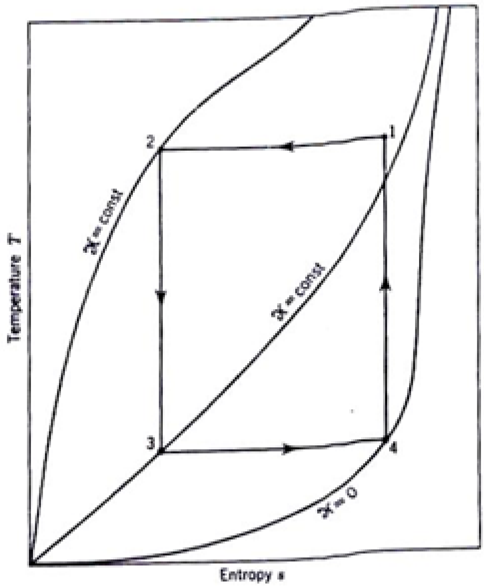
H2 = 231,300 [A/m]
J = 1.5
$Lande_g$ = 1.97
$\mu_0$ = 0.000001257 [J/A-m$^2$]
n = 2.409E+25 [per kg]
Q = 0.00001118 [W]
R = 16.662 [J/kg-K]
s2byR = 1.372
s4 = 23.1 [kJ/kg-K]
T4 = 0.22 [K]

**Thus:**

**Heat transfer rate from low temp. source = Q = 0.00001118 W …. Ans.**

---

"**Prob.4.3.36** A magnetic refrigerator uses chromium methyl ammonium alum as the working salt. Heat addition per unit mass from low temp source at 0.08 K to the working salt is 1.258 J/kg. The magnetic field intensity at the end of isothermal heat addition process is 0 A/m. Determine the magnetic field intensity at the beginning of the isothermal heat addition process. [1]"




**"Solution:"**

"Props of Chromium methyl ammonium alum:"

"Use the EES Procedure written earlier:"

Material = 3

CALL Props_Paramag(Material: MaterialName$, Lande_g, J, M, R, C)

We get:

**Unit Settings: SI K kPa kJ mass deg**

$C = 3.0500E{-}05$ [K-m$^3$/kg]

$Lande_g = 2$

Material = 3

R = 16.899 [J/kg-K]

J = 1.5

M = 0.492 [kg/mol]

MaterialName$ = 'Chromium methylammonium alum'

"Further:"

M = 0.492 [kg/mol]

N_0 = 6.023 * 10^23 "per mol …. Avogadro's No."

mu_B = 0.9273 * 10^(-23) "[A-m^2] … Bohr magneton"

mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] …permeability of free space"

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

n = N_0 / M "…no. of ions per unit mass"

T4 = 0.08[K]

H4 = 0 [A/m]

T3 = T4

Lande_g = 2

J = 3/2

R = 16.899 [J/kg-K]

Qpercycle = 1.258 [J/kg]

"Now, at zero magn. field, we have for entropy, s4:"

s4 = R * ln(2*J + 1) "..J/kg-K…. finds s4"

"And, Qpercycle:"

Qpercycle = T4 * (s4 – s3) "J / kg …. finds s3"

"To find H3, the mahnetic field at point 3:"

Xi_3 = Lande_g * mu_B * mu_0 * H3 / (2 * k * T3) " ….finds H3"

"And, get s3byR using the EES Procedure for Brillouin Function:"

CALL Brillouin_Function_entropy(Xi_3, J: B, s3byR)

"And:"

s3 = R * s3byR "J/kg-K"

"=============================================="

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

| | | |
|---|---|---|
| B = 2.689 | H3 = 94,730 [A/m] | H4 = 0 [A/m] |
| J = 1.5 | k = 1.380E-23 [J/K] | $Lande_g$ = 2 |
| M = 0.492 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m$^2$] | $\mu_B$ = 9.273E-24 [A-m$^2$] |
| n = 1.224E+24 [per-kg] | $N_0$ = 6.023E+23 | Qpercycle = 1.258 [J/kg] |
| R = 16.899 [J/kg-K] | s3 = 7.702 [kJ/kg-K] | s3byR = 0.4558 |
| s4 = 23.43 [kJ/kg-K] | T3 = 0.08 [K] | T4 = 0.08 [K] |
| $\xi_3$ = 0.9995 | | |

**Thus:**

**Magnetic field intensity at the beginning of the isoth. heat addition process:**

**H3 = 94730 A/m … Ans.**

**Prob.4.3.37** Nuclear paramagnetic materials obey the Brillouin expression, with the exception that the Bohr magneton must be replaced by the nuclear magneton ($\mu_N$) = 5.0506 × $10^{-27}$ A-m^2 and the nuclear quantum number I replaces the atomic constant, J. Determine the magnetic moment per unit mass and entropy for copper as a nuclear paramagnetic system with g = 2, I = 1/2, M = 63.54 g/mol, and R = 130.8 J/kg.K. The copper temp is 0.004 K and the applied magnetic field is 4.35 MA/m (= 5.47 T).[1]

**"EES Solution:"**

"We have:"

M = 0.06354 [kg/mol]"…for copper"

I = 1/2 "…Nuclear Quantum no. for copper"

N_0 = 6.023 * 10^23 " per mol …. Avogadro's No."

mu_N = 5.0506 * 10^(-27) "[A-m^2] .. Nuclear magneton"

```
mu_0 = 4 * pi * 10^(-7) "[J/A-m^2] …permeability of free space"

H = 4.35 * 10^6 [A/m]

k = 1.3805 * 10^(-23) [J/K] "…Boltzmann constant"

n = N_0 / M "…no. of ions per unit mass"

Lande_g = 2 "…for copper"

R = 130.8 [J/kg-K] "…for copper"

T = 0.004 [K] "…by data"

"Calculations:"

"Then, we get:"

Xi = Lande_g * mu_N * mu_0 * H / (2 * k * T)

"And, get Brillouin Function, B and sbyR using the EES Procedure:"

CALL Brillouin_Function_entropy(Xi, I: B, sbyR)

"Magnetic moment for copper:"

I_cu = (1/2) * n * Lande_g * mu_N * B

"And, Entropy, s:"

s = R * sbyR "J/kg-K"
```

**Results:**

Unit Settings: SI K kPa kJ mass deg

| | |
|---|---|
| B = 0.4621 | H = 4.350E+06 [A/m] |
| I = 0.5000 | $I_{cu}$ = 0.02212 [A-m²/kg] |
| k = 1.380E-23 [J/K] | $Lande_g$ = 2 |
| M = 0.06354 [kg/mol] | $\mu_0$ = 0.000001257 [J/A-m²] |
| $\mu_N$ = 5.051E-27 [A-m²] | n = 9.479E+24 [per kg] |
| $N_0$ = 6.023E+23 | R = 130.800 [J/kg-K] |
| s = 76.15 [J/kg-K] | sbyR = 0.5822 |
| T = 0.004 [K] | $\xi$ = 0.5 |

**Thus:**

**Magnetic moment per unit mass = I_cu = 0.02212 A-m^2/kg … Ans.**

**Entropy = s = 76.15 J/kg-K … Ans.**

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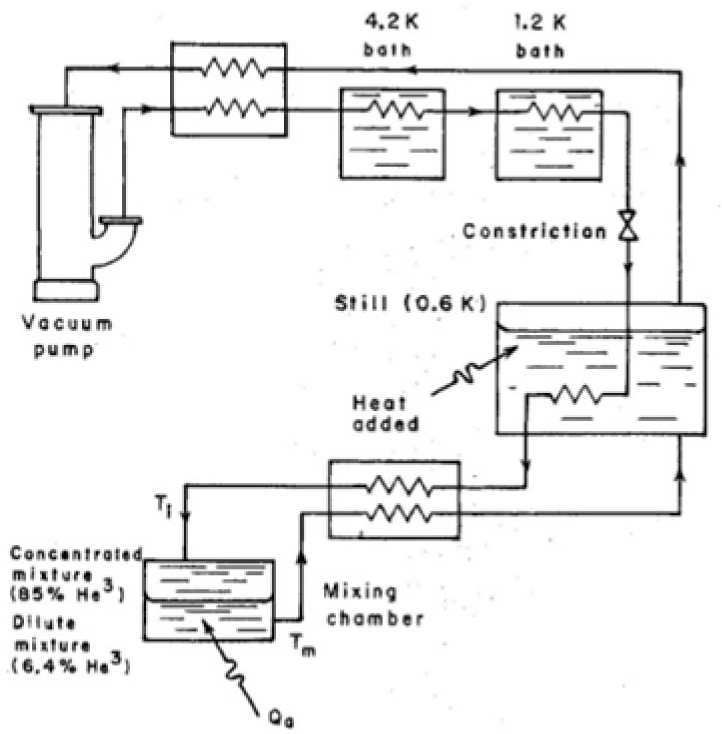
**Prob.4.3.38** A He3-He4 dilution refrigerator operates with a flow rate of $He^3$ of 1.53 × $10^{-4}$ mol/s. The liquid $He^3$ enters the mixing chamber at 0.04 K and the dilute phase leaves the mixing chamber at 0.03 K. Determine the heat transfer rate to the mixing chamber from the low temp region.[1]



**EES Solution:**

n_3 = 1.53 * 10^(-4) [mol/s]

T_i = 0.04 [K] "… temp of He3 mixture entering the mixing chamber"

T_m = 0.03 [K] "… temp of He3 mixture leaving the mixing chamber"

C_1 = 94 [J/mol-K^2]

C_2 = 12 [J/mol-K^2]

"Then, we have:"

h_m = C_1 * T_m^2 "J/mol"

h_i = C_2 * T_i^2 "J/mol"

"Therefore, heat transfer rate to mixing chamber from low temp region:"

Q_a = n_3 * (h_m – h_i) "W"

**Results:**

Unit Settings: SI K kPa kJ mass deg

| | | |
|---|---|---|
| $C_1 = 94$ [J/mol-K²] | $C_2 = 12$ [J/mol-K²] | $h_i = 0.0192$ [J/mol] |
| $h_m = 0.0846$ [J/mol] | $n_3 = 0.000153$ [mol/s] | $Q_a = 0.00001001$ [W] |
| $T_i = 0.04$ [K] | $T_m = 0.03$ [K] | |

**Thus:**

**Heat transfer rate to mixing chamber from low temp region :**

**Q_a = 0.00001001 W = 10.01 μW … Ans.**

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